MILK SECRETION

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THE STUDY OF

The Physiology and Inheritance of Milk Yield and Butter-fat Percentage in Dairy Cattle

BY

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PREFACE

A cogent truth is expressed in the trite clause, the dairy cow is the foster mother of the human race. But like most such easily remembered remarks it is but a half truth for the dairy cow not only takes up and extends the work of the human mother but furnishes absolute necessities for mankind throughout the entire life span. Milk and the dairy cow are at once problems of all the sciences, medicine, economics, chemistry, physics, and biology. The wonderful food properties of milk for bacteria as well as man early forced home to us the importance of clean milk in the sense of containing no bacteria injurious to health. Its unexcelled food properties have unconsciously made a public cry for economical milk production. The fundamental sciences have been called in to solve these problems.

But there is more to the milk problem than quality production. The recent vitamine studies have brought out in relief a fact already realized by those familiar with the problem, namely the absolute necessity of quantity. The researches of Sherman, Osbourne, Mendel, McCollum, and Bloch have shown that the child up to at least the age of fourteen years must have quantity as well as quality milk in its diet to continue its proper development and to prevent gross pathological changes. The quantity production of quality milk is a universal public health problem touching us all.

If adequately analyzed it is found that two major problems are behind the quantity production of milk and butter-fat. It is now well recognized that a cow must be bred for milk production or her yield will be small no matter what she is fed. The student of breeding equally realizes that a cow must be fed correctly or low production will result. The two fundamental problems of the dairy cattle industry are, consequently, an adequate knowledge of the laws of inheritance and of nutrition.

One regrets to admit any tardiness in his own profession, yet in justice to the nutrition workers who have studied problems of milk yield, it must be said that the progress in our knowledge of breeding seems small indeed. Figuratively speaking if we stand at the inter-

section of these two crossroads of dairy science, breeding and nutrition, and look back over them they have a markedly different appearance. The nutrition path is well beaten as if many feet had trodden it. It is strewn with achievement. The other path is merely a trail, crooked at that, looking more like a path made by the blind leading the blind. There is, however, some excuse for this lack of accomplishment. The problem is difficult in the extreme, it is beset with many physiological difficulties, it is expensive research and returns come in far more slowly than they do for most of the other agricultural problems thus far investigated. These are all questions where the public must be convinced and the public works together on a problem about as efficiently as a committee of one hundred million would be expected to do on any problem.

Purely aside from the public health significance of the problem, a significance no one would deny, the stupendous size of the breeding problem for dairy cattle is scarcely realized by those unfamiliar with the industry. In this country alone there are nearly 7,000,000 calves under one year old, over 4,000,000 heifers from one to two years of age, nearly 20,000,000 cows over two years of age, over 500,000 bulls over one year of age, or all told, over 31,000,000 dairy cattle and all these cattle have to be replaced at least every six years. Over 70 per cent of our farms derive support from this industry and the industry is growing at the rate of nearly 15 per cent a year. The present value of the dairy cattle alone is more than a billion and a half dollars and the products from them are yearly worth nearly as much. And yet this industry is built on a cow whose average yearly production is only slightly over 3000 pounds. Because of inadequate breeding and hearsay knowledge of its laws the industry gets along on a shoe-string basis making scanty profits to its owners and a slender public supply of milk which is frequently too meager for the poor to get the much needed quantity for their children.

"You can feed a cow until she bursts but she will not make milk unless she has the inheritance for doing it," said one of my good friends, himself an able investigator in the nutrition field. What then can breeding do? No one would, I think, contradict the fact that the greatest contribution to the knowledge of dairy cattle breeding would be the elimination of the poor milk yielding cows before they are born or for that matter before conception had started their existence. The work in which the writer is now engaged has

this goal. The volume here presented is in small part a beginning of the fulfillment of this desire. The hope is entertained that the future will offer the opportunity to further extend and complete the work here commenced.

The list of those who have lent their influence to further these animal husbandry investigations would indeed be long. Two men have been outstanding in the encouragement they have given the writer and in the firm, unyielding support they have given the work. To the efforts of these men, Drs. Raymond Pearl and Charles D. Woods, is largely due the completion and publication of this work.

Historically speaking this work took its origin from the year 1913 when Dr. Raymond Pearl commenced these animal husbandry investigations here at the Maine Agricultural Experiment Station. These investigations were conceived broadly. Their sole aim was the collection of critical, scientific data on the problems of the physiology and inheritance of cattle breeding. In no sense were these experiments an attempt to produce any new types of economically important animals. The guidance of the experiments was in the hands of Dr. Pearl until the beginning of the war called him elsewhere, making it impossible for him to continue the work further. At that time the writer, a student of Dr. Pearl's, who had worked in these investigations earlier (1914), took up the direction of these investigations. Building on the broad foundation already laid it became possible in 1918 to commence the analysis of the material herein presented.

Inheritance work, which involves a quantitative character, is long and tedious. The complexity of it requires the use of the keenest of analytical tools, the mathematics of probability, to render assistance in its solution. Needless to say the data presented in this volume are not all of the writer's own doing. A great measure of the effort should be credited to the mental acumen of his two assistants, Miss Marjorie Gooch and Miss Mildred R. Covell. Other assistance facilitating the work has been received from Miss Beatrice Goodine and Miss Helen A. Ring.

The results of any one investigation can scarcely be said to be final. While only one line of evidence is here presented, it should be said in justice to the work, that five lines of research have contributed their evidence to check the conclusions. These researches are the sinews making one whole. They stand or fall together. Their continued prosecution has been made possible in large measure by

a grant to the writer from the Rockefeller Institute for Medical Research. The writer here acknowledges with gratitude the financial service rendered and what is even greater, the sacrifice of personal time and energy given freely to this work by the Board of Scientific Directors and particularly by Drs. Theobald Smith and Simon Flexner.

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CHAPTER I

The Characteristics of Holstein-Friesian Milk Yield

Like races of men, breeds and their peculiarities can only be appreciated and properly interpreted when their history is known. The history portrays the forces, their action, interaction, and compressive strength, which mold the breed into a unified whole. These forces react on the whole organic complex reaching and influencing even the most minute. Should the forces react on inheritable characters, the resulting modifications will be inherited and become permanent features of the breed. It is fitting to begin the discussion of the Holstein-Friesian breed and the physiology of its milk secretion by a summary of the breed's history. For this purpose I quote the excellent discussion of Cole and Jones.[1]

The early history of the Holstein-Friesian cattle, like that of other breeds, is largely a matter of conjecture, and many of the statements in the literature cannot be accepted at their face value. For example, it has been assumed by some that the present black-and-white color originated shortly before the beginning of the Christian era by the crossing of black cattle brought by the Batavians, who settled in the region between the Rhine and the Meuse, and "pure white" cattle of the Friesians. Not only is it improbable in the first place that these early tribes possessed cattle which approximated definite breeds, uniform in color and markings, but is it exceedingly improbable from a genetical viewpoint that a pied pattern like that of the Dutch cattle should arise from the crossing of black and white stock. The statement of Hengerveld, so often quoted, to the effect that "the genealogy of Netherland cattle is pure and unadulterated, and is at least 2000 years old" can not be accepted at its face value. It is true that the region of the Netherlands has apparently possessed cattle of special value since early Christian times, or possibly before, but there is no evidence to show that there have not been intermixtures with it; in fact, there is positive evidence to the contrary. In the matter of color alone it seems fairly certain that perhaps the most general color of the cattle in the Netherlands and the surrounding provinces, until within a century or two, was red or fawn (or some shade of dun), and that black-and-white piebald as a predominating color is of comparatively

[1] Cole, Leon J. and Jones, Sarah V. H. 1920. The occurrence of red calves in black breeds of cattle. Wisconsin Agricultural Experiment Station Bulletin No. 313.

recent introduction. Red-and-white breeds still occur in Holland and the West German provinces, such as East Friesland and Oldenburg.

The fullest and most critical discussion of the color of the Netherland cattle is that of Bakker, published in 1909. He remarks that recent authors assume Netherland cattle to have been black-and-white since earliest times, but says there is no basis whatever for such an opinion. On the contrary, he believes that there were no black-and-white cattle in Holland previous to about the middle of the eighteenth century. This conclusion is based very largely on the evidence afforded by early Dutch paintings in which cattle are depicted. Bakker examined nearly 4000 paintings in the Imperial Museum at Amsterdam and found that while many of them contained cattle, no black-and-white cattle appeared prior to the second half of the eighteenth century.

TABLE 1

Colors of cattle in old Dutch pictures (from Bakker)

PERIOD	NUMBER OF CATTLE	RED	RED-AND-WHITE	DUN	"BLAAR-KOPPEN"*	"WIT-KOPPEN"*
		per cent	*per cent*	*per cent*	*per cent*	*per cent*
1500–1600	23	34.7	4.3	56.5	56.5	8.7
1600–1700	154	50.0	5.8	35.7	35.0	1.3
1700–1775†	9	22.2	55.5	11.1	66.6	0

* "Blaarkop" animals are white-faced, but with a spot of pigment encircling the eye; "witkop" is literally white-headed, the whole face, or practically the whole head, being white, as in Hereford cattle. For convenience we shall refer to the two classes together as "white-faced." The pigment accompanying these conditions may be either black or red.

† Approximate.

In the accompanying table are given the percentages of different colors in each century from 1500 until about 1775. After that period black-and-whites are numerous while recent artists use black-and-white cattle exclusively to adorn their meadow scenes.

Several interesting points came out from a study of these pictures. We note, for example, that if they may be relied upon as giving a fair indication of the predominating colors of the cattle at the time they were painted, 39 per cent of the Holland cattle in the sixteenth century were red or red-and-white, while 56.5 per cent are represented as dun, probably referring to some shade of grayish brown, perhaps somewhat like the color of many Jersey cattle. Bakker states that in some cases the color is somewhat uncertain in the paintings, so that there is considerable variation of color in animals classed in the table as dun. In the seventeenth century the proportions are not markedly different, but in the eighteenth century there is a noticeable increase in the proportion that are spotted with white. In all the pictures painted after the latter part of the eighteenth century in which cattle are included, at least one or more are black-and-white spotted.

These facts have led Bakker to conclude that the Netherland cattle are descended from the old red native breed which constituted the original cattle of all central Europe; that they were, therefore, not originally black-and-white, but that this color dates from the latter part of the eighteenth century; and that it was introduced into Holland by importations of Jutland cattle from Denmark. There is, furthermore, historical evidence of such importations to support this view. The white face marking has, however, been common in the Netherland cattle from earliest times, and is characteristic of one of the present breeds in Holland.

The black-and-white color must have found great favor in Holland, for after its appearance it seems to have become the predominating one in a relatively short time. Furthermore, the fame of the Netherland cattle, especially those of Friesland, rapidly extended them to neighboring countries, and more recently to all parts of the world. Nevertheless, as late as 1865, John H. Klippart, who made a tour of Europe for the Ohio State Board of Agriculture, reported regarding the color of these cattle at the International Fair at Stellin, Germany, that "the most in popular favor are the white, with red, grey, blue-grey, or black spots," showing that even then the breed was far from uniform.

Professor Silliman, in his "Journal of Travels" in Holland, published in 1812, says: "Innumerable multitudes of very fine cattle were grazing upon the meadows; many of them were of a pure milk-white color; others, nearly or quite black; but by far the greater number were marked by both these colors, intermixed in a very beautiful manner; and we found this fact to be general; for wherever we went in Holland, the cattle were black or white, or striped and spotted with these colors." Prof. George G. Cook is quoted as writing in 1871 that cattle are to be seen everywhere at pasture in Holland and that "their decided colors of black and white make them conspicuous objects." No mention is made of other colors. These statements confirm the conclusion that black-and-white came into predominance very rapidly after its introduction, a thing which could very naturally happen, since black is dominant to red.

At the present time there are three distinct breeds of cattle in Holland: (1) The black-and-white Friesian-Dutch, a strictly dairy breed; (2) the white-faced black Groningen cattle, of a relatively heavy beef type; and (3) the Yssel breed, red-and-white in color and intermediate in type between the other two. Importation of other breeds is not allowed. It is interesting to note the way in which these three breeds have selected among the available color and pattern characters. The Friesian-Dutch has the black piebald pattern supposed to have come in from the Jutland breed; the Groningen has the imported black color, but retains the old white-face character of so many early Holland cattle; while the Yssel (sometimes called "Oberijssel" or "Mass-Rhein-Yssel" breed) is red-and-white spotted, the red color presumably being directly descended from the old native stock. At the present day one sees practically nothing but black-and-white and red-and-white cattle in journeying through Holland; the other colors appear very largely to have been eliminated.

The cattle, now called the Holstein-Friesian, resulted from a combination of the Dutch Friesian Association and the Holstein Breeders' Association. The first herd book published by the combined association appeared in 1886. The idea of an Advanced Registry for the recording of performance was with the combined organization from the start, having been brought in by the Dutch Friesian Association. The trend toward the standardization or breeding for type began earlier, going back to the original European stock for about half a century before extensive importation into America.

Despite this breeding for type the Holstein-Friesian breed is still highly heterozygous (mixed) from a breeding standpoint. The black-and-white pattern is by no means standardized. Even red color, a factor recessive to black which has been selected against almost from the start of the two parent associations, still occasionally crops out. In the field of milk yield and butter-fat percentage much selection is said to have been practiced, the argument being based on the constant advance made by the leaders of the Advanced Registry. However, this progress is more apparent than real if the breed as a whole is considered. In fact, it is highly probable that if we could select as large a group of animals for test among the progenitors of the Holstein-Friesian cattle we would find that given equal conditions of test the cows would make records practically on a par with those made today.

There is, however, a certain homogeneity of type found in the Holstein-Friesian breed as a whole, when this breed is compared with the other breeds or with less uniform stock like the unregistered cattle of America. Data on milk yield and butter-fat percentage are quite extensive. Unfortunately, the same can not be said of the other important milk solids. The Holstein-Friesian breed as compared with the Channel Island breeds is characteristically a high milk producing, low butter-fat testing breed. The composition of the milk may be best illustrated in tabular form. Table 2 shows the data on the milk yield, butter-fat percentage and solids-not fat percentage taken from the data of the Advanced Registries of the breed.

Table 2 gives a good idea of the characteristics of Holstein-Friesian milk. Several factors influence the milk yield. These may be age, heredity, or time of calving, etc. The influence of these factors will be considered later. On the basis of pounds of butter-fat and solids-not-fat an average Holstein-Friesian milk for the 365-day period

should contain 556.5 pounds of butter-fat and 1,398 pounds of solids-not-fat. The ratio of the solids-not-fat to the butter-fat is approximately 2.5 to 1. Taken in the form of percentages, the Holstein-Friesian milk contains slightly over 12 per cent of total solids, composed of 3.43 per cent of fat and 8.61 per cent of solids-not-fat. The mean age of the group is slightly more than four years. The importance of the butter-fat is of course known to all. The solids-not-fat contain the important constituents casein, albumin, ash, milk sugar or lactose, and a number of minor substances. While relatively little emphasis is placed on the solids-not-fat they are as important as, if not more important than, the butter-fat.

A comparison of the milks of different breeds brings out several facts concerning Holstein-Friesian milk. Table 3 presents this information gathered from many sources, chief among which are the

TABLE 2

Average quantity and quality of milk yield and age at test of Holstein-Friesian Advanced Registry cattle

Mean milk yield (pounds)	16,233
Mean butter-fat percentage	3.428
Mean solids-not-fat percentage	8.612*
Mean total solids percentage	12.040
Mean age at test (years)	4.57

* All data, 335 individuals including one which is doubtful.

papers of the Agricultural Experiment Stations and the analyses of public chemists. Each tabulated value is, in general, the mean of a considerable number of observations and may be considered close to the true value.

These figures are not entirely satisfactory, representing, as they do, data collected under a great variety of conditions. This heterogeneity is unfortunate. Two errors are easily discernible: The fat percentage of the Holstein-Friesian is about 0.1 per cent too low, and the fat percentage of the Guernsey from the writer's published data on 10,644 animals for a year's test is 5.04 instead of the 4.53 of the above list. However, this is the only material available where any number of animals are tested for their total solids and percentage of fat. It is believed that even with these discrepancies the table will give a fair comparative view of the average composition of the milk of the various breeds. Taken as a whole the data show that the

butter-fat percentage in the different breeds varies between 3.05 and 5.12 per cent. Similarly, the total solids are shown to vary

TABLE 3

*Mean milk constituents of the different breeds**

BREED	TOTAL SOLIDS	FAT	SOLIDS-NOT-FAT	RATIO OF SOLIDS-NOT-FAT TO BUTTER-FAT
	per cent	*per cent*	*per cent*	
Molltaler	13.22	3.86	9.39	2.43:1
Blondvich	12.75	3.67	9.09	2.48:1
Angler	12.51	3.51	9.00	2.56:1
Jeverland	11.86	3.09	8.77	2.83:1
Holland	11.54	3.05	8.04	2.63:1
East Friesian	11.80	3.09	8.71	2.81:1
Lova Rhine	12.12	3.31	8.81	2.66:1
Breitenburg	12.34	3.36	8.98	2.67:1
Red Holstein	12.07	3.27	8.80	2.69:1
Wesermarsch	11.85	3.24	8.61	2.65:1
Schwyz	12.76	3.60	9.16	2.52:1
Simmental	13.27	4.05	9.22	2.28:1
Westerwold	12.99	3.79	9.20	2.42:1
Glan	13.57	4.16	9.41	2.26:1
Alderney	13.60	3.81	9.79	2.57:1
Jersey	14.39	5.12	9.27	1.81:1
Guernsey	13.61	4.53	9.08	2.00:1
Holstein-Friesian	11.78	3.32	8.46	2.55:1
Ayrshire	12.46	3.62	8.84	2.44:1
Shorthorn	12.61	3.70	8.91	2.41:1
Polled Jersey	13.93	4.67	9.26	1.98:1
French Canadian	13.32	4.00	9.32	2.33:1
Dutch Belted	12.31	3.40	8.91	2.62:1
Brown Swiss	12.61	3.62	8.99	2.48:1
Red Polled	12.66	3.67	8.99	2.45:1
South Devon	12.93	3.72	9.21	2.47:1
Kerries	13.10	4.02	9.08	2.26:1
Dexter	12.58	3.46	9.11	2.63:1
Holstein-Friesian	12.04	3.44	8.60	2.50:1

* See Gowen, John W. 1919. Variations and mode of secretion of milk solids. Jour. Agricultural Research, vol. xvi, no. 3, pp. 79–102, for literature on this point.

between 11.54 and 14.39 per cent and the solids-not-fat between 8.04 and 9.79 per cent. This would make the average composition of the Holstein-Friesian breed rather lower than most of the other breeds,

both in the percentage of butter-fat and in the total solids. If we consider the ratio of the solids-not-fat to the butter-fat when the milk is constant, the values of the ratios vary between 1.8 and 2.81. The Holstein-Friesian breed has a high proportion of solids-not-fat. There appears to be an association between the percentage of the fat characteristic of the breed and the content of the solids-not-fat carried in the milk—, that is, the Jerseys, with their high fat percentage, also have an increased amount of the solids-not-fat over the other breeds, and the Holland, one of the breeds lowest in butter-fat also has the lowest amount of solids-not-fat. This increase does

TABLE 4

*Mean milk constituents of different species of animals**

SPECIES	TOTAL SOLIDS	FAT	SOLIDS-NOT-FAT	RATIO OF SOLIDS-NOT-FAT TO BUTTER-FAT
	per cent	*per cent*	*per cent*	
Sow	18.51	6.60	11.91	1.8:1
Goat	11.80	3.54	8.26	2.3:1
Ewe	18.52	7.17	11.35	1.6:1
Indian buffalo	16.24	6.77	9.47	1.4:1
Bitch	15.89	5.65	10.24	1.8:1
Ass	9.72	0.90	8.82	9.8:1
Mare	10.92	0.99	8.93	9.0:1
woMan	11.78	3.28	8.50	2.6:1
Cow (Holstein-Friesian)	12.01	3.44	8.61	2.5:1
Colostrum of cow	22.88	2.30	20.58	8.9:1
Colostrum of man	12.91	2.60	10.31	4.0:1

* Loc. cit.

not go up in direct proportion to the amount of fat present in the milk, as a glance at the proportion of the two will show. It will remain for a later section to show how these constituents vary within the Holstein-Friesian race.

Data have been tabulated to show the differences in the milk of different species of animals (table 4).

The data given in table 4 are open to the same criticism as those in table 3, and are to be taken with the same limitations.

The milk of the different species varies considerably both in its butter-fat and its solids-not-fat content. The lowest percentage of fat produced is 0.90 per cent, found in the milk of the ass. The milk

of the mare corresponds closely to this, 0.99 per cent. The highest percentage of butter-fat is 7.17 per cent, contained in the milk of the ewe, closely followed by that of the Indian buffalo and the sow. The milk of the dairy cow is about intermediate between these two extremes. The colostrum is lower in its fat content than either of the normal milks of the same species. The solids-not-fat content of normal milk for the included species, varies from 8.26 per cent for the goat, to 11.91 per cent for the sow. It reaches its highest value in the colostrum, where the cow's milk includes as much as 20.58 per cent. The same general association is also found in the milk of the different species that is present in the milk of the different breeds, that is, the milk of species containing a low percentage of fat contains proportionately more solids-not-fat than does the milk of the species which contains the higher percentage of fat. The species having a

TABLE 5

Standard deviations and coefficients of variation for the milk yields, butter-fat percentages, and solids-not-fat percentages of Holstein-Friesian cattle

CHARACTER	STANDARD DEVIATION	COEFFICIENT OF VARIATION
Milk yield	4039.0	24.9
Butter-fat percentage	0.309	9.0
Solids-not-fat percentage	0.364	4.2
Age	2.25	49.1

low percentage of fat in its milk has less actual solids-not-fat per hundred pounds of milk than does the species containing a higher percentage of fat.

The milk yields of Holstein-Friesian cattle are highly variable. The butter-fat percentages are only about a third to a half as variable as the milk yields. Even this variation is large in comparison to that found for most similar data. The standard deviations and coefficients of variation for these Holstein-Friesian cattle are seen in table 5.

The amount of variation is large in each case. When this variation is compared with the mean, the variation is greatest in the age of test. This extreme variation is brought about partly by the tendency to test young cows. The next most variable character is the milk yield. Milk yield is about $2\frac{1}{2}$ times as variable as the butter-fat percentage and about 6 times as variable as the solids-not-fat. The

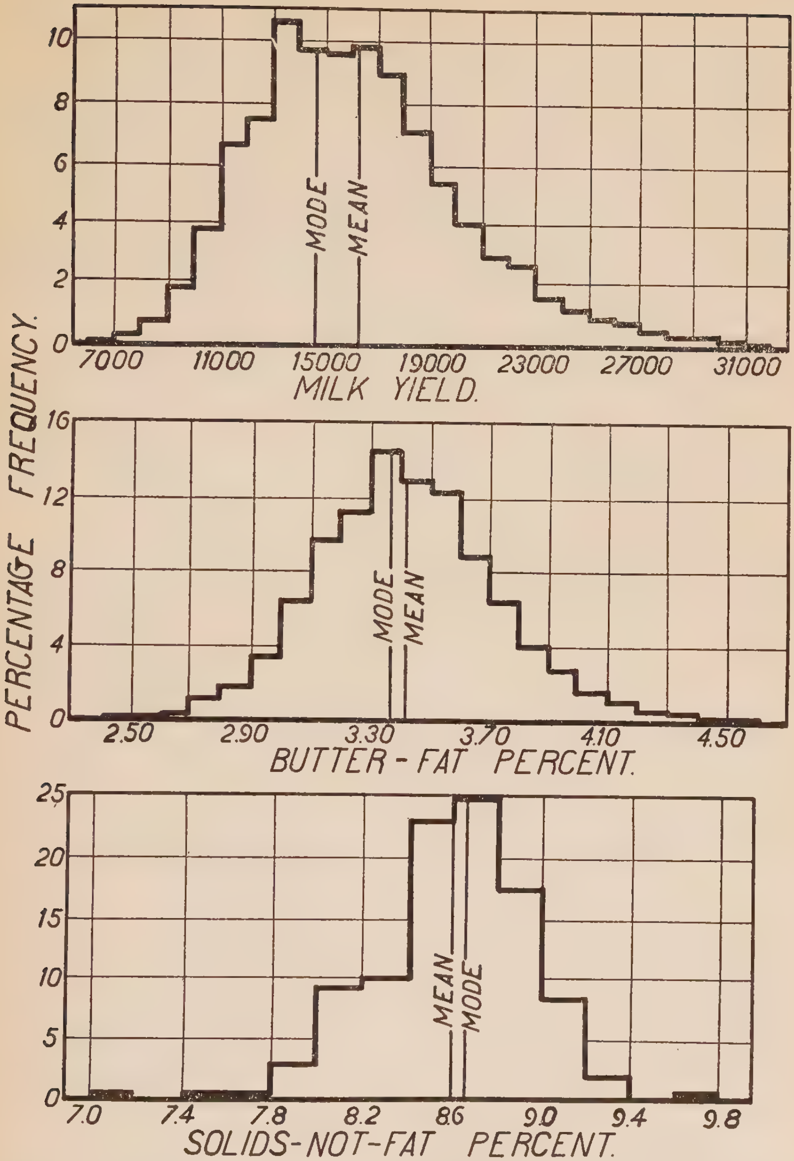

Fig. 1. Frequency Curves for Milk Yield, Butter-fat Percentage, and Solids-not-fat Percentage for Holstein-Friesian Advanced Registry Cows

butter-fat in milk is over two times as variable as the solids-not-fat. It is of no small importance to analyze these factors making this variation.

Up to the present the writer has endeavored to show how variable Holstein-Friesian milk really is, in so far as its yield and composition are concerned. However, despite its variation and the varied origin of the breed this milk has some very definite characteristics. These characteristics may best be brought out by the diagrams showing the frequency with which the Advanced Registry cows have certain

TABLE 6

Analytical constants for the solids-not-fat percentage

CONSTANTS	
Mean	8.604 ± 0.0124
Standard deviation	0.338 ± 0.009
Coefficient of variation	3.9 ± 0.1
μ_2	2.765
μ_3	−2.114
μ_4	30.484
β_1	0.2114 ± 0.1558
β_2	3.9872 ± 0.9825
β_{2-3}	0.9872
κ_1	1.3403 ± 1.6089
κ_2	0.1257 ± 0.1351
Skewness	-0.1662 ± 0.0579
Mode	8.659
Type of curve	IV
Mean age	4.49 ± 0.08
Standard deviation of age	2.27 ± 0.06

milk yields, percentages of solids-not-fat or butter-fat. Figure 1 gives the diagrams illustrating these data.

Within these data Holstein-Friesian milk yield ranges from 6000 to 32,000 pounds of milk for the 365-day period. The curve for this milk yield is slightly skew, the mode coming at 14,662 pounds of milk. The skewness is equal to 0.390. The criterion calls for a Pearson type I curve to fit the data. The other constants of the curve are given later. The second graph represents the frequency distribution of the butter-fat percentage. The range for the butter-fat percentages of the different cows is from 2.4 to 4.7 per cent. The distribution is slightly skew. This skewness is equal to 0.167. The

mode is at 3.377 per cent. The constants of the frequency curve call for a Pearson type IV curve. The third graph gives the frequency curve for the percentage of the solids-not-fat. One doubtful observation is excluded. This doubtful observation has a solids-not-fat percentage of more than 11.0. The range in per cent of the solids-not-fat extends from 7.0 to 9.0. The distribution is slightly skew—the skewness, 0.166, being practically identical with that for the butter-fat percentage. The mode is found to be at 8.659 per cent. The type of the curve is a Pearson type IV. As the constants of this curve are not given elsewhere they are shown in table 6 for the 334 individuals on which we have records of apparent accuracy.

SUMMARY

The data presented above give a picture of the diverse origin of Holstein-Friesian cattle and the characteristics of their milk as shown by their more recent Advanced Registry records. Originally these cattle came from northwestern Europe. They are the results of the admixture of red and black cattle marked by white spots. In America the black and white animals are the only ones to be allowed registry. They have been bred pure, in a registry sense, for a dozen generations in America and about the same number in Europe. From a certain point of view these cattle are a homogeneous group although even in the Advanced Registry cows the production of milk and the percentage of butter-fat are far from standardized, (made homozygous) so far as heredity is concerned. The milk yields of this breed have certain definite characteristics as compared with certain other contrasting breeds. These characteristics are a high milk yield and a lower butter-fat percentage and probably a lower solids-not-fat percentage. The constants are given for the distribution of the milk yields, butter-fat percentages, and solids-not-fat percentages for this milk. It is the purpose of this volume to attempt to analyze this variation in milk yield and butter-fat percentage into its constituent variables.

CHAPTER II

Characteristics of Modern Holstein-Friesian Pedigrees

Any Holstein-Friesian bull or cow, even though chosen at random has a history more or less complete. This history includes the registered ancestors. It includes the manner in which these ancestors were bred together with the dates of birth, ownership of the animals, and the place where they were owned. If the cow is fortunate enough to have appreciative owners or to have ancestors which had such, she or her ancestors may have records of performance. This pedigree is characteristic of this bull or cow. Other bulls or cows have similar pedigrees. These pedigrees are characteristic of them. To study the characteristics of the breed it is necessary to have a composite of the breed so that the composite may be studied. The author has studied many pedigrees singly and in this composite fashion. For this study of the composite pedigree five groups of bulls of widely differing origin have been studied. These bulls comprise first a group of 48 sires whose daughters have a high milk yield, 29 sires whose daughters have low milk yield, 22 sires whose daughters are high in butter-fat percentage, 22 sires whose daughters are low in butter-fat percentage, and 33 sires whose daughters or sons have no Advanced Registry records. From these pedigrees it is possible to gain some appreciation of the characteristics of modern Holstein-Friesian pedigrees. Four generation pedigrees have been used throughout.

ANCESTORS

The first question which naturally arises is what animals are the ancestors of the Holstein-Friesian cattle. By our herd registry system these animals are on the whole rather few in number because the system requires that present-day animals trace their pedigrees back to the early imported animals and because these early imported animals are necessarily limited by the expense of importation. The early recorded animals are the narrow neck of the bottle through which the blood of the early cattle from northwestern Europe flows

to the large American population of the Holstein-Friesian breed. Table 7 gives the number of times different ancestors were repeated in the pedigrees of this group of bulls. The repetitions are arranged in order of frequency. Only the animals appearing more than fifteen times are given.[1]

The cow, DeKol 2d, is by far the most frequently appearing animal in the pedigrees of the Holstein-Friesian cattle. In tabulating 25 cows appearing in pedigrees of these cattle DeKol 2d is found on an average, once. In one out of two pedigrees of four generations we should expect her to appear about once. Paul DeKol is the next most frequently found animal. This bull is found in one out of every 30 bulls tabulated in a pedigree or he would be expected to appear in every other pedigree. The next two bulls are also DeKol bulls, DeKol 2d's Butter Boy and DeKol 2d's Paul DeKol. These bulls appear in about one out of three pedigrees. The other bulls and cows follow in order of frequency. Only those animals which appear sixteen or more times in this pedigree group are included in this table. Those which occur less than sixteen times and more than once form a much larger group. However, even in this small list 15.7 per cent of the possible number of cows and 31.8 per cent of the possible number of bulls which could be found in four generation pedigrees appear in this table. The Holstein-Friesian breed is evidently dependent on a relatively small number of animals for its ancestors. From their origin it seems entirely probable that from an inheritance standpoint these animals were highly heterozygous. Since close inbreeding in the later generations has not been practiced, it is entirely probable as the evidence for milk yield and butter-fat percentage shows, that the breed is still heterozygous for many factors.

A critical examination of the make up of the Holstein-Friesian pedigrees shows that the animals most frequently repeated are those which are most popular. I will not say they are necessarily the best. The animal repeated infrequently or never is almost unknown to the breeders. The repeated animals are consequently those which are psychologically most important and those which should receive a good deal of study to determine if they really indicate anything when

[1] Should the reader care to study a longer list see page 123 or Studies in milk secretion IX. On the progeny performance of Holstein-Friesian sires by John W. Gowen and Mildred R. Covell. Annual report of the Maine Agricultural Experiment Station for 1921, pp. 121–252 Bulletin 300.

TABLE 7

Frequency with which ancestors are repeated in Holstein-Friesian pedigrees

ANCESTOR	SEX*	FREQUENCY OF APPEARANCE IN THE PEDIGREES	
		ACTUAL	PER CENT OF TOTAL POSSIBLE
DeKol 2nd	F	97	4.2
Paul DeKol	M	72	3.1
DeKol 2d's Butter Boy	M	52	2.3
DeKol 2d's Paul DeKol	M	50	2.2
Netherland Hengerveld	F	42	1.8
Milla's Pietertje Netherland	M	41	1.8
Sarcastic Lad	M	32	1.4
DeKol 2d's Netherland	M	31	1.3
Manor DeKol	M	30	1.3
Willem III	M	30	1.3
DeKol	F	30	1.3
Sir Abbekerk	M	29	1.3
Hengerveld DeKol	M	28	1.2
Aaggie Cornelia 5th's Clothilde Imperial	M	27	1.2
Pauline Paul	F	26	1.1
DeKol 2d's Butter Boy 3d	M	25	1.1
Maurice Bonheur	M	25	1.1
Pietertje Hengerveld	F	25	1.1
Belle Sarcastic	F	25	1.1
DeKol 2d's Prince	M	23	1.0
Pietertje Hengerveld's Paul DeKol	M	22	1.0
Magadora	F	22	1.0
Paul Mutual DeKol	M	21	0.9
Belle Korndyke	F	21	0.9
Johanna Rue 2d's Paul DeKol	M	20	0.9
A & G Inka McKinley	F	20	0.9
Paul DeKol Jr	M	19	0.8
Billy McKinley	M	18	0.8
Netherland Alban	M	18	0.8
Pontiac Korndyke	M	18	0.8
Mercedes Julip's Pietertje	F	18	0.8
Segis Inka	F	18	0.8
Duke Netherland	M	17	0.7
Homestead Jr. DeKol	M	17	0.7
Inka Princess Pietertje Netherland	M	17	0.7
King Segis	M	17	0.7
Johanna Rue 2d	F	17	0.7
Mercedes Julip's Pietertje's Paul	M	16	0.7
Manor Josephine DeKol	M	16	0.7

*M = bull, F = cow.

a pedigree is studied. The question of the relation of high milk yield or butter-fat percentage to these animals will be taken up later. At present we shall only try to give a picture of what the relation of average Holstein-Friesian pedigrees might be to each other. In the five groups of pedigrees studied, an average correlation of 0.552 for the appearances of the male ancestors was found for all possible combinations of these groups. Presented in another way this relatively high correlation shows that if we should select at random two groups of fifty bulls each and pedigree them for four generations we are justified in expecting to find not only a large proportion of the same ancestors in each of these groups but also that these ancestors would be in about the same proportion. That is, in any two groups, the pedigree most nearly representative of that group tends to be the pedigree most representative of the other group.

The pedigree of a bull has two lines, one diverging from his sire and one from his dam. We find these lines are alike in many particulars. Thus the average correlation for the appearance of the male ancestors in both the sire's and dam's sides of the pedigree is found to be 0.478. For the females this correlation was 0.462. There is then a high degree of resemblance between the animals found on the sire's and dam's sides of the pedigree. This fact could be accounted for by the probability that for any animal, the owner would be likely to use as parents the animals which were closely related. The following correlations and the lack of any appreciable inbreeding makes it doubtful if such an explanation is correct. It appears rather a characteristic of the breed itself.

Within two pedigrees there are three possible comparisons. We can compare the ancestors on the sire's side of the pedigrees, the ancestors on the dam's side of the pedigrees, and the ancestors on the sire's side on one pedigree with the ancestors on the dam's side in the other. From such a comparison it is evident that the animals in the sire's side of one pedigree tend to be those in the sire's side of any other pedigree. The correlation for the bulls is 0.496 and for the cows 0.490. In fact the sire's side of the pedigrees tend to resemble each other even more than do the sire's and dam's side of the same pedigree in the frequency with which their ancestors repeat in such pedigrees.

The dam's side of different pedigrees do not resemble each other in their ancestors so much as the sire's side of the pedigrees. The corre-

lation coefficient for the males is 0.315 and for the females is 0.317. Here is a real selective difference, the sires being a much more selected group of animals than are the cows. As will be shown further on, there is no basis in heredity for this selective difference, the dam contributing as much as the sire to the performance of the daughter. Practically, this selection is more easily practiced for the sire than for the dam as far as the possibilities of choice are concerned. On the other hand this advantage is in a great measure counterbalanced by the greater difficulty of determining the true worth of a sire because of the impossibility of getting his milk record.

The cross-correlation coefficients, sire's ancestors in one pedigree with dam's ancestors in another pedigree are; male ancestors 0.371, and female ancestors, 0.363. There is then a slightly greater resemblance between these ancestors than there is for the dam's ancestors in different pedigrees. The cross-correlations are less than those for the sire's side of the pedigrees or for the sire's and dam's side for the same pedigree.

These correlation coefficients show that the Holstein-Friesian pedigrees are much more intimately related than might be supposed at first glance. Even in four generations the animals in them are frequently the same. They repeat time and again. With what caution it is necessary to approach a study of such ancestors? These correlations show that great care is necessary at every stage if the conclusions drawn are to be sound. Thus almost any pedigree might be expected to contain DeKol 2d without any especial significance to be attached to the fact, save that she was an ancestor of most of the present day Holstein-Friesian breed.

INBREEDING

The next characteristic of a pedigree is its inbreeding. In the four generation pedigrees of this group of pedigrees relatively little inbreeding is shown. Table 8 presents the data on the mean inbreeding and its standard deviation. This inbreeding coefficient is the one developed by Pearl.[2] It is a purely objective coefficient. The basic concept of this measure of inbreeding is that inbreeding results in a narrowing of the network of descent as a result of mating together

[2] Pearl, Raymond. 1913. Tables for calculating coefficients of inbreeding. Annual report of the Maine Agricultural Experiment Station for 1913, pp. 123–138.

individuals genetically related to one another in some degree. It means that the number of potentially different germ-to-germ lines, or "blood-lines" concentrated in a given individual animal is fewer if the individual is inbred than if it is not. In other words, the inbred individual possesses fewer different ancestors in some particular generation or generations than the maximum possible number for that generation or generations. When practically measured the coefficient of inbreeding is the percentage of the difference between the maximum possible number of ancestors in a given generation and the actual number realized in the former. The coefficient may have any value between 0 and 100. When there is no breeding of relatives whatever (that is, in the entire absence of inbreeding) its value for each generation is 0. As the intensity of the inbreeding increases the value of the coefficient rises. This measure of inbreeding has to

TABLE 8

Average coefficients of inbreeding for Holstein-Friesian four generation pedigrees. Standard deviations of coefficients of inbreeding for these pedigrees

GENERATION	MEAN COEFFICIENT OF INBREEDING	STANDARD DEVIATION OF COEFFICIENT OF INBREEDING
Second	0.862 ± 0.233	4.56 ± 0.17
Third	3.161 ± 0.339	6.62 ± 0.24
Fourth	7.611 ± 0.441	8.63 ± 0.31
Total inbreeding	5.79 ± 0.51	9.95 ± 0.36

do solely with the relationship aspect of the problem. It has nothing whatever to do directly with the gametic or zygotic constitution of individuals.

The averages and standard deviations of coefficients of inbreeding for the different generations are given in table 8.

The frequency distributions of this inbreeding are distinctly J-shaped so that the probable errors do not have as much meaning as they otherwise might. One hundred and seventy-four pedigrees were available for this study. Of this number 168 showed no inbreeding in the second generation, 138 showed none in the third generation, and 69 showed none in the fourth generation. The average inbreeding though small in every case, is significant. The total inbreeding is the amount of inbreeding compared with the total possible amount, brother and sister matings. The average amount is low. The four

generation pedigrees of modern Holstein-Friesian cattle do not show that inbreeding has been practiced to any appreciable extent in the first four generations.

RELATIONSHIP BETWEEN SIRE AND DAM

The coefficient of relationship[3] is based on the concept that a state or condition of relationship or kinship between two organisms exists when these organisms have one or more common ancestors. The degree, intensity or closeness of the relationship is, in general, proportional to the number of different ancestors which the two individuals have in common, out of the whole number they might possibly have in common. The coefficient of relationship is the relation of the number of ancestors in the $n + 1$th generation which occur in

TABLE 9

Averages and standard deviations of relationship in Holstein-Friesian bulls

GENERATION	MEAN COEFFICIENT OF RELATIONSHIP	STANDARD DEVIATION OF RELATIONSHIP
Second	1.72 ± 0.47	9.12 ± 0.33
Third	4.02 ± 0.58	11.29 ± 0.41
Fourth	7.97 ± 0.71	13.87 ± 0.50

the $n + 1$th or some earlier ancestral generation in the pedigrees of the sire and dam, or in other words which are common ancestors to the total number of ancestors in the same generation of the pedigree. The coefficients of relationship give us a measure of the kinship of sire and dam. As might be expected from the results for inbreeding this kinship is small in amount. Table 9 gives this information for each generation.

The frequency curves for the coefficients of relationship of the sires and dams of these bulls are J-shaped with a high modal ordinate at zero. The average amount of relationship is small. The standard deviation of this relationship is large.

[3] Pearl, Raymond. 1917. Studies in inbreeding. VII. Some further considerations regarding the measurement and numerical expression of degrees of kinship. Amer. Nat., vol. li, pp. 545–559.

SUMMARY

These pedigree studies show that when groups of pedigrees are compared, there is a high degree of resemblance between them in the ancestors which they contain and quite apart from the groups which may be compared. This is a consequence of the manner in which the breeding of Holstein-Friesian cattle has been carried on in the past. It is in fact a descriptive feature of the breed whose significance to milk yield and butter-fat percentage will be taken up shortly.

If the single pedigrees are studied it is found that the inbreeding in them is relatively small in amount for the first four generations. Modern Holstein-Friesian cattle tend to be outbred. The relationship between the sire and dam of these pedigrees is also small in average amount. In these later generations the American Holstein-Friesian breeders have avoided mating a given bull with a closely related cow.

CHAPTER III

Conformation in Relation to 7-day Milk Yield and Butter-fat Percentage

Before the Advanced Registry, in fact before the organization of many of the breeds; observing dairymen put forth the opinion that certain points of the cow's conformation are directly associated with milk yield. This opinion, although untested was rapidly adopted as a fact into the dairy literature. The advent of the Advanced Registry has tended to materially weaken the emphasis formally given to type as a complete guide to the milk yield for the cow although such quotations as the following show that the influence of the conformation ideal is still strongly associated with the idea of performance at the pail,—"when judging direct fitness . . . . for dairy purposes all breeding and ancestoral records may be disregarded as all practical evidences of utility and quality are largely visible on the exterior of the animal."

Undoubtedly conformation will strongly influence the breeding and sale of dairy cattle for many years to come. In view of this fact it is of much importance to have an appreciation of the true influence of type to performance. Furthermore the problem is of critical biological import for such data present an opportunity to resolve the different physiological processes into terms of correlation between structure and function. Toward this end the writer has analyzed a large series of score cards on Jersey cattle.[1] It is proposed in this chapter to present the results of another investigation on the relation of actual measurement of certain body parts of Holstein-Friesian cattle to the performance of these cows in 7-day milk yield and butter-fat percentage.

[1] Gowen, John W. 1920. Conformation and its relation to milk producing capacity in Jersey cattle. Journal of Dairy Science, vol. 3, no. 1, pp. 1–32.

Gowen, John W. 1921. Studies on conformation in relation to milk producing capacity in cattle. II. The personal equation of the cattle judge. Journal of Dairy Science, vol. 4, no. 5, pp. 359–374.

These data are contained in the early volumes of the Holstein-Friesian Advanced Registries. The measurements give evidence of being taken with care to their accuracy. Besides the measurements they also present a carefully worded description of other parts of the cow. The confusion of differences in wording between different men has thus far prevented the writer from making an analysis of this portion of the data in relation to the performance of these cattle.

There are 385 of these records, the cows ranging in age from one year and six months to ten years and six months. All records for measurement were within the year when the 7-day lactation record was made. These records had measurement records on height at shoulders, height at hips, body length, rump length, body width, thurl width, and body girth. The weight, either actual or estimated was given in 339 of them. All records have the 7-day milk yields, butter-fat percentages, and ages recorded. The method to be followed in making these measurements is described by these rules:

> The animal must be brought to a stand on a level place and in a natural position, the feet squarely under the body, the head at a medium height and the neck straight. The two items of height are taken perpendicularly from the ground to the top of the animal, the one immediately over the knee and center of the shoulder, and the other over the hook bone to the center of the back; the length of body is taken from the extreme front of the shoulder point to the extreme rear and highest point of the rump, diagonally in a straight line; the length of the rump, from the extreme front side of the hook bone to the extreme of the rump as described above; the width of the hips, from the outside of one hook bone to the outside of the other in a straight line; the width of the thurl from the outside of one thurl bone to the outside of the other, also in a straight line; the girth by a tape closely fitting the smallest circumference of the chest; in the latter measurement the tape must be drawn so closely that a slight movement of it will move the skin of the animal. If the head is lowered from the natural position, this measurement will invariably be erroneous. And if the neck is turned to the right or left, the measurement of length of body will also be erroneous. Then follows the ascertaining of the weight, which must invariably be obtained by reliable scales; the date of service is then ascertained of the owner and condition described. The date of the examination should also be recorded in some convenient place in the memorandum book.

Figure 2 shows the methods by which these measurements are taken. Only the inspectors can tell us how accurately the records were made. The analysis of the records, showing as it does some-

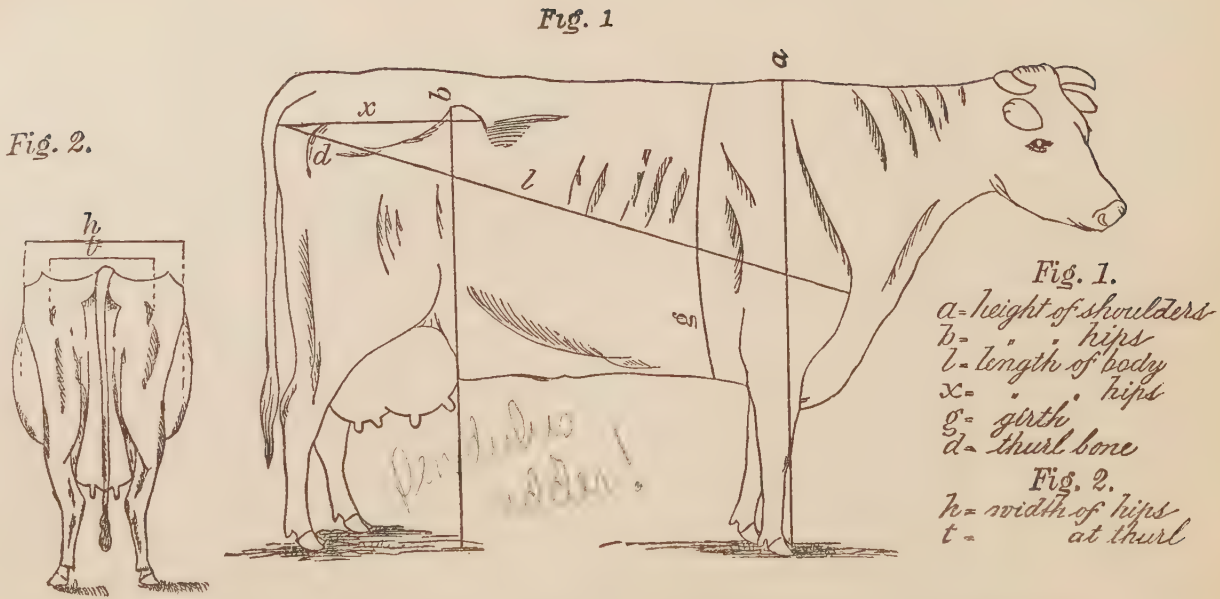

Fig. 2. Diagram Showing the Various Measurements Described in Advanced Register

All measurements are taken in straight lines excepting girth. The measurement "b" is to full height of the chine.

thing of the care of measurement, indicate that the records are for the most part accurate.

In view of the printing costs it does not seem feasible to print the correlation tables. The constants of variation for the different parts are given in table 10.

The averages for age, butter-fat percentage, and milk yield show that this group of cows is younger, has a higher butter-fat test, and is less productive of milk than are the cows of the whole Advanced Registry.

The series of means of the different parts of the body give a statement of what might be called the average Holstein-Friesian cow.

TABLE 10

Means, standard deviations, and coefficients of variation for the physical measurements of Holstein-Friesian cows

CHARACTERS	MEAN	STANDARD DEVIATION	COEFFICIENT OF VARIATION
Age	3.90±0.08	2.22 ±0.05	56.92±1.78
Butter-fat percentage	3.53±0.01	0.361±0.009	10.23±0.25
Milk yield	338.0 ±2.0	72.6 ±1.8	21.48±0.55
Shoulder height	52.8 ±0.1	1.94 ±0.05	3.67±0.09
Hip height	53.8 ±0.1	1.91 ±0.05	3.55±0.09
Body length	62.0 ±0.1	3.73 ±0.09	6.02±0.15
Rump length	20.4 ±0.1	1.44 ±0.04	7.06±0.17
Body width	21.3 ±0.1	1.72 ±0.04	8.07±0.20
Thurl width	19.0 ±0.1	2.63 ±0.06	13.84±0.34
Body girth	73.2 ±0.2	4.59 ±0.11	6.41±0.16
Weight	1088 ±6	164.0 ±4.2	15.07±0.40

This average cow may or may not be a desirable specimen. It would be interesting to see a reconstruction based on these measurements. It might be that we should find the same condition of affairs as that found for the typical American soldier, that is, that the average is disproportionate for some of the parts.

The weights of the cows are based on estimated and actual records of weights. There are 161 records of actual weight and 178 of estimated weight. The actual weights have a mean of 1078, a standard deviation of 162, and a coefficient of variation of 15.0. The distributions are consequently closely similar to each other.

The coefficients of variation for age, butter-fat percentage, and milk yield are essentially the same as those which have been found elsewhere. The parts of conformation are essentially dependent on

TABLE 11

Coefficients of variation for characters comparable to those of conformation

CHARACTER	COEFFICIENT OF VARIATION	AUTHORITY
Weight of spleen (English males)....	38.2	Greenwood (1)
Milk production (Jersey random sample herd)......................	25.5	Gowen (2)
Milk production (Guernsey Advanced Registry)........................	24.7	Gowen (2)
Milk production (Holstein-Friesian Advanced Registry)...............	24.2	Gowen (3)
Milk production (Jersey Registry of Merit)...........................	23.2	Unpublished
Heart weight (English males).......	22.2	Greenwood and Brown (4)
Milk production..................	21.5	These data
Weight of kidneys (English males)...	21.0	Greenwood and Brown (4)
Weight of liver (English males).....	20.8	Greenwood and Brown (4)
Body weight (English males)........	18.9	Greenwood and Brown (4)
Rev. maximum daily milk yield (for given age).......................	17.9	Gavin (5)
Head—eyes full and placid..........	17.3	Gowen (6).
Weekly milk yield (Ayrshire cattle).	17.0	Pearl and Miner (7)
Breathing capacity (English males).	16.6	Pearson (8)
Weight (Holstein-Friesian)..........	15.1	These data
Width between thurl (Holstein-Friesian)...........................	13.8	These data
Weight (domestic fowl)..............	12.7	Curtis (10)
Back—straight to hip-bones.........	12.6	Gowen (6)
Udder—broad, level or spherical....	12.4	Gowen (6)
Milk veins—large, tortuous, and elastic..............................	12.2	Gowen (6)
Fore udder—full and well-rounded...	11.9	Gowen (6)
Tail—thin, long, with good switch...	10.9	Gowen (6)
Udder—large size and not fleshy....	10.6	Gowen (6)
Head—medium size.................	10.1	Gowen (6)
Thigh—flat and well cut out........	9.9	Gowen (6)
Neck—thin, rather long, with clean throat...........................	9.9	Gowen (6)
Teats—good and uniform...........	9.7	Gowen (6)
Rump—long and level...............	9.3	Gowen (6)
Yield of mixed milk (daily fluctuations............................	9.0	Pearl and Miner (11)
General appearance.................	8.4	Gowen (6)
Rear-udder—well rounded...........	8.4	Gowen (6)

TABLE 11—*Continued*

CHARACTER	COEFFICIENT OF VARIATION	AUTHORITY
Hip-bones—high and wide apart...	8.2	Gowen (6)
Width of hips (Holstein-Friesian)...	8.1	These data
Body—wedge shape.................	7.9	Gowen (6)
Lung capacity......................	7.5	Gowen (6)
Rump length (Holstein-Friesian)....	7.1	These data
Body girth (Holstein-Friesian)......	6.4	These data
Body length (Holstein-Friesian).....	6.0	These data
Mature cows, 800 to 1000 pounds....	5.6	Gowen (6)
Length of forearm (English males)	5.2	Pearson and Lee (9)
Length of femur (French males).....	5.0	Pearson (8)
Total score on conformation........	4.9	Gowen (6)
Stature (English males).............	3.9	Pearson and Lee (9)
Shoulder height.....................	3.7	These data
Hip height..........................	3.6	These data

(1) Greenwood, M. 1904. A first study of the weight, variability, and correlation of the human viscera, with special reference to the healthy and diseased heart. Biometrika, vol. iii, pp. 63–84.

(2) Gowen, John W. Studies in milk secretion. V. On the variations and correlations of milk secretion with age. Genetics, vol. 5, pp. 111–188, March, 1920.

Gowen, John W. VI. On the variations and correlations of butter-fat percentage with age in Jersey cattle. Genetics, vol. 5, pp. 249–324, May, 1920.

(3) Gowen, John W. 1919. Variations and mode of secretion of milk solids. Jour. of Agri. Research, vol. xvi, no. 3, pp. 79–102.

(4) Greenwood, M. and Brown, T. W. 1913. A second study of the weight variability and correlation of human viscera. Biometrika, vol. ix, pp. 473–485.

(5) Gavin, W. F. 1913. The interpretation of milk records. Jour. Roy. Agri. Soc., vol. lxxiii, pp. 1–22.

(6) Gowen, John W. 1920. Conformation and its relation to milk producing capacity in Jersey cattle. Jour. Dairy Science, vol. iii, no. 1, pp. 1–32.

(7) Pearl, Raymond, and Miner, John R. 1919. The variation of the milk of Ayrshire cows in quantity and fat content of their milk. Ann. report. Maine Agricultural Experiment Station, 1919, pp. 57–64.

(8) Pearson, Karl. 1899. Mathematical contributions to the theory of evolution. V. On the reconstruction of the stature of prehistoric races. Phil. Trans. R. Soc. London, vol. 192A, pp. 162–244, 2 pl.

(9) Pearson, Karl and Lee, Alice. 1897. Mathematical contributions to the theory of evolution. On the relative variation and correlation in civilized and uncivilized races. Proc. R. Soc. London, vol. lxi, no. 375, pp. 343–357.

(10) Curtis, M. R. 1914. A biometrical study of egg production in the domestic fowl. IV. Factors influencing the size, shape, and physical condition of eggs. Arch. Entwicklungsmech. Organ., Bd. 39 Heft 2/3; pp. 217–327, 18 fig., pl. 6–10.

(11) Pearl, Raymond and Miner, John R. 1919. Variation of Ayrshire cows in the quantity and fat content of their milk. Jour. Agr. Research, vol. xvii, no. 6, pp. 288–322.

the anatomical structure of the cow. They are essentially morphological characters. The variation of such characters has been found to be low in amount. These measurements are no exception to this rule. The range of the coefficients of variation are from 3.55 to 15.07. Some appreciation of what this range means is obtained by an examination of table 11.

Table 11 presents a picture of the range of variation normally found in characters similar to those of conformation and milk yield. The information is interesting in many ways. The data are arranged in order of the amount of the variation.

The variation of the amount of secretion of milk is important to the dairyman and to the medical man. It will be noted that milk yield is one of the most variable characters known. The standard deviation is a fourth to a fifth of the mean yield of the product and the range for different individuals is about three-fourths the mean value on either side of this mean. Looking at these facts from a medical viewpoint we note that this huge variation in secretion is for one of the most important glands of the body. It brings out the fact, already recognized but not fully appreciated, that in all forms of glandular secretion we may expect wide variations between individuals in the amount, and for that matter in the quality, of the secretory product. In the light of the coming recognition of the importance of all these secretory glands to the health and psychological reactions, if not actual life, of the individual this variation between individuals is enlightening as to the probably resulting variations in these individuals due to differences in the secretory activity of their glands. If we can show, as I think it has been shown in the subsequent pages, that the amount and quality of this secretion is quite largely controlled by heredity, the explanation of much in the inheritance of certain defects may well have been found.

From a comparison of the work of Greenwood and of Greenwood and Brown with the results here presented on milk yield we note that the variation of milk yield is of much the same magnitude as the variation in the size of the soft organs in the body. When the variations of the organs which depend for their size on the hard parts or more particularly skeletal structures like stature, length of forearm, and length of femur are compared with those of the soft organs and milk secretion it is seen that the variation is very much less for these bony parts. The physiological characters are clearly more variable than the morphological ones.

The characters used in score-card judging are essentially divisible into two classes, physiological and morphological. Those characters are presented in this table for the Jersey Registry of Merit cattle scores. The judges who made these scores have undoubtedly taken cognizance of the increases in many of them as age advances and compensated for such increase in recording their judgment on the cows. The amount of variation would consequently be expected to be less than it would have been had age been left out of consideration. The data on the Holstein-Friesian cattle are actual measurements. In general the comparison of these actual measurements with the scores shows that the scores of the judges on conformation are actually more variable than are the parts of the cows from which their score is taken. In other words the judgment by eye is on the whole less reliable than estimates based on the yard stick.

The comparable results found on other forms in conjunction with those found for these Holstein-Friesian cattle are worthy of record. The weight of these Holstein-Friesian cows has practically the same variation as the weight of men or of the domestic fowl. Shoulder or hip height in cattle is as variable as the stature of men. Rump length, body girth, and body length are slightly more variable than the length of the forearm or femur.

The comparison of the variation of milk yield, where the milk yield is for individual cows, with the variation of yield for the mixed milk of a herd is a feature of no little significance. As noted by Pearl and Miner[2] the variation for herd's milk yield is about 9, whereas the coefficients for milk yield give values of 17 to 25. In secular variation in the amount or quality of the mixed milk of a large herd, individuality of the animal as a source of variation is entirely eliminated. The observed variation must, therefore, be due to the combined action of all the external environmental influences which affect in greater or less degree the milk yield of every cow.

On the other hand, the constants of variation for milk yield determined in this paper are based upon the diversity or variation in weekly yield exhibited among a large number of different cows. Here one primary factor in the causation of the observed variation must be the individuality of the animal with respect to milking

[2] Loc. cit.

ability. By individuality in this sense is meant the genotype of the individual with regard to the character named. But in the causation of the variation in milk yield as here discussed there must be involved the combined influence of the individuality of the animal plus that of all the environmental factors which act in producing variation in the mixed milk of the herd, since each of these causes influences every individual animal while it is making its individual record.

It is therefore possible to make comparison here between observed variations (as measured by the coefficient), due, on the one hand, to environmental influences alone and, on the other hand, to the genotypic differences plus environmental influences. The difference should represent in a general way that part of the observed variation due to genotypic differences.

The figures as they stand suggest that roughly about one-half of the variation (measured by the coefficients of variation) in milk production results from the varying genotypic individuality of the animals with respect to this character, and the other half results from the varying external circumstances to which cows are subjected during lactation and which have an effect upon the flow of milk. Or, to put the matter in another way, if the conclusion just stated were true it would mean that if a large number of cows were placed in environmental circumstances which were at once ideal and uniform we should expect the variation exhibited in milk production to be roughly about one-half of that which we actually find when we measure this variation under ordinary circumstances. This point is well taken and will be considered further when other evidence on the relation of the milk yields or butter-fat percentages of different lactations are considered.

CORRELATION OF BODY MEASUREMENTS TO MILK YIELD AND BUTTER-FAT PERCENTAGE

The association of the size of different body parts to milk yield or butter-fat percentage is a most interesting point to the dairyman. The correlation coefficients measuring these relations are given in table 12. These correlations do not take age into account. Thus we know milk yield increases with age, therefore, if any of the other measurements increase with the age of the cow (we know they do)

then any positive correlation between the size of the body part and milk yield will be increased considerably through their joint correlation with age. This fact will be analyzed further but should be kept in mind in considering the data of table 12.

TABLE 12

Correlation of the body measurements with the age, butter-fat percentage, and milk yield of the cow

CHARACTERS CORRELATED	CORRELATION COEFFICIENT
Milk yield with age	0.592±0.022
Milk yield with shoulder height	0.380±0.029
Milk yield with hip height	0.344±0.030
Milk yield with body length	0.590±0.022
Milk yield with rump length	0.391±0.029
Milk yield with body width	0.520±0.025
Milk yield with thurl width	0.214±0.033
Milk yield with body girth	0.476±0.027
Milk yield with weight	0.652±0.021*
Butter-fat percentage with shoulder height	0.036±0.034
Butter-fat percentage with hip height	−0.024±0.034
Butter-fat percentage with body length	0.030±0.034
Butter-fat percentage with rump length	0.061±0.034
Butter-fat percentage with body width	0.070±0.034
Butter-fat percentage with thurl width	0.159±0.034
Butter-fat percentage with body girth	0.129±0.034
Butter-fat percentage with weight	0.054±0.037*
Age with shoulder height	0.357±0.030
Age with hip height	0.266±0.032
Age with body length	0.601±0.022
Age with rump length	0.440±0.028
Age with body width	0.571±0.023
Age with thurl width	0.350±0.030
Age with body girth	0.512±0.025
Age with weight	0.674±0.021*
Age with butter-fat percentage	0.087±0.034

*The correlations for the 161 actual weights are milk yield with weight 0.623; butter-fat percentage and weight 0.033; age and weight 0.699. Other results agree closely with the whole group—actual and estimated.

Table 12 reveals several facts of no little importance to the selection of dairy cattle. Taking the first nine variables which are correlated with milk yield we note that every one of them has a rather large positive relation to the performance of the cow at the pail. As will

be shown later a fairly large element in this correlation is the fact that the age of the cow materially influences the performance of the cow in milk production and at the same time and to nearly like measure causes a growth in the size of the body parts. On the other hand if we have little or no knowledge of the age of the cow and select out of a group of such cows the largest, say in body length or girth, we are quite sure of choosing the cows which are producing the most milk. It matters little to the practical dairyman perhaps whether this selection is based on the selection of the size of the cow or that the size of the cow directly selects the older cows and consequently the cows which produce most milk. From the physiologist's standpoint it does matter and we shall try to differentiate the influence of the two variables.

The next eight constants show the relation of the body parts to the butter-fat percentage. Butter-fat percentage has little relation to age so that this variable plays little part in the association of the size of the body parts to the cow's performance in butter-fat percentage. Only two of the correlations, thurl width and body girth have any relation to butter-fat percentage and in the case of each of these the relation is of doubtful significance. We may conclude therefore that the size of the body parts shows little or nothing about the butter-fat percentage which the cow may produce.

The last nine variables show the relation of age to the body parts and to butter-fat percentage. The size of the body parts is quite closely related to the age of the cow, the older the cow the greater the average size of the parts. It would be of interest to digress and consider this fact further from the viewpoint of when a cow may be said to reach maturity, but as this volume is primarily aimed to deal with the functions of the mammary gland and another is planned to deal with the general physiology, it seems desirable to stick strictly to the text and leave this point to the volume to come. Age, then, is a variable causing a marked change in milk yield or in the size of the body parts but having little influence on butter-fat percentage. We know the relation of age to milk yield is skew and cursory examination of the tables indicates that the relation of age and the size of the body parts is also skew. We shall, however, consider that the regression lines are straight and that the correlation coefficients measure the relation between the variables fairly accurately. Such assumptions do not take care of all the in-

fluence of age. On the other hand the end result is not far wrong and the major conclusion derived from such a treatment of the data will be correct to at least a first approximation.

To determine the relation of the body parts to milk yield for cows of the same age we shall use the partial correlation coefficients where age is held constant. Table 13 shows these partial correlation coefficients.

The first eight correlation coefficients of table 13 show the relation of the milk yields of the cows with the size of the different parts of the body for a constant age. While it is probable that some slight influence of age is still unaccounted for, the amount of this age influence remaining should change the correlation coefficients but little. Live weight and body length are the variables closely associated with milk yield even for cows of constant age. Either of the variables predicts fairly well the probable yield of the cow. The variables body width, body girth, height at hips and shoulder are in the order named important in indicating the probable performance of the cow. The variables, rump length and thurl width are not important in indicating milk yield and probably are not expected to be by those who suggested their measurement, since most teaching suggests that these variables are important chiefly in facilitating gestation. The whole trend of these results shows that a cow must have size to produce quantities of milk. This size may cost more to maintain but in any case size is a component part of large yields.

The correlation coefficients for the relation of type to milk are larger than those found for the previous study[3] of the relation of score to milk yield in Jersey cattle. This difference might be interpreted as due to one or more differences in the data. In the first place the score represented the judgment of a man on his own mental yardstick as to the approach to the ideal milk type for the cow. This yardstick may vary in the individual and certainly does vary between individuals. We might therefore interpret the difference in these results as favoring measurement in some common unit of measure like an inch for judging cattle for milk yield instead of the present method of judging. The scores were taken by a number of different men. It has been possible to show that the different men differed markedly in their ability to judge cattle by the score card and select

[3] Loc. cit.

those which were high in milk yield. It was further shown that when a judge was able to select the high milking cows by one point

TABLE 13

Partial correlation coefficients showing the relation of body parts, milk yield, butter-fat percentage, and age where one term is constant

CHARACTERS CORRELATED	CONSTANT CHARACTER	COEFFICIENT
Milk yield with shoulder height.........	Age	0.224±0.033
Milk yield with hip height..............	Age	0.240±0.032
Milk yield with body length.............	Age	0.364±0.030
Milk yield with rump length............	Age	0.180±0.033
Milk yield with body width.............	Age	0.275±0.032
Milk yield with thurl width.............	Age	0.009±0.034
Milk yield with body girth..............	Age	0.250±0.032
Milk yield with weight..................	Age	0.425±0.028
Milk yield with age.....................	Shoulder height	0.528±0.025
Milk yield with age.....................	Hip height	0.553±0.024
Milk yield with age.....................	Body length	0.368±0.030
Milk yield with age.....................	Rump length	0.508±0.026
Milk yield with age.....................	Body width	0.421±0.028
Milk yield with age.....................	Thurl width	0.565±0.023
Milk yield with age.....................	Body girth	0.461±0.027
Milk yield with age.....................	Weight	0.273±0.032
Age with shoulder height................	Milk yield	0.177±0.033
Age with hip height.....................	Milk yield	0.083±0.034
Age with body length...................	Milk yield	0.387±0.029
Age with rump length...................	Milk yield	0.281±0.032
Age with body width....................	Milk yield	0.382±0.029
Age with thurl width	Milk yield	0.284±0.032
Age with body girth....................	Milk yield	0.325±0.031
Age with weight........................	Milk yield	0.471±0.027
Butter-fat percentage with shoulder height...............................	Age	0.005±0.034
Butter-fat percentage with hip height...	Age	−0.049±0.034
Butter-fat percentage with body length.	Age	−0.028±0.034
Butter-fat percentage with rump length.	Age	0.025±0.034
Butter-fat percentage with body width..	Age	0.025±0.034
Butter-fat percentage with thurl width..	Age	0.138±0.034
Butter-fat percentage with body girth....	Age	0.099±0.034
Butter-fat percentage with weight.......	Age	−0.006±0.034

he was in general able to select the high milking cows by the other points of conformation. A real vital difference thus exists in the men who are judging cattle. Furthermore if the ability of the best

judges is compared with the results here presented we note that even the best men could not select the high milking cows better than they could be selected on the basis of actual measurement with a ruler. There is no personal bias in a ruler as a standard, in view of which fact the ruler seems to be the safest judge.

The relation of age to milk yield is lowered but little by making any but the variables body height and live weight, constant. The correlation of age with milk yield is lowered quite a good deal by making either of these variables constant.

The eight correlation coefficients for the relations of the body parts to age with milk yield constant show that even when the milk yield is constant there still remains some correlation, 0.083 to 0.471, between age and these parts of conformation. From these facts the conclusion may be drawn that there is an independent relation as well as a joint relation of milk yield, age, and the parts of conformation: shoulder height, hip height, body length, rump length, body width, thurl width, body girth, and weight.

As might perhaps be expected, butter-fat percentage has practically no relation to the eight body measurements when age is constant. That is, conformation is nearly if not entirely worthless for predicting butter-fat percentage. This fact has been recognized perhaps, but its significance has not been always considered. Probably over half the cows of this country earn their way by the butter-fat they produce. The amount of this butter-fat depends jointly on the milk yield and butter-fat percentage. Conformation, as has been noted, may to a certain extent predict milk yield but this is only one of the important variables, for the high milk-yielding cow may be the one of very low test. The problem reduces down to the cold fact that it takes milk records and the Babcock test to really know much about the cow's ability to produce.

Some importance is attached to the manner of testing the cow in relation to what the conformation might show as to her milk yield. A short chapter on the 7-day and 365-day test is given elsewhere We may for sake of this analysis compare the average correlation coefficient of the 7-day test with the 365-day test where the 7-day test was a part of the 365-day test and where they were in separate lactations, with the correlation coefficients for these parts of conformation as analyzed above. We may also include the average correlation coefficient for the relation of the 365-day test for one lactation with that of another. These data are given in table 14.

Figure 3 presents the data of table 14 in graphical form. The length of the bars represents the degree of reliance (correlation) which may be placed in the given variable in predicting the probable milk yield of the cow. The partial correlation coefficients for the relation of the part of conformation to milk yield are given since the correlation coefficients for the 7-day and 365-day milk yields are based on groups where the age interval was small thus eliminating any influence which the age might have.

Figure 3 shows clearly the superiority of a lactation record of even so short a duration as seven days to any part of conformation, thus far studied, for predicting milk yield. These data support the

TABLE 14

Conformation versus an Advanced Registry record as a means of predicting milk yield

CHARACTERS CORRELATED	CORRELATION COEFFICIENT
365-day with 365-day milk yield	0.660
7-day with 365-day milk yield (same lactation)	0.598
7-day with 365-day milk yield (different lactation)	0.462
Weight with 7-day milk yield	0.425
Body length with 7-day milk yield	0.364
Body width with 7-day milk yield	0.275
Body girth with 7-day milk yield	0.250
Hip height with 7-day milk yield	0.240
Shoulder height with 7-day milk yield	0.224
Rump length with 7-day milk yield	0.180
Thurl width with 7-day milk yield	0.009

conclusion drawn from the study of the Jersey cattle scores. In these Jersey data the range of the correlation coefficients between the points of conformation studied and 365-day milk yield was 0.194 to −0.070. This range was even lower than that found for the body parts of these Holstein-Friesian cattle, 0.425 to 0.009. The difference in results may very likely be due to the fact that in the Jersey the year lactation record was used while in the Holstein-Friesian the seven-day record was used. It is unfortunate perhaps that as yet we have not been able to analyze satisfactorily the description features relating conformation of such parts as the udder to milk yield in the Holstein-Friesian cattle, for the Jersey records showed these parts to be most closely linked to milk yield. Thus

in these Jerseys only one part of the conformation has a minus value when correlated with the milk production of the cows. Put in words

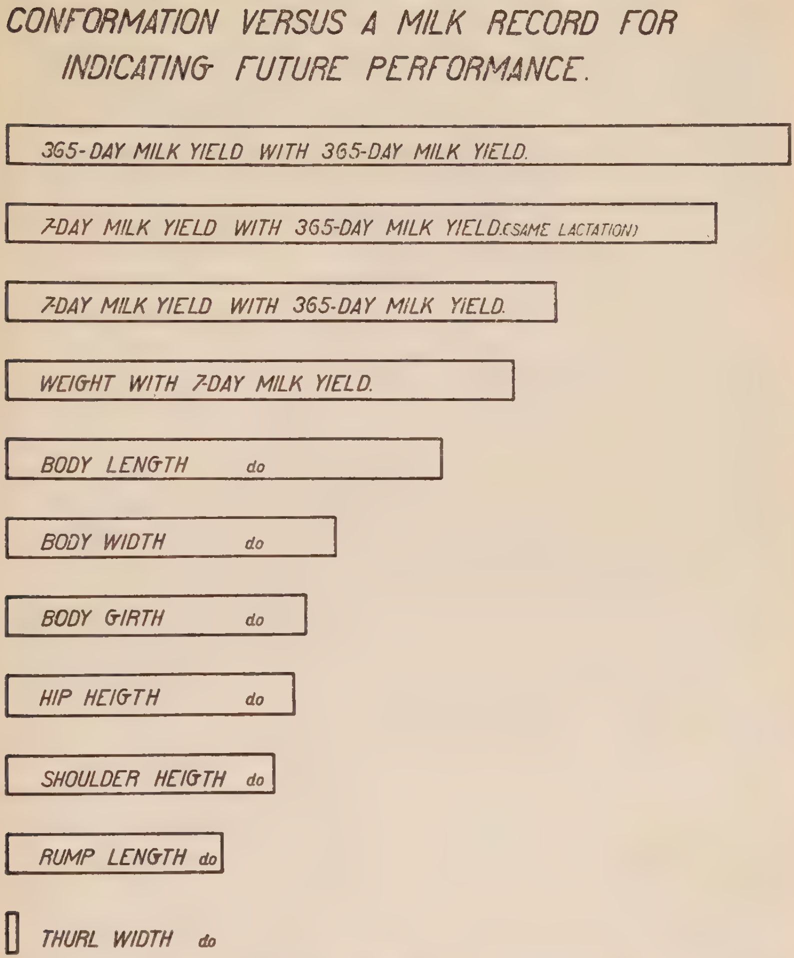

Fig. 3. Relation of Conformation to 7-day Milk Yield and of 7-day Milk Yield to 365-day Milk Yield

this means that as judged by these men the cows which had the backs straight to hip bones were very slightly poorer milk producers than those which were not so straight.

All correlation coefficients but two were more than three times their probable error. There are then 15 parts contributing toward the ideal conformation which are indicative of the high milk producers. The total score most nearly represented the milk producing capacities of the cow. Of the separate divisions into which conformation is divided, the milk veins—large, tortuous, and elastic—distinguish the high producer from the low producer most accurately. This conclusion finds interesting confirmation in the work of Aldrich and Dana.[4] This work dealt with nearly 600 cows. Among the items considered in relation to milk flow were:

a. Size of wells
b. Diameter of milk veins
c. Length of milk veins

The correlation coefficients on these data where accurate measurements in English units were used, were (*a*). Aldrich 0.151 $\pm$ 0.031, Dana 0.262 $\pm$ 0.027; (*b*) Aldrich 0.193 $\pm$ 0.036, Dana 0.282 $\pm$0.027; (*c*) Aldrich 0.003 $\pm$ 0.066, Dana 0.211 $\pm$ 0.042. These correlations where actual measurements are used are quite close to the results obtained from the data of this paper. The conclusion seems justified, therefore, that the condition of the milk vein is to some extent an indicator of milk yield.

Close seconds to the condition of the milk veins as an indicator of the cow's possible milk yield are the size and character of the udder and the shape of the rear udder. The body size and shape together with the general appearance of the cow are next most indicative of her milk yield. It is interesting to note that the size of the udder plays a much more important part in the yield than does the shape and contour of the udder. Likewise it seems that the appearance of the udder is more important than is the general form of the body taken as a whole. These facts substantiate the view that a cow with a large capacious barrel and large, elastic udder is likely to be a good milker. In so far as the data allow comparison these facts are substantiated by the Holstein-Friesian data just presented.

[4] Aldrich, A. M., and Dana, J. W. 1917. The relation of the milk veins system to production. Bul. 202, Vermont Agricultural Experiment Station, pp. 1–24, fig. 3.

The writer does not wish to be interpreted as favoring the abolition of breeding for type and placing all emphasis on breeding for production. From the standpoint of a cattle judge and an admirer of the breed it is desirable if not essential to have breeding for type going hand in hand with breeding for production. It seems to me that the facts above show that there need be no clash between the two for the results show that to a slight degree the desirable type is the type for milk yield. On the other hand it is equally clear that type is to a considerable degree independent of milk yield. Such being the case if we are to have type and milk yield in the one individual and for each individual in the herd then the breeding operations must take both into consideration. The task of breeding for type is, however, far easier than is the task of breeding for milk yield because of the tremendous advantage of being able to see type in each individual, both male and female, whereas with milk yield we can only see it in the female and after long periods of milking.

SUMMARY

An analysis of the relation of conformation to 7-day milk yield is presented in this section. It is shown that the body parts, shoulder height, hip height, body length, rump length, body width, thurl width, body girth, and weight have a fair degree of correlation with milk yield even when the age of the cow is constant. No correlation is found between these body parts and the butter-fat percentage which the cow is able to produce.

CHAPTER IV

Age of the Cow and Its Influence on Her Milk Yield, Milk Solids, and Milk Solids Percentage

For a good many years it has been well known that the age of the cow affects markedly the milk yield of the cow. The true form of the function describing this effect has only recently been determined. The function is logarithmic, rising at an ever decreasing rate as the age of the cow advances until the age of maximum production is reached. From this point the curve declines at an ever increasing rate as age advances in years. This curve is distinctly different from the curve used to demark the limit above which a cow must produce to enter the Advanced Registry. This standard called for the production by Holstein-Friesian cows of not less than 250.5 pounds of butter-fat for the 365-day period at two years old and for every day that she exceeds two years of age the requirement in butter-fat was increased one-tenth of a pound. This increased requirement of 0.1 pound of butter-fat daily for each day's increase in age continued till the requirement reached 360 pounds at the age of five years after which no further increase was made. Such a standard is a linear function, that is, the cow's possibilities of butter-fat production must commence at 250.5 pounds as a two-year-old and must go on increasing one-tenth of a pound each day, one uniform step up each day, until she is five years old when the cow is supposed to be at her maximum productivity. The requirement is for butter-fat but it might just as well have been milk yield for the butter-fat is but a multiple of milk yield and butter-fat percentage and the butter-fat percentage is almost uninfluenced by age.

The Holstein-Friesian Association has collected and is collecting a fairly large number of these 365-day records for milk yield, butter-fat percentage, and age on their cattle. These records are arranged alphabetically according to the names of the cows in the Advanced Registry. Such an arrangement gives no clue as to any relationship which may exist between the attributes, age and yearly milk yield or age and yearly butter-fat percentage. The data have been

rearranged in a correlation table to allow the determination of these relations.

Two thousand five hundred and eighty-six complete 365-day records of the milk yield and age, and butter-fat percentage and age were available for study. These cows range from an age of one and one-half to fifteen and one-half years. The milk productions range from 6000 to 32,000 pounds for the year period. The butter-fat percentages range from 2.4 to 4.7 per cent. The size of the ranges obviously requires a grouping of the material into classes. These classes have been chosen as one-half year for age commencing at one and one-half years. The class interval for milk yield was taken as 1000 pounds of milk, commencing at 6000 pounds for the year. The class interval for the butter-fat percentage was chosen as 0.1 per cent.

The resulting correlation tables are shown in tables 15 and 24. Table 15 shows the association of milk yield with age and table 24 the association of butter-fat percentage with age.

The mean milk yield for this group of cows is 16,234 pounds for the year period. The standard deviation or the amount which they vary is 4039 pounds and the coefficient of variation of the milk yield is 24.9. The mean age at which these cows are tested is 4.57 years. The standard deviation for the ages is 2.25 years and the coefficient of variation is 49.1.

Observation of table 15 shows that there is a noticeable relation of milk secretion to age. The correlation coefficient as deduced from these data is $+0.4332 \pm 0.0108$. The extreme skewness of the data tends to reduce the size of the correlation coefficient as compared to the true relationship which does exist. In view of this fact only slight dependence can be placed in its absolute value. However, since the true amount of relationship would be increased rather than decreased were the skewness removed, it follows that there is a distinctly significant association of milk secretion to age. As our object is not the amount of the correlation which exists between these variables age and milk yield but is the form and equation to the curve which describes the relation we need not pause longer on this phase of the subject.

Proceeding to the calculations of the mean milk production within the age groups as seen in table 15 we find the mean 365-day milk yields to be those shown in table 16.

TABLE 15

Correlation surface for the variables age at commencement of test and milk yield in 365 days. Holstein-Friesian Cattle

MILK YIELD	AGE AT COMMENCEMENT OF TEST (YEARS)																											Total
	1.5–2.0	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0–15.5	
5,000–6,000																													
6,000		1																											1
7,000	3	4	1																										8
8,000	6	9	3	1	1																								20
9,000	10	20	10	6	2	2																							50
10,000	6	39	24	9	10	6	1	2	1	2				1				1											102
11,000	11	67	38	13	16	8	8	5	2	2	2	2						1											175
12,000	13	62	28	10	24	14	10	10	6	2	3	1	5	2	1			1	2			1	1						196
13,000	7	79	42	35	21	18	19	14	11	7	5	6	4	1	4		1	1		1		1	1	1					279
14,000	6	45	31	26	24	20	22	19	16	8	8	3	6	3	5	4	2	6	1		1	1						1	258
15,000	4	44	39	18	23	12	22	16	9	11	14	10	7	5	5	6	5			1				1				1	253
16,000	3	37	26	20	28	22	19	25	19	13	8	8	4	8	3	4	1	4	2	2	1	2							259
17,000		30	26	16	21	22	18	29	13	20	7	10	1	6	6	4	3	2	3										237
18,000	1	18	11	20	18	13	14	22	17	16	8	8	1	8	6	1		1	2										185
19,000		8	7	10	14	12	9	10	14	14	12	8	4	5	6	6	1	2			1								143
20,000	1	4	7	4	9	6	13	7	7	10	14	4	5	2	6	6	1				1		1						108
21,000		3	1	7	5	6	9	6	8	5	2	8	5	5		1	3	1	2				1						78
22,000		5	2	3	5	6	4	2	10	9	4	5	5	4			3	2	1		1	1							72
23,000				2	5	2	4	4	4	6	5	2	2	3	3		1												43
24,000				3		3	4	2	2	2	4	3	1	4	2		2												32
25,000						1	2	2	5	3	2	2		1	1	2	2	1			1								25
26,000					1	1	1	2	2	2	3	2	4	1		1	1												21
27,000					1			5	1	3	1	2		3															16
28,000				1		1	1	2		1		1	1	1						2									11
29,000								1	1	1	1		2		1			1											8
30,000						1		1	1		1																		4
31,000												1			1														2
32,000–33,000																													
Total	71	475	296	204	228	176	180	186	149	137	104	86	57	63	50	35	26	24	13	6	6	6	4	2				2	2,586

These observational means are shown as small circles in figure 4. The ordinates are the pounds of milk produced and the ages are the abscissas. From these observational means the logarithmic curve, shown as the smooth curve of figure 4, is calculated. The equation to this curve is:

$$y = 9432.0 + 2069.6a - 128.9a^2 + 1548.4 \log (a - 1.25) \qquad (1)$$

where y is equal to milk yield and a is the age of the cow in years.

This curve strikes through the observations very well. The agreement between the observed and the theoretical curve is especially close for the milk yields of the younger cows. The considerable variation of the older cows in the observational mean curve is due to lack of numbers for the age classes. It will be observed that even here the theoretical curve strikes through the observations well.

TABLE 16

Mean 365-day milk yield of Holstein-Friesian cows at different ages

AGE AT TEST	MEAN MILK YIELD	AGE AT TEST	MEAN MILK YIELD
years		*years*	
1.5 to 2.0	12,007	8.0 to 8.5	19,405
2.0 to 2.5	13,774	8.5 to 9.0	18,560
2.5 to 3.0	14,264	9.0 to 9.5	18,414
3.0 to 3.5	15,623	9.5 to 10.0	19,654
3.5 to 4.0	15,860	10.0 to 10.5	17,292
4.0 to 4.5	16,528	10.5 to 11.0	17,500
4.5 to 5.0	16,972	11.0 to 11.5	19,833
5.0 to 5.5	17,511	11.5 to 12.0	19,833
5.5 to 6.0	18,178	12.0 to 12.5	16,000
6.0 to 6.5	18,675	12.5 to 13.0	17,000
6.5 to 7.0	18,760	13.0 to 13.5	14,500
7.0 to 7.5	18,977	13.5 to 14.0	
7.5 to 8.0	18,939	15.0 to 15.5	15,000

These results show that milk yield rises at an ever decreasing rate as the age of the cow increases until the age of maximum productivity is reached, from this age of maximum productivity the milk yield declines at an ever increasing rate as age increases. It is rather obvious that this phenomena is an expression of growth and

senescence and has been so interpreted by us.[1] Brody[2] et all have recently fitted curves of the type $M_t = A\ (ae^{-k_1t} - be^{-k_2t})$ to data on age and butter-fat production. They then draw the interesting conclusion, lacking, however, further supporting evidence that the whole process of milk secretion depends on the course of two monomolecular reactions, which are going on simultaneously during the cow's life and find expression as growth and senescence of butter-fat yield. The difficulty with this interpretation is in the fact that curves of this type are curves also fitting economic data which by no stretch of the imagination can be considered as due to any chemical reaction behind them.

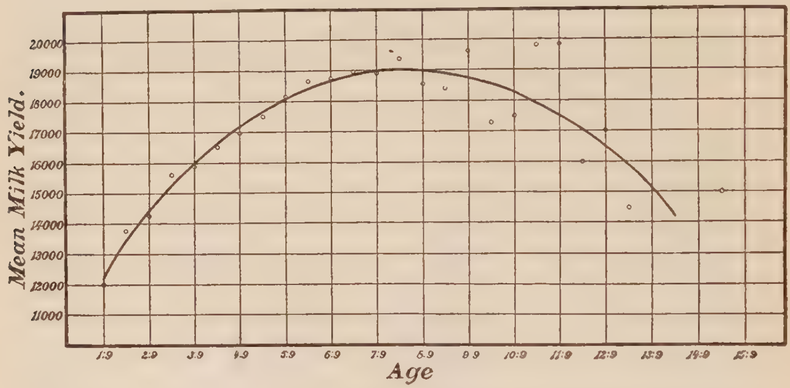

Fig. 4. Observational and Fitted Curves Showing the Relation of 365-day Milk Yield to Age for Holstein-Friesian Cattle

The observational curve is represented by small circles. The smooth curve shows the fitted logarithmic curve for milk yield. Age is in years and months.

The view that the age and milk yield relation is due to growth and senescence is an hypothesis which the writer has had foremost in his mind. For some time evidence has been collected in the attempt to throw direct light on this problem.

[1] Gowen, John W. 1920 Studies in milk secretion. V. On the variation and correlation of milk secretion with age. Genetics, vol. 5, pp. 111–188.

[2] Brody, Samuel, Ragsdale, A. C., and Turner, C. W. 1923. The rate of growth of the dairy cow. IV. Growth and senescence as measured by the rise and fall of milk secretion with age. Jour. General Physiology, vol. vi, pp. 31–40.

By differentiation of the logarithmic equation for the milk yield it is possible to find the age at which the maximum productivity of these cows occurs. This is shown to be eight years, four months, and twenty-nine days. The average milk yield at this age is 19,043 pounds of milk for the year period. While it is true that the change of mean milk yield is slight between the ages 6.5 years to 9.75 years, still it is equally true that the milk production of these Advanced Registry cows increases considerably over that given at five years. Such an increase is obviously unfair to those cows which are tested at five years in competition with those cows tested at seven or eight years.

TABLE 17

Correction factors for standardization of Holstein-Friesian Advanced Registry milk yield to the probable production at 8.25 years

AGE OF COW AT TEST	MULTIPLICATION FACTOR TO CORRECT TO 8.25 YEARS' PRODUCTION	AGE OF COW AT TEST	MULTIPLICATION FACTOR TO CORRECT TO 8.25 YEARS' PRODUCTION
years		*years*	
1.75	1.562	7.5	1.006
2.0	1.480	8.0	1.001
2.5	1.365	8.5	1.000
3.0	1.282	9.0	1.002
3.5	1.217	9.5	1.009
4.0	1.166	10.0	1.018
4.5	1.125	10.5	1.032
5.0	1.091	11.0	1.050
5.5	1.064	11.5	1.073
6.0	1.043	12.0	1.101
6.5	1.026	12.5	1.134
7.0	1.014	13.0	1.175

This information may be used to correct the milk records of cows at different ages to the probable milk yield of these cows at a standard age. The standard age in our work has been considered 8.25 years, or the age of practically maximum milk yield. The correction used is based on the ratio of the mean milk yield at 8.25 years to the mean milk yield at the given age. This is a multiplication factor. In case others wish to use these factors they are tabled above.

From these data it is possible to form a table showing the probable milk yield of a cow at 8.25 years when the milk yield of a cow is given at another age. This information is presented in table 18.

TABLE 18

Milk yields of Holstein-Friesian Advanced Registry cows corrected to an 8.25 year basis

AGE	MILK YIELD AT GIVEN AGE												
	6,000	8,000	10,000	12,000	14,000	16,000	18,000	20,000	22,000	24,000	26,000	28,000	30,000
	MILK YIELD AT 8.25 YEARS												
years													
2.0	8,882	11,843	14,804	17,765	20,726	23,687	26,647	29,608	32,569				
2.5	8,190	10,919	13,649	16,379	19,109	21,839	24,569	27,299	30,028				
3.0		10,253	12,816	15,371	17,943	20,506	23,070	25,633	28,196	30,759			
3.5		9,739	12,173	14,608	17,043	19,477	21,912	24,347	26,781	29,216			
4.0			11,662	13,994	16,326	18,659	20,991	23,323	25,656	27,988	30,320		
4.5			11,249	13,499	15,748	17,998	20,248	22,498	24,747	26,997	29,247	31,497	33,746
5.0			10,914	13,097	15,280	17,463	19,646	21,829	24,011	26,194	28,377	30,560	32,743
5.5			10,644	12,773	14,902	17,031	19,160	21,289	23,418	25,547	27,676	29,804	31,933
6.0			10,430	12,516	14,602	16,687	18,773	20,859	22,945	25,031	27,117	29,203	31,289
6.5			10,263	12,316	14,368	16,421	18,473	20,526	22,579	24,631	26,684	28,736	30,789
7.0			10,140	12,167	14,195	16,223	18,251	20,279	22,307	24,335	26,363	28,391	30,419
7.5			10,056	12,067	14,078	16,089	18,101	20,112	22,123	24,134	26,145	28,156	30,168
8.0			10,009	12,012	14,013	16,015	18,017	20,019	22,021	24,023	26,025	28,027	30,029
8.5			9,999	11,999	13,999	15,999	17,999	19,999	21,999	23,998	25,998	27,998	29,998
9.0			10,025	12,030	14,035	16,040	18,044	20,050	22,054	24,059	26,064	28,069	30,074
9.5			10,086	12,103	14,121	16,138	18,155	20,172	22,189	24,207	26,224	28,241	30,258
10.0			10,185	12,221	14,258	16,295	18,332	20,369	22,406	24,443	26,480		
10.5			10,322	12,386	14,451	16,515	18,580	20,644	22,708	24,773	26,837		
11.0			10,502	12,602	14,702	16,803	18,903	21,003	23,104	25,204	27,304		
11.5			10,727	12,873	15,018	17,164	19,309	21,455	23,600	25,746			
12.0			11,005	13,206	15,407	17,608	19,809	22,010	24,211				

If the correction factors of table 17 or the corrected milk yields of table 18 are to be used for more than an approximation to a result it will be necessary to obtain many intermediate points, especially for the younger cow. These points may be obtained from the equation to the curve.

Table 18 gives a rather good picture of how age influences milk yield. At the top of the table is given the milk which the cow actually produces, 6000, 8000, etc., pounds, in a year. The left hand column gives the ages at which this production occurs. In the rows opposite these ages are given the expected milk yields of these cows at 8.25 years of age, or the age of probable maximum production for the cow. To illustrate, if a two-year-old cow produces 16,000 pounds, what will be her equivalent production at 8.25 years. We find the 16,000 column at the top of the table under "Milk yield at given age;" then we find the 2.0 year age at the top of the left hand column under "Age;" following this 2.0 row across the table to column 16,000 we find the expected milk yield at 8.25 years for this cow to be 23,687 pounds. Similarly any other expected milk yield may be obtained from an actual Holstein-Friesian Advanced Registry record of 365 days in length.

It would, of course, be surprising if the milk yield of a cow when she was eight years three months old was exactly that given in table 18. There are many reasons for this difference. The circumstances under which different tests are made vary greatly. The condition of the cow is by no means always the same. We can only consider the result as an average value subject to a variation of probably at least 10 per cent of the predicted lactation record.

This variation of the actual 8.25 year records from the expected records on the basis of the actual lactation records of other ages brings up the next series of questions. The first of these is the change of the standard deviations of the milk yields of these Advanced Registry cows as the age of the cow advances. Table 19 gives the raw standard deviations for each age as given in table 15.

These standard deviations of milk yield do not follow a straight line. The analysis of their general trend shows that a parabola fits the observations fairly well. The observed and fitted curves are shown in figure 5. The equation to this curve is

$$\text{Standard deviation} = 1681.6 + 652.1 \text{ age} - 44.64 (\text{age})^2 \qquad (2)$$

where *age* is in years and the *standard deviation* is given in pounds of milk. From the above we may conclude that as the cow advances in age the milk yield tends to have a greater variation until the age of about maximum milk yield is reached. From this point the variation in the milk yield declines somewhat.

It is of interest to examine the constants of variation of this same population of Holstein-Friesian cows when the milk yields were

TABLE 19

Variation of milk yield for each six months of age

AGE*	STANDARD DEVIATIONS	
	Observed	Theoretical
years	*pounds*	*pounds*
1.75	2637	2686
2.25	2883	2923
2.75	2864	3137
3.25	3440	3329
3.75	3412	3499
4.25	3729	3646
4.75	3517	3771
5.25	3928	3873
5.75	3867	3953
6.25	3794	4010
6.75	4010	4045
7.25	4086	4058
7.75	4735	4048
8.25	4058	4016
8.75	3987	3962
9.25	3111	3885
9.75	3987	3786
10.25	4406	3664
10.75	3038	3520

* Age is given as the mid-point of each class-interval.

corrected by equation (1) for the effect of age. Table 20 furnishes this information.

The constants derived from this table are given in table 21.

The constants of table 21 furnish some information relative to the Advanced Registry of the Holstein-Friesian breed. The milk which these cows are expected to produce when in their mature form (8.25 years) is 19288.6 pounds of milk for the year period. The

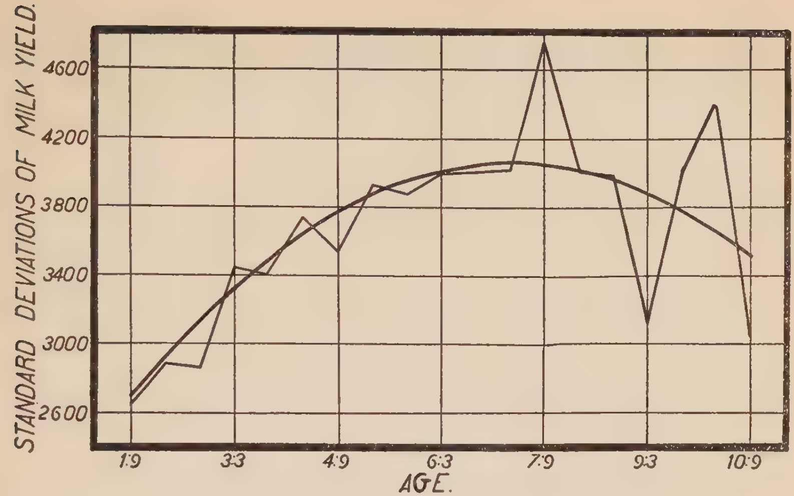

Fig. 5. Observed and Fitted Curves Showing the Variation of Milk Yield with Age

Age is in years and months

TABLE 20

Frequency distributions of corrected milk yield of the 365-day period for Holstein-Friesian Advanced Registry cows

MILK YIELD	365-DAY FREQUENCY	MILK YIELD	365-DAY FREQUENCY
pounds		*pounds*	
9,000–10,000	1	23,000	131
10,000	12	24,000	96
11,000	27	25,000	68
12,000	62	26,000	55
13,000	110	27,000	42
14,000	166	28,000	32
15,000	238	29,000	17
16,000	210	30,000	18
17,000	262	31,000	12
18,000	243	32,000	4
19,000	280	33,000	2
20,000	217	34,000	1
21,000	163	35,000–36,000	1
22,000	155		
		Total............	2625

standard deviation shows that there is a wide scatter in the milk yields of the individual cows around this average milk yield. This variation amounts to 4147.6 pounds. The mean milk yield and the standard deviation of this milk yield for these corrected records is

TABLE 21

Physical constants for the corrected 365-day milk yield of Holstein-Friesian cows, the frequencies for which are shown in table 20

MEAN	STANDARD DEVIATION	COEFFICIENT OF VARIATION	OCTILE						
			First	Second	Third	Fourth	Fifth	Sixth	Seventh
19,288.6 ±54.6	4147.6 ±38.6	21.50 ±0.21	14,700	16,192	17,604	18,924	20,137	21,863	24,207

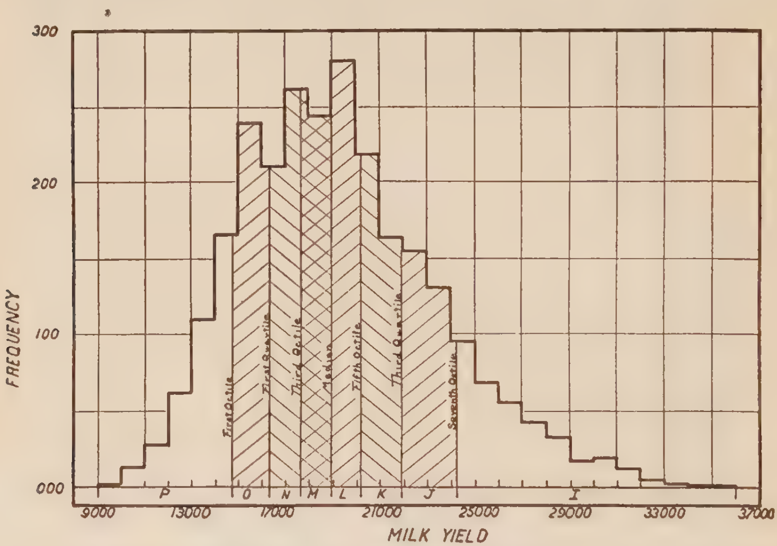

FIG. 6. HISTOGRAM SHOWING THE FREQUENCY DISTRIBUTION OF CORRECTED MILK YIELD FOR HOLSTEIN-FRIESIAN ADVANCED REGISTRY COWS

The octile lines divide these distributions into eight equal parts

practically that of the eight years and three months age on the uncorrected records. This is, of course, as it should be and acts as a check on the work. Figure 6 shows the variation of the corrected milk yields and the octile division lines.

7-DAY MILK YIELD AND AGE

Some years ago the writer formed tables showing the relation of 7-day milk yield and butter-fat production to age. These data are believed to include all entries previous to volume 25 of the Advanced Register. While the author hopes to be able to present shortly much more extensive data on nearly 150,000 entries, it is believed that in absence of these tables the earlier ones contain a good deal of information that is valuable. Table 22 shows the relation of 7-day milk yield to age. This table is the result of combining a larger table having 15 pounds of milk yield as the class unit. All calculations were made from the larger table.

These tables find an important place in answering such questions as these. The cow, May Queen Segis, produced 510 pounds of milk at five years eleven months of age. Is such a cow an animal from which to breed? By referring to table 22 we note that the range of milk yield for cows five and one-half years to six years is from 245 pounds to 815 pounds and that a cow producing 510 pounds is well above the average production. Such a cow would consequently be a valuable animal in most herds.

A milk record does not mean much unless something is known of the age of the cow. In other words, information must be available to describe the age changes for milk yield. Something must also be known about the standard deviation of milk yield since this, too, changes markedly with age. Table 23 gives the means and standard deviations of milk yield for the different age groups, together with curves which have been found to fit the data.

The curve for mean milk yield plotted against age is shown in figure 7. The rough line represents the actual mean 7-day milk yield for each age. The smooth curve gives the theoretical mean curve derived from the equation fitted to these actual observations. The curve is:

$$\text{7-day milk yield} = 323.7 + 3.408a - 0.8296a^2 + 199.5 \log (a - 1.25) \qquad (3)$$

where a is the age at test in years. The age of maximum production is eight years ten months and twenty days for the 7-day records.

The standard deviations of milk yield as given in columns 4 and 5 of table 23 are shown graphically in figure 8. The observational standard deviations are fitted by the curve:

$$\text{7-day standard deviation of milk yield} = 53.0 + 2.63a - 0.217a^2 + 18.82 \log (a - 1.25) \qquad (4)$$

where a is the age in years.

Correlation surface showing the relation of 7-day milk yield to age

AGE	7-DAY MILK YIELD IN POUNDS																								
	170–200	200	230	260	290	320	350	380	410	440	470	500	530	560	590	620	650	680	710	740	770	800	830	860	890–920	Total
years																										
1.0–1.5			1	1		1							1													4
1.5	7	108	217	252	216	164	77	33	14	4	2	1														1095
2.0	8	129	420	665	813	721	476	270	115	45	16	9	4	2												3693
2.5	1	26	147	336	450	446	348	271	144	89	41	16	8	6	1											2330
3.0			29	117	213	321	338	315	258	205	116	79	30	6	5	7	2									2041
3.5			5	48	135	264	309	336	299	235	145	84	48	21	11	7	2	1								1950
4.0				6	43	116	200	279	283	254	176	126	59	46	21	14	3	1								1627
4.5				5	23	74	181	236	248	261	183	155	87	63	29	11	5	4								1565
5.0				1	9	40	88	179	210	198	151	131	84	56	22	11	7	4	2	2						1195
5.5			1	1	5	32	72	155	193	190	148	141	90	51	24	23	5	7	3			1				1142
6.0				1	6	22	65	114	136	136	127	97	76	48	23	13	10	3	2	2	1					882
6.5					6	29	54	92	149	141	112	96	74	47	21	13	8	6	1				1			850
7.0			1	1	3	20	56	80	119	104	97	70	51	35	25	9	4	6	3					1		685
7.5				1	8	20	46	84	91	111	85	44	47	27	14	8	6	3	2							597
8.0					2	15	30	37	81	74	56	48	35	25	11	3	9	5	2						1	434
8.5				1	3	13	30	47	76	67	53	56	32	21	14	9	7	1	1	1						432
9.0					4	5	15	26	40	54	29	31	22	11	7	3	1	1								249
9.5					1	13	20	31	45	37	30	22	16	14	9	3	2									243
10.0					2	7	9	28	17	22	19	15	8	13	3	4	1	1								149
10.5					2	3	8	20	30	16	19	13	14	9	1				1	1						137
11.0					1	3	6	12	10	14	5	4	10	3	2		1	1								72
11.5						1	10	12	14	8	8	5	4	2	2	1										67
12.0						1	6	6	4	6	6	2	2	2	1					1						37
12.5					1	3	5	4	4	9	5	1	2	1												35
13.0							3	3	5	2	3	2	2													20
13.5					1		2	3	5	2	3	3	1	1	1											22
14.0						1	1	2	1		2	2	1													10
14.5–15.0				1	1	1		1			2															6
Total...	16	263	821	1437	1948	2336	2455	2676	2591	2284	1639	1253	808	510	247	139	73	44	17	7	1	1	1	1	1	21,569

BUTTER-FAT PERCENTAGE AND AGE

Table 24 shows the association of 365-day butter-fat percentage with age corresponding to table 15 which shows the relation of milk yield and age. The interval chosen for age is the same in both tables. The interval for butter-fat percentage is 0.1 per cent.

TABLE 23

Means and standard deviations of milk yield for the different age groups

AGE	MEAN		STANDARD DEVIATION	
	Actual	Theoretical	Actual	Theoretical
1.75	290	272	50.5	51.3
2.25	316	332	54.5	57.8
2.75	340	367	61.6	61.8
3.25	385	391	69.7	64.8
3.75	404	409	68.9	67.2
4.25	433	423	69.2	69.1
4.75	447	435	72.6	70.7
5.25	458	444	70.9	72.1
5.75	465	451	73.4	73.1
6.25	468	458	75.7	74.0
6.75	468	462	71.8	74.8
7.25	467	465	78.2	75.3
7.75	460	467	70.2	75.6
8.25	472	469	80.3	75.8
8.75	469	470	78.0	75.9
9.25	466	469	69.7	75.7
9.75	459	468	74.5	75.5
10.25	463	467	79.4	75.1
10.75	461	465	72.2	74.6
11.25	458	462	79.4	73.9
11.75	446	458	68.1	73.1
12.25	456	453	75.1	72.2
12.75	430	449	61.7	71.2
13.25	445	444	56.8	70.0
13.75	454	438	69.0	68.8
14.25	447	431	64.5	67.5
14.75	380	425	84.2	66.0

The mean butter-fat percentage is 3.428; the standard deviation is 0.309. If we compare the coefficient of variation with that for the milk the coefficients are found to stand in the relation of 1 to 2.7. From this it may be argued that butter-fat percentage is much

less variable within this group of Holstein-Friesian cows than is the milk yield. This conclusion is in practical agreement with that found for other data comparing milk yield and butter-fat percentage.

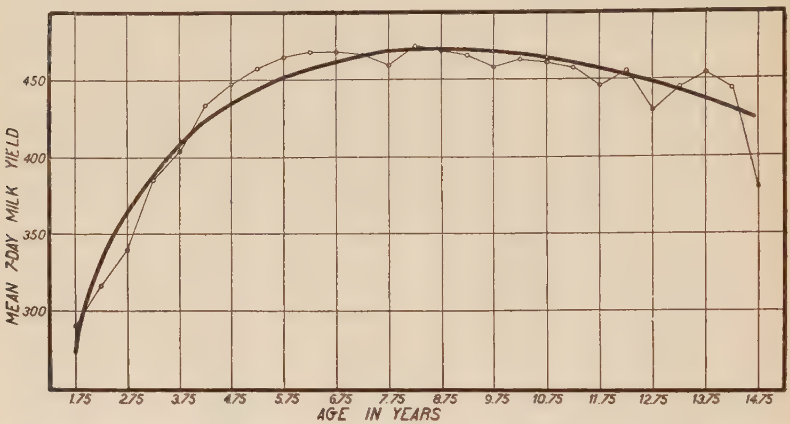

Fig. 7. Curves Showing the Relation of 7-day Milk Yield to Age

The actual curve is represented by the rough line, the fitted observations by the smooth curve.

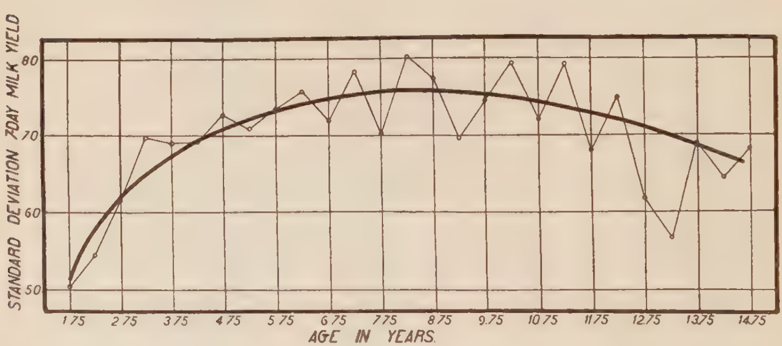

Fig. 8. Curves Showing the Relation of the Standard Deviations of 7-day Milk Yield to Age

The rough line represents the observational standard deviations, the smooth curve the theoretical observations found from the equation given above.

TABLE 24

Correlation surface showing the association of age and the butter-fat percentage of the 365-day record for Holstein-Friesian cows

Butter-fat percentage	Age at commencement of test																												Total
	1.5–2.0	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0–15.5	
2.3–2.4																													
2.4		1					1																						2
2.5		1										1																	2
2.6			1	2			3									1													7
2.7	1	5	4	2	5	2		2	3		1	1	3	1															30
2.8		8	3	3	6	4	3	4	3	1	2	1	3	2		1	2	1											47
2.9	2	17	6	10	7	6	9	11	1	6	3	1	1	1	2		2	1				1						1	88
3.0	5	30	18	11	17	10	10	9	10	9	8	8	2	7	1	5	1	2			1	1	1					1	167
3.1	4	45	21	22	26	15	19	26	9	16	9	13	7	3	7	2	3	1	1	1		1							251
3.2	4	60	24	10	21	28	24	22	25	15	10	10	4	8	6	5	2	4	2	3		1		1					289
3.3	14	54	43	36	37	22	15	28	23	19	21	18	5	11	12	5	5	2	2		1	1	2						376
3.4	6	60	42	21	33	18	22	15	25	20	14	13	6	11	9	5	5	5	3	2		1							336
3.5	13	59	31	31	22	16	32	20	20	14	15	9	9	6	4	2	4	4	4		2								317
3.6	9	45	31	17	23	16	13	16	13	16	7	2	6	3	4	4		1			1		1						228
3.7	3	45	29	14	10	16	11	9	4	4	3	1	7	4	1	1		2											164
3.8		21	19	10	6	7	8	8	5	7	2	1	1	4	1	4			1										105
3.9	4	7	8	7	8	6	4	6	5	4	4	2	1				1	1						1					69
4.0	4	4	6	3	2	5	5	3	1		4	2									1								40
4.1	1	6	5	1	2	1		3	1	1	1	2	1	1	1		1												28
4.2	1	2	3	1	2			1	1	1		1	1		2														16
4.3		3	2		1	1		2		2				1															12
4.4		2		1		3				1																			7
4.5				1			1	1		1																			4
4.6–4.7				1																									1
Total........	71	475	296	204	228	176	180	186	149	137	104	86	57	63	50	35	26	24	13	6	6	6	4	2				2	2,586

The correlation coefficient between butter-fat percentage and age is -0.0675 ± 0.0133. The correlation coefficient is consequently slightly significant.

The mean butter-fat percentages for the different age groups as exhibited in table 24 are shown in table 25.

These observational means are shown as the small circles in figure 9. The ordinates are the percentages of butter-fat and the abscissas are the ages. These observations on butter-fat percentage clearly are linear in their relation to age. Such being the case the ordinary

TABLE 25

Mean 365-day butter-fat percentage of Holstein-Friesian cows at different ages

AGE AT TEST	MEAN BUTTER-FAT PERCENTAGE	AGE AT TEST	MEAN BUTTER-FAT PERCENTAGE
years		*years*	
1.5 to 2.0	3.489	8.0 to 8.5	3.399
2.0 to 2.5	3.432	8.5 to 9.0	3.416
2.5 to 3.0	3.483	9.0 to 9.5	3.376
3.0 to 3.5	3.442	9.5 to 10.0	3.342
3.5 to 4.0	3.398	10.0 to 10.5	3.388
4.0 to 4.5	3.449	10.5 to 11.0	3.442
4.5 to 5.0	3.409	11.0 to 11.5	3.300
5.0 to 5.5	3.417	11.5 to 12.0	3.533
5.5 to 6.0	3.410	12.0 to 12.5	3.200
6.0 to 6.5	3.443	12.5 to 13.0	3.350
6.5 to 7.0	3.414	13.0 to 13.5	3.600
7.0 to 7.5	3.366	13.5 to 14.0	
7.5 to 8.0	3.415	15.0 to 15.5	3.000

regression formula may be used to fit this curve. The equation to this curve is:

$$\text{Butter-fat percentage} = 3.470 - 0.009 \text{ age} \qquad (5)$$

There is consequently a slight decrease in the butter-fat percentage which a Holstein-Friesian cow is capable of giving as the age of that cow increases. That this change is slight may be seen from the fact that the decrease in butter-fat percentage from the age of one year and nine months to fifteen years and nine months is only 0.130 per cent as shown by the fitted curve of figure 9.

The low correlation of butter-fat percentage with age justifies the use of the raw butter-fat percentage record without further correc-

tion. The frequency distribution of these butter-fat percentages is consequently the same as that given in table 24 save for the addition of a few individuals subsequently recorded. The physical constants derived from the distribution are given in table 26.

The distribution showing graphically the variation of the butter-fat percentage in the Holstein-Friesian breed is presented in figure 10.

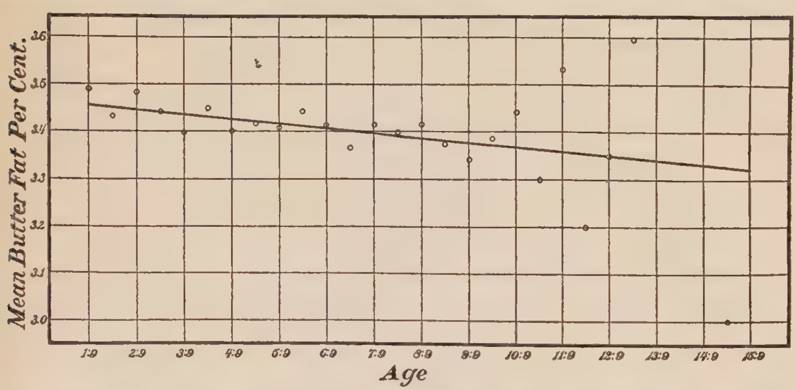

FIG. 9. OBSERVATIONAL AND FITTED CURVES SHOWING THE RELATION OF 365-DAY BUTTER-FAT PERCENTAGE TO AGE FOR HOLSTEIN-FRIESIAN CATTLE

The observational curve is represented by small circles. The smooth curve shows the fitted curve for butter-fat percentage. Age is in years and months.

TABLE 26

Physical constants for the butter-fat percentage of Holstein-Friesian cows

MEAN	STANDARD DEVIATION	COEFFICIENT OF VARIATION	OCTILE						
			First	Second	Third	Fourth	Fifth	Sixth	Seventh
3.430±0.004	0.315±0.003	9.18±0.09	3.090	3.217	3.320	3.406	3.501	3.614	3.784

SOLIDS-NOT-FAT PERCENTAGE AND AGE

Table 27 shows the relation of the percentage of solids-not-fat to milk yield. It will be noticed that there is one value (in parentheses) far removed from the distribution of the other entries. It seems desirable to exclude this value since it is probable that some error has crept into the determination of the solids for this test. In the calculations which follow, this value is not included.

The means and standard deviations of the solids-not-fat percentage and of age are given in the first chapter, table 6. The correlation coefficient for the relation of solids-not-fat percentage and age is -0.161 ± 0.036. The correlation ratio is 0.234 ± 0.035 and the correlation ratio squared minus the correlation coefficient squared is 0.028 ± 0.012, or it is reasonable to assume a linear regression line. The mean solids-not-fat percentage for each array of age is given in table 28.

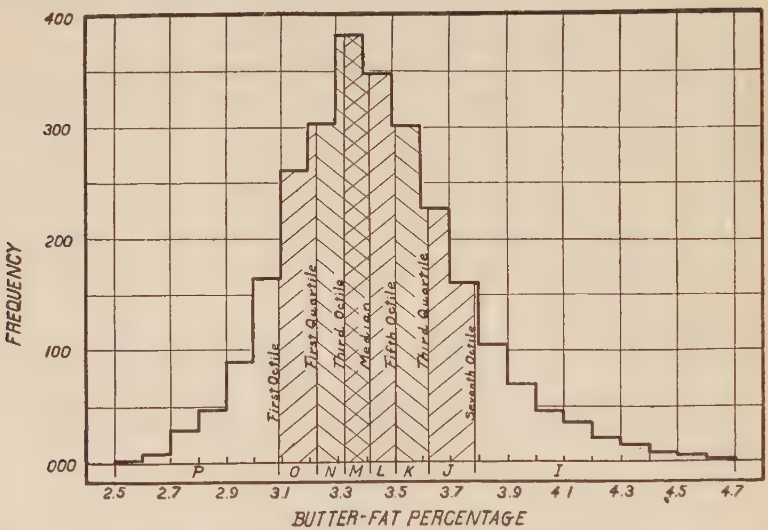

FIG. 10. HISTOGRAM SHOWING THE FREQUENCY DISTRIBUTION OF 365-DAY BUTTER-FAT PERCENTAGE FOR HOLSTEIN-FRIESIAN ADVANCED REGISTRY COWS

The octile lines divide these distributions into eight equal parts.

The raw average solids-not-fat percentages are given in column two of table 28. The ages are the mid-points of each class of table 27. Column three gives the calculated averages for the solids-not-fat percentages for each age. The equation from which these calculated values are derived is:

$$\text{Solids-not-fat percentage} = 8.71 - 0.024 \text{ age} \tag{6}$$

where the age is in years. Figure 11 shows the observed and fitted percentages of solids-not-fat for each age.

TABLE 27

Correlation surface for the variables, age at test and percentage of solids-not-fat, for the semi-official year records of Holstein-Friesian cows

AGE AT TEST	PERCENTAGE OF SOLIDS-NOT-FAT																					Total
	7.0–7.2	7.2–7.4	7.4–7.6	7.6–7.8	7.8–8.0	8.0–8.2	8.2–8.4	8.4–8.6	8.6–8.8	8.8–9.0	9.0–9.2	9.2–9.4	9.4–9.6	9.6–9.8	9.8–10.0	10.0–10.2	10.2–10.4	10.4–10.6	10.6–10.8	10.8–11.0	11.0–11.2	
years																						
1.5 to 2.5	1			1		2	7	15	22	18	12	4										82
2.5 to 3.5					3	6	1	14	18	15	3			1								61
3.5 to 4.5						4	9	15	14	7	6	1										56
4.5 to 5.5			1		1	4	6	11	8	10	1	1										43
5.5 to 6.5					2	5	3	10	9	4	1											34
6.5 to 7.5					1	5	4	5	4		3											22
7.5 to 8.5						2	1	4	1	2												10
8.5 to 9.5					1	2	1	3	5	2	1										(1)	16
9.5 to 10.5					1	1		1	2		1											6
10.5 to 11.5					1					1												2
11.5 to 12.5							2		1													3
Total........	1		1	1	10	31	34	78	84	59	28	6		1							1	335

TABLE 28

Average solids-not-fat percentage for each age at commencement of test

AGE	MEAN SOLIDS-NOT-FAT PERCENTAGE	
	Observed	Calculated
years		
2	8.71	8.66
3	8.63	8.64
4	8.62	8.61
5	8.56	8.59
6	8.51	8.56
7	8.46	8.54
8	8.50	8.52
9	8.55	8.49
10	8.50	8.47
11	8.40	8.44
12	8.43	8.42

BUTTER-FAT AND AGE

The correlation surface showing the relation of Holstein-Friesian butter-fat yield in pounds for the 365-day lactation period with the age when the lactation commenced is given in table 29. This table was made at a later date and with slightly different data from those of the tables for milk yield and butter-fat percentage.

The average butter-fat production of these cows is 559 pounds for the year period. The standard deviation of the butter-fat is 146 pounds. The correlation coefficient of butter-fat and age is 0.376. The regression is a skew one so that this correlation does not indicate the complete relation of butter-fat and age. The correlation coefficient is sufficiently large, however, to indicate that the two variables are quite closely associated.

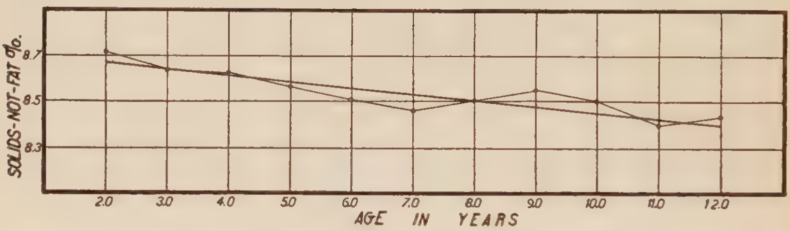

Fig. 11. The Relation of Percentage of Solids-not-fat to Age

The raw averages are shown as circles, the straight line is derived from equation (6).

From table 29 we find the average butter-fat productions for each age group to be those shown in column two of table 30. The third column of table 30 shows the butter-fat yields on the basis of the curve fitted to the raw means shown in column two.

Figure 12 shows the raw mean butter-fat productions for each age as small circles. The smooth curve is a logarithmic curve determined from these raw means. The equation to this curve is:

$$\text{Butter-fat production} = 352.2 + 60.56a - 4.056a^2 + 76.3 \log (a - 1.25) \quad (7)$$

where a is the age in years. This curve fits the data well. From the equation we may determine the age of maximum butter-fat production. This age is shown to be eight years and twenty-three days. The average production at this age is 641 pounds.

TABLE 29

Correlation surface showing the relation of age to butter-fat production

AGE	BUTTER-FAT PRODUCTION IN POUNDS																				TOTAL
	250–300	300	350	400	450	500	550	600	650	700	750	800	850	900	950	1000	1050	1100	1150	1200–1250	
years																					
1.5–2.0	6	7	13	14	12	5	4	2													63
2.0	10	40	64	95	95	67	36	23	19	5	6	1		1	1						463
2.5	3	11	47	44	55	49	33	24	12	14	5	2									299
3.0	1	10	16	31	33	46	29	10	13	9	6	3	2	1				1			211
3.5		6	18	31	45	30	28	27	22	4	5	3	1	1							221
4.0		3	18	13	21	29	21	29	16	11	3	8	2		2			1			177
4.5		1	11	18	22	32	21	26	16	12	5	9	2	3		1					179
5.0			10	15	21	39	35	18	17	9	8	6	4	1	3	3	1		1	1	192
5.5			6	5	18	17	22	21	18	13	11	5	3	4	3	2	1				149
6.0			5	6	13	14	25	20	16	16	8	8	5	3	4	1	2				146
6.5			2	5	11	18	12	20	14	9	7	3	6	2	2	2	2				115
7.0			1	4	12	17	11	13	13	5	4	8	6	2	1						97
7.5			1	4	5	7	8	6	4	13	4	4	1	1	2						60
8.0			2	4	5	10	7	14	11	5	3	6	4	3	2	2	1				79
8.5				4	8	5	6	9	9	5	2	3	2		1						54
9.0				1	3	8	7	5	6	3	2	1	2								38
9.5				2	4	4	2	2	5	1	2	3	1								26
10.0				5	5	3	3	4	3	2	2					1					28
10.5				2	2	1	2	1	1												9
11.0					2	3			1					2		1					9
11.5				1	1			1	1	1			1								6
12.0			1		1	3		1													6
12.5					1					1											2
13.0						1															1
13.5																					
14.0																					
14.5																					
15.0				1	1																2
15.5																					
16.0																					
16.5–17.0			1																		1
Total..	20	78	216	305	396	408	312	276	217	138	83	73	42	24	21	13	7	2	1	1	2633

As the butter-fat production is simply the multiple of the milk yield and butter-fat percentage it seems a work of supererogation to carry this analysis further, as the chapters to come will only deal with the two basic variables and not the butter-fat as such.

TABLE 30

Mean and fitted mean butter-fat productions for each age group

AGE	BUTTER-FAT PRODUCTIONS	
	Raw means	Fitted means
years		
1.75	423	422
2.25	473	468
2.75	503	501
3.25	530	529
3.75	534	553
4.25	572	573
4.75	581	590
5.25	599	604
5.75	633	616
6.25	646	626
6.75	644	633
7.25	632	637
7.75	646	640
8.25	665	640
8.75	623	638
9.25	620	634
9.75	625	627
10.25	584	620
10.75	530	609
11.25	675	596
11.75	634	581
12.25	509	565

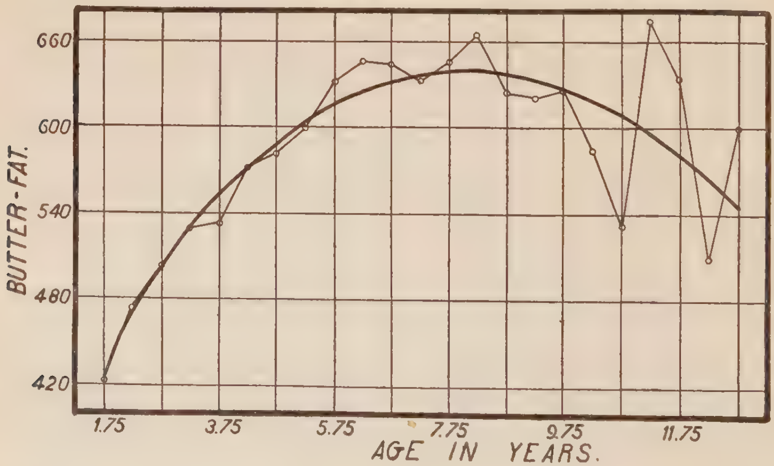

FIG. 12. RAW AND FITTED MEAN BUTTER-FAT PRODUCTION FOR EACH AGE

7-DAY BUTTER-FAT YIELD AND AGE

Like the previous data on 7-day milk yield the information on the relation of the 7-day butter-fat to age was taken from the first 24 volumes of the Advanced Register. The table given below shows the distribution of butter-fat yield for each age test. Such a table finds a use similar to that for milk yield in that it enables the reader to obtain easily some appreciation of the relative worth of an animal as compared to the rest of those in the Advanced Registry. Thus at nine years three months Tilly Alcartra had a butter-fat test of 32.6 pounds or as table 31 shows the record is well up with the leaders in the breed, a natural expectation considering that she has one of the best butter-fat records of the breed for the 365-day period.

From the data of table 31 the means and standard deviations of butter-fat yields for each age are obtained and shown in table 32.

The first column of table 32 shows the mean age of the groups of cows. The second column gives the actual butter-fat yields of each group for the 7-day period. The theoretical curve showing the relation of the butter-fat yields to age is derived by fitting the observations of column two by the method of least squares. The equation to the curve is:

$$\text{7-day butter-fat} = 11.68 + 0.020a - 0.031a^2 + 7.89 \log (a - 1.25) \qquad (8)$$

where a is the age in years. The theoretical curve is shown in the third column. The age of maximum butter-fat production is found by differentiation of this curve to be eight years two months and twenty-six days. At this age the mean production of butter-fat is 16.4 pounds. The observational and fitted curves are shown in figure 13.

SOLIDS-NOT-FAT AND AGE

Table 33 shows the mean solids-not-fat for each age of the array. These raw means are based on relatively small numbers of individuals so that there is a good deal of extraneous variation due to random sampling. The mean solids-not-fat of the group is 1308 pounds with a standard deviation of 259 pounds.

Table 33 shows the raw mean solids-not-fat and the fitted mean solids-not-fat for each age group starting at one year and nine months. The curve is on the whole remarkably smooth considering

Correlation surface showing the relation of 7-day butter-fat to age

AGE	7-DAY BUTTER-FAT IN POUNDS																													TOTAL
	7.0–8.0	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35.0–36.0	
years																														
1.0–1.5		1		1	1						1																			4
1.5	160	257	193	189	126	81	44	22	12	8	2	1																		1,095
2.0	158	545	673	664	570	429	265	164	94	69	36	16	3	4		1	1	1												3,693
2.5	12	220	386	411	357	333	208	148	80	94	38	18	10	11	4															2,330
3.0	1	13	173	229	292	271	260	256	168	158	88	62	31	13	5	4	6	6	3		1	1								2,041
3.5			33	191	281	290	275	249	166	152	109	69	48	39	19	16	2	10		1										1,950
4.0			2	46	144	211	218	220	186	205	114	99	57	55	19	20	14	7	4	1	3	2								1,627
4.5					57	223	232	224	181	188	136	93	77	60	34	22	17	14	3	2	1		1							1,565
5.0			1		2	121	175	160	141	163	121	86	69	57	31	26	13	16	8	3	1								1	1,195
5.5					2	125	136	167	139	152	131	100	50	47	31	23	12	9	5	4	4	3	2							1,142
6.0						99	97	118	112	121	96	71	52	48	26	13	7	12	5	1	1	3								882
6.5						87	97	124	120	133	79	62	49	37	25	10	7	10	7	2			1							850
7.0					1	78	93	85	93	87	74	45	37	34	15	15	10	8	3	3	1		2	1						685
7.5						87	82	87	76	79	67	32	28	18	16	7	5	8	2	2	1									597
8.0				1		50	42	58	50	48	48	50	29	16	15	8	3	10	2	1		1	1				1			434
8.5						48	63	72	42	48	46	38	22	20	9	8	5	7	1	1	1					1				432
9.0						32	26	41	31	35	35	16	10	9	6	2		3	2		1									249
9.5						34	37	42	31	23	31	14	12	8	5	4	1	1												243
10.0						27	18	13	20	26	11	6	9	10	4		1	2	2											149
10.5						20	20	27	17	22	12	4	6	3	4	1			1											137
11.0						14	11	8	7	12	6	6	3	3	1		1													72
11.5						15	16	7	12	6	1	4		5	1															67
12.0						8	7	2	6	5	3	3		2				1												37
12.5						8	4	5	7	7	1	1	2																	35
13.0						4	4	6	2	1	1			1	1															20
13.5						3	1	5	3	4		4	2																	22
14.0						2		4		3				1																10
14.5–15.0						3	1	2																						6
Total	331	1036	1461	1732	1833	2703	2432	2316	1796	1849	1287	900	606	501	271	180	105	125	48	21	15	10	7	1		1	1		1	21,569

the limited nature of the data. The equation describing the relation of age to the solids-not-fat is:

$$\text{Solids-not-fat} = 604 + 267a - 17.4a^2 - 158 \log (a - 1.25) \qquad (9)$$

TABLE 32

Means and standard deviations of butter-fat production for each age

AGE	MEAN BUTTER-FAT YIELD	
	Actual	Theoretical
years		
1.75	10.0	9.2
2.25	11.0	11.6
2.75	11.8	12.9
3.25	13.5	13.5
3.75	14.2	14.5
4.25	15.3	15.0
4.75	15.8	15.4
5.25	16.4	15.7
5.75	16.4	15.9
6.25	16.5	16.1
6.75	16.4	16.2
7.25	16.4	16.3
7.75	16.0	16.4
8.25	16.7	16.4
8.75	16.4	16.4
9.25	16.1	16.3
9.75	15.8	16.2
10.25	16.1	16.1
10.75	15.6	16.0
11.25	15.7	15.8
11.75	15.0	15.7
12.25	15.4	15.5
12.75	15.0	15.2
13.25	14.9	15.0
13.75	15.8	14.7
14.25	15.3	14.4
14.75	13.4	14.1

where a is the age in years. Figure 14 shows the observational means and the fitted means as determined from the above equation.

From the equation given above we find that the age of maximum production is equal to seven years four months and one day. This age is slightly less than that found for the milk yield or the butter-fat.

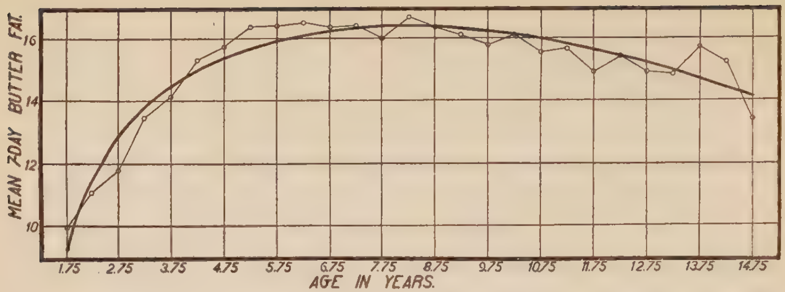

FIG. 13. OBSERVATIONAL AND FITTED CURVES SHOWING THE RELATION OF 7-DAY BUTTER-FAT PRODUCTION TO THE AGE OF THE COW

The rough line shows the actual observations. The smooth line represents the curve fitted to these observations.

TABLE 33

Mean solids-not-fat and standard deviation of solids-not-fat at successive ages

AGE	SOLIDS-NOT-FAT	
	Observed mean	Calculated mean
years		
1.75	1050	1066
2.25	1146	1117
2.75	1143	1179
3.25	1278	1241
3.75	1359	1298
4.25	1309	1349
4.75	1361	1394
5.25	1458	1431
5.75	1437	1460
6.25	1465	1482
6.75	1392	1496
7.25	1460	1501
7.75	1850	1499
8.25	1530	1488
8.75	1470	1468
9.25	1483	1440
9.75	1400	1404
11.03	1250	1276

This difference might be expected, however, in view of the greater negative correlation of solids-not-fat per cent with age.

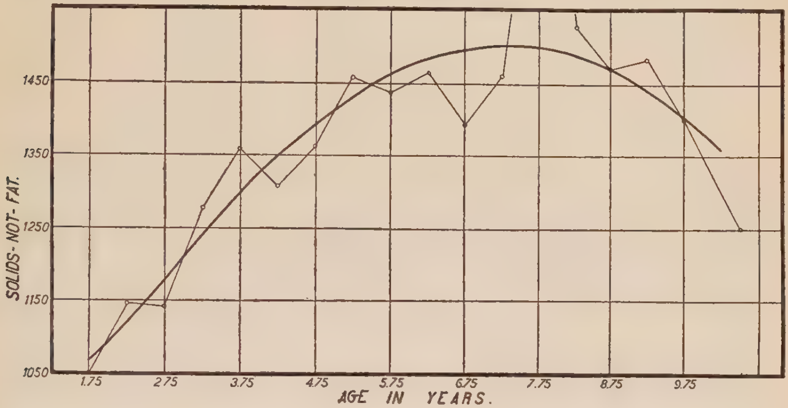

FIG. 14. OBSERVATIONAL AND FITTED CURVES FOR THE RELATION OF THE SOLIDS-NOT-FAT TO THE AGE OF THE COW

SUMMARY

The data to determine the relation of milk yield, butter-fat, solids-not-fat, butter-fat percentage, and solids-not-fat percentage to the age of the cow are presented in this section. The equations for the relations are given. Correction factors and tables for the 365-day records are presented for determining the equivalent maximum production from the milk yield at another age.

CHAPTER V

Mode of Secretion of Milk and Milk Solids

Before 1850 the prevailing opinion held that the milk solids were filtered out by the mammary gland from the blood serum. This view was shown to be incorrect by the fact that lactose is not present in the blood and the fat percentage of the serum is not sufficient to account for the fat in a single milking. To replace this old theory, three major hypotheses have been put forth to account for the secretion of the mammary gland:

1. Cells of the gland break loose bodily and disintegrate in the alveoli to form the milk solids.

2. The portion of the cells toward the alveoli becomes loaded with solids, breaks loose from the basal portion, and disintegrates to form the milk solids.

3. The cells of the mammary gland secrete the materials of the milk solids without themselves breaking down.

In opposition to the first theory, it may be said that no such extensive cell multiplication is witnessed in the mammary gland as would be necessary to replace the cell destruction called for by this theory. This disintegration, as pointed out by Heidenhain[1] for the milk produced by some cows in one day would require the replacing of all cells in the udder at least five times a day, a replacement of cells unprecedented in our knowledge of cell division.

The second theory, suggested by Langer and ably supported by Heidenhain,[2] Steinhaus[3] and Brouha[4] lays its foundation on histological evidence. According to this evidence, the gland cells lengthen

[1] Heidenhain, R. 1883. Die Milchabsonderung. Hermann, Handbuch der Physiologie. Bd. 5, F. 1, pp. 380.

[2] Loc. cit.

[3] Steinhaus, Julius. 1892. Die Morphologie der Milchabsonderung. Arch. Anat. u. Physiol., 1892, Sup. Bd. Physiol. Abt., pp. 54–68, pl. 5–7.

[4] Brouha. 1905. Recherches sur les diverses phases du developpement et de l'activite de la mamella. Arch. Biol., t. 21, pp. 459–603, pl. 18–20. Travaux renseignes, pp. 591–596.

out into the lumen of the alveoli. The projecting ends of these cells become loaded with nutrients similar to milk solids. These projecting ends disintegrate to allow the escape of these solids. The basal portions, including a nucleus, are left to rebuild the cell and to enable it to repeat the process. Steinhaus says that, in order to support this rebuilding, mitotic divisions are frequent, and that the daughter nuclei which lie on the outer portion of the cell often degenerate.

The third theory lays its stress on analogy with the other secretory glands without other supporting evidence than the negative evidence of Bertkau,[5] who says the disintegration appearing in the secretory cells is due to imperfect fixation and that no necrobiosis of any kind appeared.

Before examining the new evidence to be presented on this question it seems desirable to consider a difficulty of the method used by the above investigators. Examination of the cow's udder shows that the cells of the actively lactating mammary gland of a Holstein-Friesian cow are quite small. Considered in the light of this small size, it is likely that observations on the distal end of a cell might be called by one observer the destruction of this portion and by another, the cell in its natural shape. This difficulty in the way of the histologist is not to be minimized.

The variation of the constituent parts of milk and the milk quantity offers a means of approach to the problem. The variables which we propose to study are milk yield, age, amount of solids-not-fat and butter-fat. Tables for the interrelation of these variables have been presented elsewhere.[6] While reëxamination of the data has added perhaps a half dozen observations and corrected slight errors in others the end results are so nearly the same that it does not seem desirable to go to the expense of reproducing the correlation tables. The constants as now obtained are given in table 34.

The interrelation of three of these variables, milk yield, butter-fat, and solids-not-fat, is very close indeed. The correlation of age to the other three is much less. Considerable importance is attached to the significance of the differences between these correlation

[5] Bertkau, F. 1907. Ein Beitrag zur Anatomie und Physiologie der Milchdruse. Anat. Anz., Bd. 30, No. 7/8, p. 161–180, 7 fig. Literatur, pp. 179–180.

[6] Gowen, John W. 1919. Variations and mode of secretion of milk solids. Jour. Agri. Research, vol. xvi, no. 3, pp. 79–102.

coefficients. The difference between the correlation coefficients for milk yield and solids-not-fat 0.9655, and milk yield and butter-fat 0.8927, is 0.0728 ± 0.0079 or the difference is 9.2 times its probable error.[7] The correlation between milk yield and solids-not-fat is consequently significantly greater than the correlation between the milk yield and the butter-fat. The importance of this should be emphasized because of what is to follow. The correlation of the solids-not-fat with the butter-fat is not significantly different from the correlation of the milk yield with the butter-fat. The correlation of the milk yield with the solids-not-fat is significantly larger than the correlation of the solids-not-fat with the butter-fat.

TABLE 34

Constants showing the means, standard deviations, coefficients of variation, and correlation coefficients for milk yield, age, solids-not-fat, and butter-fat

Characters	Mean	Standard deviation	Coefficient of variation
Milk yield	15205 ±108	2945 ±77	19.4±0.5
Age	4.53± 0.08	2.29± 0.06	50.5±1.6
Solids-not-fat	1308 ± 9	259 ± 7	19.8±0.5
Butter-fat	519 ± 4	104 ± 3	20.0±0.5
	Correlation coefficient	**Correlation ratio**	
Milk yield and age	0.4771±0.0284	0.6006±0.0235	0.5021±0.0275
Milk yield and solids-not-fat	0.9655±0.0025	0.9681±0.0023	0.9666±0.0024
Milk yield and butter-fat	0.8927±0.0075	0.9047±0.0067	0.9022±0.0069
Age and solids-not-fat	0.4240±0.0302	0.5967±0.0237	0.4688±0.0287
Age and butter-fat	0.4564±0.0291	0.5628±0.0251	0.4747±0.0285
Solids-not-fat and butter-fat	0.9158±0.0059	0.9277±0.0051	0.9232±0.0054

The correlations of the age with the other three variables are about half the size of the correlations of the three variables among them-

[7] This difference is probably not so significant as would appear from the probable error. *Soper et al.* have shown that the distribution for correlation coefficients of 0.9 and *n* of 400 is not exactly Gaussian. The probable error should be somewhat larger consequently. See Soper, H. E., Young, A. W., Cove, B. M., Lee, A., Pearson, K., on the distribution of the correlation coefficient of small samples. Biometrika, vol. xi, pp. 328–413.

selves. The correlation ratios for age and the other variables show that at least one of the regression lines is not linear. The greatest departure from linearity occurs for the relation of milk yield to age. We are going to handle the problem as if the regressions were linear, however, in view of the difficulty in handling the partial correlation ratios of the higher orders. While all the influence of age on the other variables will not be accounted for by this method, the method will present a result which will be correct to at least a first approximation. In fact it seems probable in view of the fact that age is correlated with the three variables to nearly the same degree, in no

TABLE 35

Partial correlation coefficients showing the relation of two of the four variables—milk yield, age, solids-not-fat, and butter-fat for a constant third variable

VARIABLES CORRELATED	CONSTANT VARIABLE	CORRELATION COEFFICIENT
Milk yield × age	Butter-fat	0.174±0.036
Milk yield × age	Solids-not-fat	0.289±0.034
Milk yield × butter-fat	Age	0.863±0.009
Milk yield × butter-fat	Solids-not-fat	0.082±0.037
Milk yield × solids-not-fat	Age	0.959±0.003
Milk yield × solids-not-fat	Butter-fat	0.820±0.012
Age × butter-fat	Milk yield	0.077±0.037
Age × solids-not-fat	Milk yield	−0.161±0.036
Age × butter-fat	Solids-not-fat	0.187±0.036
Age × solids-not-fat	Butter-fat	0.017±0.037
Butter-fat × solids-not-fat	Milk yield	0.463±0.029
Butter-fat × solids-not-fat	Age	0.896±0.007

case are the differences significant, that the variable age might be dropped altogether.

The first order correlation coefficients showing the relation of two of the four variables for a constant third are given in table 35.

Several significant points are brought out in table 35. Let us consider first the three variables, milk yield, butter-fat, and solids-not-fat, with linear regression lines. When the correlation between the butter-fat and solids-not-fat for a constant quantity of milk is thus measured, it is found that the partial correlation coefficient is 0.463 ± 0.029. This correlation shows that the production by the mammary gland of butter-fat and of solids-not-fat are correlated functions. This correlation being plus it means that an increase in

the liberation of either constituent of cow's milk means a coincident increase in the other. This conclusion is important, as it shows that the factors responsible for the increase in the content of butter-fat for a given volume of milk are in a high degree responsible for the increase in the solids-not-fat content in this same milk. It means that the physiology of the mammary gland in elaborating the milk solids is such that the release of a certain amount of butter-fat to the milk also releases a proportionate amount of solids-not-fat.

The correlation coefficient for butter-fat with milk yield for a constant solids-not-fat is practically zero, 0.082 ± 0.037. That is, all the correlation between milk yield and butter-fat yield is due to the correlation of these variables with the solids-not-fat. Looked at from another angle, milk yield and butter-fat vary independently in milks of a constant solids-not-fat. Or from still another standpoint, milk yield is controlled largely by the solids-not-fat output and solids-not-fat output is controlled by milk yield. The solids-not-fat at the same time controls part of the butter-fat output. The solids-not-fat has a correlation of 0.463 with butter-fat for a constant milk, or in terms of variation, when the solids-not-fat and milk yield are constant the variation of the butter-fat is reduced to 41.4 as against 103.8 where the solids-not-fat and milk yield are not constant. Although age has a rather large correlation with these variables it is rather insignificant in its influence on milk yield as compared with the other highly correlated variables already discussed.

The second order correlation coefficients for the relation of two of the variables for the other two constant are given in table 36. This table shows that milk yield has a very close relation to solids-not-fat even when ages of the cows and their butter-fat yields are made constant. The milk yield and butter-fat vary practically independently when the solids-not-fat and the ages are made constant. A fair sized correlation remains between the milk yields and ages even when the solids-not-fat and butter-fat yields are made constant.

The solids-not-fat and the butter-fat yields are closely correlated, 0.48, even though the milk yield and age are made constant. The solids-not-fat decline with age for cows of constant milk yield and butter-fat percentage.

Curiously enough in view of the negative correlation of milk yield with butter-fat percentage and of age with butter-fat percentage the relation of butter-fat and age for a constant milk yield and solids-not-fat is positive. The correlations are all small and scarcely significant, however.

From these data it is clear that the major variable influencing milk yield is the solid-not-fat. The solids-not-fat is composed of several different substances generally classified under the titles of sugar, ash, casein, and albumin. All of these substances may be jointly responsible for milk yield or it may be that only one of them is responsible. Gaines[8] and others have put forward the view that milk yield is due to the osmotically active substances in milk. On

TABLE 36

Second order correlation coefficients for the relation of two variables for the other two constant, the variables being, milk yield, butter-fat, solids-not-fat, and age

VARIABLES CORRELATED	CONSTANT VARIABLES	SECOND ORDER CORRELATION COEFFICIENTS
Milk yield and solids-not-fat	Butter-fat and age	0.828 ± 0.012
Milk yield and butter-fat	Solids-not-fat and age	0.030 ± 0.037
Milk yield and age	Solids-not-fat and butter-fat	0.280 ± 0.034
Solids-not-fat and butter-fat	Milk yield and age	0.481 ± 0.028
Solids-not-fat and age	Milk yield and butter-fat	-0.223 ± 0.035
Butter-fat and age	Milk yield and solids-not-fat	0.172 ± 0.036

the other hand if we should adopt the cell destruction theory either for the whole cell or for part of it, the protein elements would probably be the controlling factors for milk yield. It takes other evidence to settle the problem. This evidence is being collected.

From the data presented it is possible to form prediction equations to determine the average milk yield, etc., of cattle when other information such as solids-not-fat is known. As these equations are frequently useful they are given below for milk yield, butter-fat and solids-not-fat. It should be remembered that all of the influence of age is not corrected for by linear equations.

[8] Gaines, W. L. 1922. The inheritance of fat content of milk in dairy cattle. Proc. Amer. Soc. Animal Production for 1922, pp. 29–32.

$$\text{Milk yield} = 865 + 11.0 \text{ solids-not-fat} \quad (10)$$
$$\text{Milk yield} = 2056 + 25.33 \text{ butter-fat} \quad (11)$$
$$\text{Milk yield} = 748 + 10.46 \text{ solids-not-fat} + 1.50 \text{ butter-fat} \quad (12)$$
$$\text{Milk yield} = 2137 + 24.19 \text{ butter-fat} + 113.29 \text{ age} \quad (13)$$
$$\text{Milk yield} = 902 + 10.57 \text{ solids-not-fat} + 106.43 \text{ age} \quad (14)$$
$$\text{Milk yield} = 827 + 0.554 \text{ butter-fat} + 10.41 \text{ solids-not-fat} + 104.6 \text{ age} \quad (15)$$
$$\text{Solids-not-fat} = 14.0 + 0.085 \text{ milk yield} \quad (16)$$
$$\text{Solids-not-fat} = 120.0 + 2.289 \text{ butter-fat} \quad (17)$$
$$\text{Solids-not-fat} = 1090.5 + 48.0 \text{ age} \quad (18)$$
$$\text{Solids-not-fat} = 0.062 \text{ milk yield} + 0.665 \text{ butter-fat} - 13 \quad (19)$$
$$\text{Solids-not-fat} = 8.04 + 0.087 \text{ milk yield} - 5.38 \text{ age} \quad (20)$$
$$\text{Solids-not-fat} = 121 + 2.28 \text{ butter-fat} + 0.859 \text{ age} \quad (21)$$
$$\text{Solids-not-fat} = 0.066 \text{ milk yield} + 0.681 \text{ butter-fat} - 6.633 \text{ age} - 16 \quad (22)$$
$$\text{Butter-fat} = 41.6 + 0.031 \text{ milk yield} \quad (23)$$
$$\text{Butter-fat} = 39.8 + 0.366 \text{ solids-not-fat} \quad (24)$$
$$\text{Butter-fat} = 425.3 + 20.69 \text{ age} \quad (25)$$
$$\text{Butter-fat} = 28.5 + 0.0045 \text{ milk yield} + 0.3227 \text{ solids-not-fat} \quad (26)$$
$$\text{Butter-fat} = 42.6 + 0.0308 \text{ milk yield} + 1.791 \text{ age} \quad (27)$$
$$\text{Butter-fat} = 41.1 + 0.3523 \text{ solids-not-fat} + 3.767 \text{ age} \quad (28)$$
$$\text{Butter-fat} = 36.1 + 0.0017 \text{ milk yield} + 0.3370 \text{ solids-not-fat} + 3.59 \text{ age} \quad (29)$$

By way of illustrating the use of these equations we may take the case of a cow producing 1500 pounds of solids-not-fat and determine the probable milk yield of the cow for the year period. Equation (10) gives the necessary data, $11.0 \times 1500 = 16{,}500 + 865 = 17{,}365$ pounds of milk for the year period.

Equation (15) is perhaps the most interesting since it includes all four variables, milk yield being the predicted variable. By the use of this equation we may find the relative influences of the butter-fat as distinct from solids-not-fat or age in determining milk yield, and in the same way of solids-not-fat distinct from the other two and of age apart from the other two. Let two cases be assumed, a cow whose solids-not-fat, butter-fat, and age were respectively 2650 pounds, 1000 pounds, and 10 years, should produce 30,000 pounds of milk of which the solids-not-fat would determine 27,573 pounds, the butter-fat 554 pounds, the age 1046 pounds, and extraneous unknown causes 827 pounds. Or, the solids-not-fat would determine 91 per cent of the milk yield, the butter-fat 1.8 per cent, the age 3.5 per cent, and unknown causes 2.7 per cent. Another cow might produce 9000 pounds of milk with 750 pounds of solids-not-fat, 300 pounds butter-fat and may be 2 years of age. Under those circumstances the 9000 pounds of milk would be proportioned as follows; the solids-not-fat

account for 7800 pounds of milk, the butter-fat 166 pounds of the milk, the age 209 pounds, and extraneous causes 827 pounds. The solids-not-fat account for 87 per cent, the butter-fat 1.8 per cent, and the age 2.3 per cent. These ranges include practically the range of the breed in Advanced Registry milk yield. It is clear that the solids-not-fat control the major portion of the milk yield.

VARIATION IN MILK YIELD CONTROLLED BY THE THREE VARIABLES

The problem may be looked at from another angle—the amount of variation in milk yield which these three variables solids-not-fat, butter-fat, and age control. The data just presented allow the determination of some of these relations. The amount of variation re-

TABLE 37

Amount of variation in milk yield controlled by the solids-not-fat, butter-fat, and age, singly and jointly

STANDARD DEVIATION OF MILK YIELD	VARIABLES CONSTANT	VARIATION CONTROLLED BY CONSTANT VARIABLES
2945	None	000
761	Solids-not-fat	2184
1326	Butter-fat	1619
2588	Age	357
758	Solids-not-fat and butter-fat	2187
731	Solids-not-fat and age	2214
1306	Butter-fat and age	1639
730	Solids-not-fat, butter-fat, and age	2215

maining in one variable when a correlated variable is constant is equal to $\sigma\, 1\sqrt{1-r_{12}{}^2}$. The amount of variation remaining in one variable when the two other correlated variables are constant, is $\sigma\, 1\sqrt{1-r_{12}{}^2}\ \sqrt{1-r_{13\cdot 2}{}^2}$ and for the case where three other variables are constant, is $\sigma\, 1\sqrt{1-r_{12}{}^2}\sqrt{1r-_{13\cdot 2}{}^2}\sqrt{1-r_{14\cdot 23}{}^2}$. The amount of variation in milk yield controlled by the three other variables is shown in table 37.

The standard deviation for milk yield, when all correlated variables vary as they will, is 2945 pounds of milk. This standard deviation is over one thousand pounds of milk less than the standard deviation for all the Advanced Registry animals, or this group of cows is probably less variable in their milk yield than those of the whole Advanced Registry.

If the solids-not-fat yield be made constant the variation in the milk is reduced to 761 pounds, or the variation of the solids-not-fat account for 2184 pounds of the variation of the milk yield. In the same way if the butter-fat is made constant the variation in milk yield is reduced to 1326 pounds and we might consider the amount of milk controlled by the butter-fat to be 1619 pounds. Again the variation of the milk yield when the age is controlled is equal to 2588 pounds or the pounds of variation which we might consider as controlled by age are 357. Hastily considered it might seem possible by simply adding these amounts to obtain the variation in milk yield controlled by these three variables, but $2184 + 1619 + 357 = 4160$ pounds or a total of more than the original variation of the milk yields, 2945, an obvious absurdity. The variation controlled by the butter-fat, for instance, is also partly controlled by the solids-not-fat. That is, the variation of butter-fat and solids-not-fat is in part a joint variation so that some of the variation controlled by one is also controlled by the other.

Taking the variables two at a time the solids-not-fat and butter-fat when both are made constant reduce the variation of the milk yield to 758 pounds; the solids-not-fat and age made constant reduce the variation of the milk yield to 731 pounds; and the butter-fat and age made constant reduce the variation to 1306 pounds. The degree of control of the three pairs of variables are in order 2187, 2214 and 1639 pounds of milk. It is clear that the solids-not-fat part of the combination controls the major portion of the variation, the age next, and the butter-fat the least. This is illustrated when all three variables are made constant, the variation becoming 730 pounds and the degree of control 2215 pounds of milk.

The variation of the solids-not-fat is the major cause of the variation in milk yield as shown by these records. These data make it appear altogether probable that we must look to the solids-not-fat as the major cause of the variation in milk yield and to some extent as the cause of the variation in the butter-fat yield.

RELATION OF THE CONCENTRATION OF THE MILK SOLIDS TO MILK QUANTITY

The possible association of the concentration of the milk solids and milk quantity is important to what is coming in other chapters.

The data for this discussion, as presented elsewhere,[9] and as extended with the passage of time, may be briefly summarized as follows. The Holstein-Friesian cows show a correlation coefficient between the milk quantity and butter-fat percentage of -0.101 ± 0.013 for 2633 cows on the 365-day test. The correlation coefficient, while highly significant, shows that milk quantity has but little influence on the concentration of butter-fat in the milk of this breed. This conclusion finds confirmation in some of the work of Ellinger's on Red Danish cattle.[10] These cattle are quite comparable to the Holstein-Friesian in ancestry and consequently should be expected to have a comparable variation. The data appear to bear out this expectation in showing but a slight relation between milk yield and butter-fat percentage for these cattle. The Holstein-Friesian cattle seem to differ from the island breeds in this regard, as the island cattle have a correlation coefficient between their milk yield and butter-fat percentage of nearly three times that of the Holstein-Friesian cattle. The solids-not-fat percentage appears to have little relation to the milk quantity for Holstein-Friesian cow's milk, the correlation coefficient being but -0.055 ± 0.037. This conclusion might have been expected on other grounds.

SUMMARY

The current theories of milk secretion call for either a (1) complete or (2) partial destruction of the udder cells concerned in the formation of the milk solids or (3) the formation of milk as a true secretion. The evidence for and against these theories is reviewed. Further evidence is presented to show that the solids-not-fat are the controlling elements in milk yield.

[9] Loc. cit.

[10] Ellinger, Tage U. 1923. The variation and inheritance of milk characters. Proc. Nat. Acad. Sciences, vol. 9, no. 4, pp. 111–116.

CHAPTER VI

The Permanence of a Cow's Milk Yield or Butter-fat Percentage

To be worthy of consideration a record for milk yield or butter-fat percentage must do two major things: first it must predict with reasonable accuracy what the production of that cow will be in subsequent lactations; second, it should indicate, to some degree at least, what milk production may be expected of that cow's offspring. Only recently has the data been analyzed to determine how closely a record of a cow does these two things. An abstract and an extension of a recent bulletin[1] presenting the analysis of the first postulate will be given in this section.

Concisely, if two cows, of which one produces 30,000 and the other 10,000 pounds of milk as six-year-olds, each have exactly 18,000 pounds for their milk yields as seven-year-olds, and remain equal in their production for the rest of their working lives, it is obvious that as an indication of producing capacity the original six-year-old records are worthless. If the illustration be extended to include all the animals which have tests and it is found that the high milk producing cows of one lactation are no better than the low ones in subsequent lactations and vice versa, then no value can be attached to the productivity of a cow in any lactation, for such a record has no predictive value. Fortunately, the supposition of equal production of the 30,000-pound cow and the 10,000-pound cow in future lactations is not correct. There is a predictive value to a milk record. The 30,000-pound cows have higher average production in subsequent lactations than do the 10,000-pound cows. The problem with which we shall deal, is the degree of accuracy of this prediction and the variation which may be expected from one test to another. The 365-day records may be taken first.

[1] Gowen, Marie S., and Gowen, John W. 1922. Studies in milk secretion. XIII. Relation between milk yield and butter-fat percentage of the 7-day and 365-day tests of Holstein-Friesian Advanced Registry. Annual report of the Maine Agricultural Experiment Station for 1922, Bulletin, 306, pp. 21–60.

THE RELATION BETWEEN THE 365-DAY TEST AND RETEST RECORDS FOR MILK YIELD AND BUTTER-FAT PERCENTAGE

The data for the year tests have been divided into age groups. The reader will remember that Holstein-Friesian milk yield has been shown to increase at an ever declining rate until it reaches the maximum at eight years and five months. From this point the milk yield declines slightly as age advances. Since the age effect is greatest between 1.75 years and five years, the attempt was made to make as many age divisions, during this time, as the data would allow. At the more advanced ages, where the age effect is less, the divisions are made broader.

The mean milk yields for the first tests at the different ages, as given in table 38, are quite variable. Their average has a reasonable agreement with the milk yields for the whole breed as given in Chapter IV. This agreement would indicate that no selection of high milking cows for subsequent retest has been made.

If the mean milk yields of the retest cows are compared with the mean milk yields of the first test cows, the same ages being included in each group, it is found that the retest cows exceed the first test cows in milk yield. The excess milk yield for each age group is scarcely significant. Taken for all groups, where the data are weighted as the square root of n, the average difference between the test and retest milk yields is 833 pounds. This difference is probably significant. While not large such a difference points to the development of the udder as a factor to be taken into consideration in Advanced Registry testing.

The age of the Holstein-Friesian Advanced Registry cow does not materially affect the butter-fat percentage contained in her milk. Since this is the case the butter-fat percentage could be lumped together for age without influencing the results. However, since these age divisions were used for the division of milk yield, it seems desirable to continue their use throughout the investigation. The constants for the butter-fat percentages are comparable with those found for the whole breed.

TABLE 38

Physical constants for the variation of 365-day milk yield and butter-fat percentage of test and retest cows

AGE		MILK YIELD			BUTTER-FAT PERCENTAGE		
Test	Retest	Mean first test	Standard deviation first test	Coefficient of variation first test	Mean first test	Standard deviation first test	Coefficient of variation first test
years	*years*						
1.75– 3	3– 4	14,033±266	2,647±188	18.9±1.4	3.34±0.02	0.238±0.017	7.12±0.50
2 – 3	4– 5	12,935±301	3,026±213	23.4±1.7	3.43±0.03	0.332±0.023	9.67±0.71
2 – 3	5– 8	13,221±294	2,856±208	21.6±1.7	3.42±0.03	0.290±0.021	8.47±0.56
3 – 4	4– 5	15,167±600	3,771±424	24.9±3.0	3.53±0.05	0.307±0.035	8.70±1.02
3 – 4	5– 8	16,722±422	3,757±299	22.5±1.8	3.44±0.04	0.352±0.028	10.21±0.80
4 – 5	5–11	16,523±375	3,650±265	22.1±1.7	3.34±0.03	0.330±0.024	9.88±0.73
5 –11	6–13	19,157±382	4,726±269	24.7±1.5	3.42±0.02	0.284±0.017	8.31±0.46
		RETEST	RETEST	RETEST	RETEST	RETEST	RETEST
1.75– 3	3– 4	16,567±441	4,389±312	26.5±2.0	3.36±0.03	0.297±0.021	8.85±0.65
2 – 3	4– 5	17,652±421	4,237±298	24.0±1.8	3.43±0.03	0.329±0.023	9.61±0.71
2 – 3	5– 8	20,081±437	4,244±309	21.1±1.6	3.45±0.03	0.317±0.023	9.20±0.66
3 – 4	4– 5	17,000±621	3,905±439	23.0±2.7	3.47±0.05	0.300±0.034	8.65±1.02
3 – 4	5– 8	20,722±553	4,922±391	23.8±2.0	3.48±0.04	0.319±0.025	9.17±0.72
4 – 5	5–11	19,733±433	4,209±306	21.3±1.6	3.41±0.04	0.343±0.025	10.06±0.73
5 –11	6–13	20,586±356	4,417±252	21.5±1.2	3.39±0.02	0.284±0.016	8.38±0.46

THE CORRELATION OF ONE LACTATION RECORD WITH THAT OF ANOTHER

While these physical constants are important, our problem pertains chiefly to the association of one lactation record with that of another. The constants measuring this correlation are given in table 39.

The correlation coefficients for the milk yield of the 365-day test with the milk yield of a subsequent 365-day lactation are all rather high for data dealing with milk yield. The average correlation coefficient for this relation is 0.667. The weighted mean correlation

TABLE 39

Constants measuring the relation between the records for 365-day milk yield and butter-fat percentage of different lactations

AGE		MILK YIELD			BUTTER-FAT PERCENTAGE		
Test	Retest	Correlation coefficient	Correlation ratio	$\eta^2 - r^2$	Correlation coefficient	Correlation ratio	$\eta^2 - r^2$
years	*years*						
1.75–3	3–4	0.733±0.047	0.837	0.162±0.074	0.756±0.043	0.787	0.049±0.043
2 –3	4–5	0.705±0.050	0.780	0.112±0.063	0.789±0.038	0.859	0.115±0.063
2 –3	5–8	0.480±0.079	0.784	0.385±0.100	0.593±0.067	0.681	0.112±0.065
3 –4	4–5	0.777±0.062	0.919	0.240±0.136	0.783±0.062	0.843	0.097±0.094
3 –4	5–8	0.702±0.057	0.890	0.299±0.103	0.702±0.057	0.767	0.094±0.066
4 –5	5–11	0.584±0.068	0.769	0.250±0.089	0.640±0.061	0.767	0.178±0.079
5 –11	6–13	0.689±0.042	0.790	0.148±0.057	0.741±0.036	0.772	0.047±0.034
Mean.....		0.667	0.824		0.715	0.782	

coefficient for milk yield is 0.660. The correlation ratios are, as would be expected, slightly more than the correlation coefficients. The test for the linearity of the regressions, correlation ratio squared minus the correlation coefficient squared, justifies the use of linear equations.

The yearly butter-fat percentage of one lactation measures the butter-fat percentage of another lactation more accurately than a milk yield of one lactation measures the milk yield of another. The difference is not significant, however. The correlation coefficients in relation to the correlation ratios show that the regression lines are satisfactorily described by linear functions.

The relation of the performance of one lactation to another may be accounted for by heredity or environment or both acting jointly. Heredity is, of course, permanent throughout the life of the cow. Environment is variable and only that part of environment which tends to make the performance constant between lactations can account for the consistence of the milk yields of dairy cattle. Were we able to determine all possible correlations between heredity, permanent environment, and milk yield or butter-fat percentage we should expect the total correlation to be equal to that given above, 0.667 for milk yield and 0.715 for butter-fat percentage as a first approximation. As will be shown later this degree of correlation may be accounted for by these factors.

In another place (Chapter IV) the separate prediction equations for milk yield and butter-fat percentage have been formed for these variables. As pointed out there and as may be repeated here there is no apparent evidence to show that the relation of one lactation to another is different from one lactation to another. It is consequently entirely reasonable to consider that the average correlation coefficient is the most accurate measure of the relation of the performance of one lactation to that of another. Age has already been shown to materially influence the mean milk production of dairy cattle, a logarithmic curve describing the relationship. The standard deviation of milk yield is also influenced by age, a parabola describing the relationship. With the knowledge of these curves we could make an equation to predict milk yield at one age when the milk yield of another is known. This equation is:

$$\text{Milk yield } (A \text{ age}) = C - 0.667\frac{D}{E}F + 0.667\frac{D}{E}\text{ milk yield } (a \text{ age}) \quad (30)$$

when $C = 9432 + 2070A - 128.9A^2 + 1548 \log (A - 1.25)$
$D = 1682 + 652A - 44.6A^2$
$E = 1682 + 652a - 44.6a^2$
$F = 9432 + 2070a - 128.9a^2 + 1548 \log (a - 1.25)$
A = age to which milk yield is predicted
a = age when actual milk yield was procured.

This equation furnishes a means of correcting milk records for the influence of age and also for those fluctuating environmental factors which make a record low or high. It seems probable that the results

from the use of such an equation in predicting individual milk yields would be more nearly those actually produced by the cows than would those derived from only the correction on the basis of the age curve for milk yield. This would be particularly true for the extremely high or low producers in any one lactation. The case may be illustrated as follows: A cow produces 6000 pounds as a two-year-old, what will be her probable production as a 8.25-year-old? On the basis of the above equation where $C = 19{,}041$, $D = 4023$, $E = 2807$ and $F = 12{,}862$ the probable milk yield would be 12,480 pounds. On the basis of the mean curve the production would be 8880 pounds. For a cow producing 22,000 pounds as a two-year-old the equation above gives 27,779 pounds and that of the mean curve gives 32,560 pounds. The first case differs by 3600 pounds and the second by 4781 pounds. These are extreme cases. The difference between the two methods declines as the average milk yield for any age is approached. The difference becomes zero. The correction of any record for age is subject to a large probable error. The standard deviation for milk yield at 8.25 years is 4023. The standard deviation of records corrected for age is consequently $4023\sqrt{1-0.667.^2}$ or 2997 or the probable error is 2000 pounds. In other words it is equiprobable that the results are $12{,}480 \pm 2000$ or $27{,}779 \pm 2000$. Or put in another way it is equally likely that the first result lies inside 10,480 to 14,480 pounds or outside this range and the higher results lies between 25,779 to 29,779 pounds or outside this range. On the basis of these calculations it would be quite unlikely that the results from the mean production curve, 8880 or 32,560 pounds would be correct.

There is a difference in the logic behind the two methods of correcting milk records for age. Milk records corrected for age are corrected only for age as indicated by the changes in mean milk yield with the advancing age. Milk records where the attempt is made to predict what a cow's subsequent lactation records will be are corrected not only for age but also for those temporary environmental factors which are brought to play on the first record and will probably be brought to play on the second record. Where the aim is a correction for age the first method appears to the writer to be the desirable one. Where the aim is the prediction of a milk yield in a subsequent lactation the second method should have the preference.

For these reasons in all the work in this paper the age corrections for milk yields have been based on the strictly mean curve proportioned according to the mean milk yields at the age of the record and the age to which correction is made. I have gone into this subject rather at length because it is necessary to know exactly how corrections are made and the philosophy behind them, for the use of arithmetic without due consideration is unsatisfactory.

The method of correlation is valuable to the man who wants to know from one lactation record what the cow is likely to produce in another lactation. The cost of printing is too high to enable us to present a very extensive table. In view of these facts we have chosen to determine the productions for only the ages of two, three, four, five, six, eight, ten, and twelve years. These groups are sufficiently inclusive to give most of the information desired in practical work. Table 40 presents these data.

The use of this table may be illustrated by the case of V. P. I. Norma Veeman. This cow had a 365-day record of 10,141 pounds at two years and five months of age, her expected record at five years is found on the 10,000 line under column five years. The record is 14,843. The actual record was 14,369 pounds of milk. The expected record has a probable error of 1922 pounds at the foot of column for five years. In other words it was equally probable that the record of V. P. I. Norma Veeman would be between 12,921 to 16,765 or outside of this range. The range necessary to include 99 per cent of all the cows whose first production was 10,141 would be 14,843 $\pm$ 7354 or 7489 to 22,197 pounds. The limits of this range are given in the bottom row of the table.

If the record is made at three years the second group of columns should be used, if made at four years, the third group of figures. Thus if a cow has a record of 12,000 as a four-year-old and we desire to know the probable production as an eight-year-old we find the 12,000 row in the first column and follow that in the 17th column where the milk yield of 15,764 is found to be the probable milk yield. The probable error of this milk yield found in the next to the last row of the column is 2031 pounds. This method corrects not only for age but also for those fluctuating environmental factors which tend to make a record low or high.

The butter-fat percentage has but a slight relation to the age of the cow. In fact during the whole life of the cow, from two years to

fifteen years of age the difference in test is, on the average, 0.13 per cent. Such a difference is easily within the errors of the data for any one cow. The standard deviations of the butter-fat percentages also show practically no relation to the age of the cow. In view of these facts we may neglect the influence of age on butter-fat percentage and determine an equation to predict the butter-fat percentage of a cow in a future lactation from that of a previous lactation by using the mean butter-fat percentage and standard deviation as they are for the whole breed, 3.428 and 0.309. The equation for butter-fat percentage of one lactation from that of another is thus:

$$\text{Butter-fat percentage} = 0.977 + 0.715 \text{ butter-fat percentage (known)} \quad (31)$$

Thus if a cow gave a milk testing 3.8 on the 365-day test, the expected butter-fat percentage of this cow in a future lactation would be 3.69 per cent. There would be variation from this figure, 3.69, if, say 10 such cows were predicted and then tested in the subsequent lactation. The figure 3.69 would be approximately their average, however. The variation which might be expected from this average would be dependent on the standard deviation. This standard deviation is equal to

$$0.309\sqrt{1 - 0.715^2} \text{ or } 0.216$$

The probable error of the 3.69 prediction would then be equal to 0.15 or for any one cow there would be an equal chance of the butter-fat test being between 3.54 and 3.84 and outside that range, either below 3.54 or above 3.84, when the first test of the cow is 3.8. This equation is to be used for the prediction of one lactation's butter-fat percentage from that of another. It is not, for the same reasons as those indicated for the milk yield, to be used to correct for age in inheritance studies. Here the nearly zero correlation between age and butter-fat percentage justifies the use of these records without any correction for age.

For the convenience of those who wish to use these results on their own data the prediction of the butter-fat percentages and the range necessary to include 50 and 99 out of every 100 cows tested are given in table 41.

The entry and reentry records of Princess Segis Walker 161344 may be taken to illustrate the use of this table. As a two-year-old this cow's 365-day test was 3.85 per cent fat. In column one we

TABLE 40

Expected milk yields of cows at given lactation periods predicted from a previous lactation. The probable errors and 99 per cent range are also given

365-DAY MILK YIELD	365-DAY MILK YIELD AT 2 YEARS (SEE FIRST COLUMN) Expected milk yield at ages indicated							365-DAY MILK YIELD AT 3 YEARS (SEE FIRST COLUMN) Expected milk yield at ages indicated					
	3 years	4 years	5 years	6 years	8 years	10 years	12 years	4 years	5 years	6 years	8 years	10 years	12 years
6,000	9,581	10,499	11,207	11,754	12,433	12,597	12,284						
8,000	11,119	12,199	13,025	13,648	14,353	14,373	13,748	11,273	12,042	12,624	13,312	13,414	12,953
10,000	12,657	13,899	14,843	15,542	16,273	16,149	15,212	12,747	13,620	14,268	14,978	14,956	14,223
12,000	14,195	15,599	16,661	17,436	18,193	17,925	16,676	14,221	15,198	15,912	16,644	16,498	15,493
14,000	15,733	17,299	18,479	19,330	20,113	19,701	18,140	15,695	16,776	17,556	18,310	18,040	16,763
16,000	17,271	18,999	20,297	21,224	22,033	21,477	19,604	17,169	18,354	19,200	19,976	19,582	18,033
18,000	18,809	20,699	22,115	23,118	23,953	23,253	21,068	18,643	19,932	20,844	21,642	21,124	19,303
20,000	20,347	22,399	23,933	25,012	25,873	25,029	22,532	20,117	21,510	22,488	23,308	22,666	20,573
22,000	21,885	24,099	25,751	26,906	27,793	26,805	23,996	21,591	23,088	24,132	24,974	24,208	21,843
24,000								23,065	24,666	25,776	26,640	25,750	23,113
26,000								24,539	26,244	27,420	28,306	27,292	24,383
28,000								26,013	27,822	29,064	29,972	28,834	25,653
30,000													
Probable error ±	1,626	1,797	1,922	2,004	2,031	1,879	1,547	1,797	1,922	2,004	2,031	1,879	1,547
99 per cent range ±	6,220	6,873	7,354	7,664	7,768	7,187	5,919	6,873	7,354	7,664	7,768	7,187	5,919

365-DAY MILK YIELD	365-DAY MILK YIELD AT 4 YEARS (SEE FIRST COLUMN)					365-DAY MILK YIELD AT 5 YEARS (SEE FIRST COLUMN)				365-DAY MILK YIELD AT 6 YEARS (SEE FIRST COLUMN)			365-DAY MILK YIELD AT 8 YEARS (SEE FIRST COLUMN)		365-DAY MILK YIELD AT 10 YEARS (SEE FIRST COLUMN)
	Expected milk yield at ages indicated					Expected milk yield at ages indicated				Expected milk yield at ages indicated			Expected milk yield at ages indicated		Expected milk yield at ages indicated
	5 years	6 years	8 years	10 years	12 years	6 years	8 years	10 years	12 years	8 years	10 years	12 years	10 years	12 years	12 years
6,000															
8,000	11,506	12,067	12,748	12,885	12,517	13,081	13,773	13,844	13,307	13,341	13,536	13,048	13,126	12,714	12,523
10,000	12,934	13,555	14,256	14,279	13,665	14,471	15,181	15,148	14,381	14,793	14,788	14,078	14,360	13,730	13,621
12,000	14,362	15,043	15,764	15,673	14,813	15,861	16,589	16,452	15,455	16,145	16,040	15,108	15,594	14,746	14,719
14,000	15,790	16,531	17,272	17,067	15,961	17,251	17,997	17,756	16,529	17,497	17,292	16,138	16,828	15,762	15,817
16,000	17,218	18,019	18,780	18,461	17,109	18,641	19,405	19,060	17,603	18,849	18,544	17,168	18,062	16,778	16,915
18,000	18,646	19,507	20,288	19,855	18,257	20,031	20,813	20,364	18,677	20,201	19,796	18,198	19,296	17,794	18,013
20,000	20,074	20,995	21,796	21,249	19,405	21,421	22,221	21,668	19,751	21,553	21,048	19,228	20,530	18,810	19,111
22,000	21,502	22,483	23,304	22,643	20,553	22,811	23,629	22,972	20,825	22,905	22,300	20,258	21,764	19,826	20,209
24,000	22,930	23,971	24,812	24,037	21,701	24,201	25,037	24,276	21,899	24,257	23,552	21,288	22,998	20,842	21,307
26,000	24,358	25,459	26,320	25,431	22,849	25,591	26,445	25,580	22,973	25,609	24,804	22,318	24,232	21,858	22,405
28,000	25,786	26,947	27,828	26,825	23,997	26,981	27,853	26,884	24,047	26,961	26,056	23,348	25,466	22,874	23,503
30,000															
Probable error ±.	1,922	2,004	2,031	1,879	1,547	2,004	2,031	1,879	1,547	2,031	1,879	1,547	1,879	1,547	1,547
99 per cent range ±............	7,354	7,664	7,768	7,187	5,919	7,664	7,768	7,187	5,919	7,768	7,187	5,919	7,187	5,919	5,919

find 3.8 per cent and in column two opposite this, 3.69 per cent as the probable test of a second 365-day test. In column three the range 3.55 to 3.84 shows that it is equally probable that the second record of Princess Segis Walker will lie inside the range 3.55 to 3.84 or outside this range (that is, either below or above it). In column 4 the range 3.14 to 4.26 shows the range within which 99 per cent of the second test cows will be found. The actual test of Princess Segis Walker at three years old was 3.64.

TABLE 41

Probable butter-fat percentages of one lactation from those of a previous lactation. Ranges necessary to include 50 and 99 per cent of the cows whose first test is that indicated in the first column

BUTTER-FAT PERCENTAGE		RANGE NECESSARY TO INCLUDE	
First lactation	Subsequent lactation	50 per cent of the cows	99 per cent of the cows
2.6	2.84	2.69–2.98	2.28–3.40
2.8	2.98	2.84–3.12	2.43–3.54
3.0	3.12	2.98–3.27	2.57–3.68
3.2	3.27	3.12–3.41	2.71–3.83
3.4	3.41	3.27–3.55	2.86–3.97
3.6	3.55	3.41–3.69	3.00–4.11
3.8	3.69	3.55–3.84	3.14–4.26
4.0	3.84	3.70–3.98	3.28–4.40
4.2	3.98	3.84–4.12	3.43–4.54
4.4	4.12	3.98–4.27	3.57–4.68

RELATION BETWEEN THE 7-DAY AND 365-DAY MILK YIELDS AND BUTTER-FAT PERCENTAGES IN THE SAME LACTATION

Tables 42 and 43 present the information for milk yields and butter-fat percentages of the Holstein-Freisian cows with 7-day and 365-day tests where the 7-day test is a part of the 365-day test.

If the average 7-day milk yields of table 42 are compared with the average 7-day milk yields of all Holstein-Friesian cows, recorded up to volume 25 of the Advanced Register, it is found that the 7-day test cows tested for 365-day records produce about 50 pounds of milk more than all the Advanced Registry cows. The mean milk yield of the 365-day tests for the different age groups is, on the whole, lower than the mean milk yield found for all these age groups including all the animals with 365-day tests recorded in the Advanced Registry

up to volume 31. Individually considered these differences are not significant in comparison with their probable errors. The consistency of the difference probably indicates that the 365-day records are slightly lower in milk yield. This is in line with what will be shown on a subsequent page,—that there is a slightly significant increase of milk yield from dam to daughter in this breed.

TABLE 42

Physical constants for the variation of 7-day and 365-day milk yield where the 7-day test is a part of the 365-day test

AGE	7-DAY MILK YIELD			365-DAY MILK YIELD		
	Mean	Standard deviation	Coefficient of variation	Mean	Standard deviation	Coefficient of variation
years						
1.5 and 2.0	304±4	48±3	15.8±1.1	11,811±239	2,585±169	21.9±1.5
2.0 and 2.5	353±2	67±2	19.1±0.5	13,548±100	2,776±70	20.5±0.5
2.5 and 3.0	371±3	68±2	18.3±0.6	14,160±134	2,817±95	19.9±0.7
3.0 and 3.5	381±5	82±4	21.6±1.0	14,667±173	2,702±122	18.4±0.8
3.5 and 4.0	430±5	76±3	17.8±0.8	15,333±189	3,109±134	20.3±0.9
4.0 and 4.5	465±6	79±4	17.0±0.9	15,901±289	3,861±205	24.3±1.3
4.5 and 5.0	467±6	83±5	17.8±1.0	17,402±252	3,272±178	18.8±1.1
5.0 and 5.5	494±7	84±5	16.9±1.0	17,694±318	3,995±225	22.6±1.4
5.5 and 6.0	508±7	87±5	17.2±0.9	17,625±291	3,861±206	21.9±1.2
6.0 and 6.5	495±7	71±5	14.3±1.0	18,157±299	3,165±211	17.4±1.2
6.5 and 7.0	513±9	91±6	17.8±1.3	18,653±378	3,926±267	21.0±1.5
7.0 and 7.5	503±9	80±6	16.0±1.2	18,450±368	3,449±260	18.7±1.5
7.5 and 8.0	516±13	92±9	17.8±1.8	19,043±631	4,486±446	23.6±2.5
8.0 and 8.5	542±15	91±10	16.9±2.0	18,111±480	3,016±339	16.7±2.0
8.5 and 9.0	510±13	80±9	15.7±1.9	17,647±492	3,009±348	17.0±2.0
9.0 and 9.5	485±10	55±7	11.4±1.4	18,400±290	1,665±205	9.1±1.1
9.5 and 11.0	516±14	83±10	16.0±1.9	16,471±640	3,913±453	23.8±2.9
11.0 and 15.0	508±19	69±13	13.5±2.8	17,667±1,571	5,706±1,111	32.3±6.8

The coefficients of variation give the comparative values of the variations of the 7-day and 365-day tests. The 7-day tests are proportionately less variable than the 365-day tests. The conclusion to be drawn from this fact is not sharply defined. It may be argued that since the 7-day test is less variable than the 365-day test it is a better measure of the cow's worth, or again the argument may be that the 365-day test is more variable than the 7-day test because it tests out the producing capacities of all cows more thoroughly. The

choice of either horn of this dilemma does not help much toward the true statement of the facts desired; i.e., the real worth of the two tests.

Table 43 presents the data for the relation of 7-day butter-fat percentage to the 365-day butter-fat percentage when both are of the same lactation.

Table 43 shows that the 7-day butter-fat percentage is in excess of the 365-day butter-fat percentage. This fact is a bone of contention, for it is argued that if the average butter-fat percentage of the 7-day test is higher than the year butter-fat percentage then reliance cannot be put upon the 7-day butter-fat percentage as an indicator of the probable 365-day butter-fat percentage. Unfortunately such reasoning is by no means necessarily true since it may be that all classes of cows, high or low testing, have their 7-day and 365-day butter-fat raised proportionately. The question is consequently on the relation between the two variables and not primarily a question of means.

The standard deviations of the butter-fat percentage for the 7-day test are also higher than those of the 365-day test. This would indicate that either the 7-day test differentiates the cow's true producing capacities more accurately than the 365-day test, or that the 7-day test of some cows may be increased in their butter-fat percentage through a trick of feeding or management while their equally good sisters are not increased. Here again the answer to the question comes in a study of the relations between the tests.

The relation which exists between the 7-day and 365-day tests is shown by the correlation coefficients and correlation ratios of table 44. These constants measure the degree of reliance which may be placed on the 7-day test.

The correlation coefficients for the relation of the 7-day and 365-day lactation records for milk yield and butter-fat percentage are practically all of significantly large size. They are in fact some of the largest correlation coefficients known for the variation of economic characters. No consistant variation of the correlations with age is to be noted. Under these conditions the most reasonable value for the correlation coefficients is the average value. The value of this correlation coefficient for the milk is 0.583 and for the butter-fat percentage is 0.531. To a slight degree, then, the 7-day milk yield is a better measure of the cow's performance in the year test than the 7-

TABLE 43

Physical constants for the variation of the 7-day and 365-day butter-fat percentage when both tests are in the same lactation

AGE	7-DAY BUTTER-FAT PERCENTAGE			365-DAY BUTTER-FAT PERCENTAGE		
	Mean	Standard deviation	Coefficient of variation	Mean	Standard deviation	Coefficient of variation
years						
1.5 and 2.0	3.59±0.04	0.386±0.025	10.75±0.73	3.50±0.03	0.293±0.019	8.38±0.53
2.0 and 2.5	3.63±0.02	0.500±0.013	13.80±0.36	3.42±0.01	0.297±0.008	8.67±0.23
2.5 and 3.0	3.68±0.03	0.527±0.018	14.33±0.48	3.51±0.01	0.310±0.010	8.86±0.31
3.0 and 3.5	3.70±0.03	0.507±0.022	13.71±0.65	3.49±0.02	0.336±0.015	9.64±0.46
3.5 and 4.0	3.62±0.03	0.466±0.020	12.89±0.57	3.42±0.02	0.312±0.013	9.15±0.39
4.0 and 4.5	3.69±0.04	0.526±0.028	14.25±0.76	3.47±0.03	0.364±0.019	10.49±0.54
4.5 and 5.0	3.68±0.04	0.491±0.027	13.33±0.72	3.41±0.03	0.337±0.018	9.88±0.55
5.0 and 5.5	3.81±0.05	0.594±0.033	15.61±0.92	3.48±0.03	0.346±0.019	9.94±0.57
5.5 and 6.0	3.64±0.04	0.550±0.029	15.12±0.82	3.40±0.02	0.270±0.014	7.95±0.43
6.0 and 6.5	3.72±0.04	0.428±0.029	11.52±0.81	3.39±0.02	0.222±0.015	6.53±0.47
6.5 and 7.0	3.65±0.03	0.361±0.025	9.90±0.69	3.42±0.02	0.245±0.017	7.16±0.48
7.0 and 7.5	3.60±0.06	0.529±0.040	14.72±1.16	3.38±0.03	0.296±0.022	8.76±0.68
7.5 and 8.0	3.73±0.09	0.631±0.063	16.92±1.74	3.46±0.03	0.236±0.023	6.82±0.70
8.0 and 8.5	3.57±0.05	0.333±0.037	9.35±1.02	3.37±0.03	0.176±0.020	5.24±0.56
8.5 and 9.0	3.51±0.08	0.502±0.058	14.28±1.65	3.41±0.05	0.315±0.036	9.26±1.05
9.0 and 9.5	3.58±0.15	0.835±0.103	23.33±2.98	3.39±0.06	0.349±0.043	10.29±1.24
9.5 and 11.0	3.52±0.05	0.328±0.038	9.31±1.05	3.43±0.05	0.314±0.036	9.15±1.05
11.0 and 15.0	3.80±0.09	0.321±0.063	8.46±1.57	3.40±0.09	0.342±0.067	10.05±1.97

day record for the butter-fat percentage is for the 365-day butter-fat percentage.

The correlation coefficient for the 365-day test and retest records is about 0.1 higher than the correlation coefficient for the 7-day with the 365-day test where the 7-day record is a part of the 365-day test.

TABLE 44

Constants of correlation for the relation of the variables 7-day and 365-day milk yield and butter-fat percentage of the same lactation

AGE	7-DAY MILK YIELD AND 365-DAY MILK YIELD SAME LACTATION			7-DAY BUTTER-FAT PERCENTAGE AND 365-DAY BUTTER-FAT PERCENTAGE SAME LACTATION		
	Correlation coefficient	Correlation ratio	$\eta^2 - r^2$	Correlation coefficient	Correlation ratio	$\eta^2 - r^2$
years						
1.5– 2.0	0.653±0.053	0.799	0.213±0.076	0.601±0.059	0.684	0.108±0.057
2.0– 2.5	0.679±0.019	0.684	0.007±0.006	0.559±0.025	0.592	0.039±0.014
2.5– 3.0	0.624±0.029	0.668	0.057±0.022	0.549±0.033	0.597	0.055±0.022
3.0– 3.5	0.355±0.056	0.529	0.538±0.046	0.539±0.045	0.719	0.226±0.054
3.5– 4.0	0.623±0.037	0.663	0.051±0.027	0.509±0.045	0.581	0.078±0.033
4.0– 4.5	0.732±0.035	0.759	0.040±0.029	0.540±0.053	0.686	0.179±0.057
4.5– 5.0	0.447±0.062	0.524	0.075±0.040	0.548±0.054	0.653	0.126±0.051
5.0– 5.5	0.584±0.052	0.664	0.101±0.048	0.636±0.047	0.731	0.130±0.054
5.5– 6.0	0.574±0.050	0.658	0.103±0.046	0.579±0.050	0.713	0.173±0.057
6.0– 6.5	0.634±0.056	0.676	0.054±0.043	0.381±0.081	0.442	0.050±0.041
6.5– 7.0	0.587±0.063	0.704	0.151±0.069	0.406±0.080	0.551	0.139±0.067
7.0– 7.5	0.477±0.082	0.541	0.065±0.053	0.628±0.065	0.705	0.102±0.065
7.5– 8.0	0.794±0.052	0.812	0.030±0.048	0.499±0.106	0.678	0.210±0.115
8.0– 8.5	0.518±0.116	0.735	0.273±0.142	0.643±0.093	0.715	0.099±0.095
8.5– 9.0	0.521±0.119	0.570	0.054±0.074	0.498±0.123	0.706	0.251±0.142
9.0– 9.5	−0.115±0.172	0.819	0.657±0.165	0.505±0.130	0.942	0.632±0.168
9.5–11.0	0.740±0.074	0.847	0.169±0.122	0.907±0.029	0.952	0.084±0.091
11.0–15.0	0.835±0.083	0.972	0.246±0.237	0.030±0.275	0.218	0.047±0.116
Mean...	0.583	0.701		0.531	0.659	

The 365-day would consequently appear to measure the cow's capacity slightly better than would the 7-day test.

It is frequently desirable to have some means of estimating the 365-day record from the 7-day record of the same lactation. A generalized equation for this prediction is here presented. The equation would be of the form:

$$\text{365-day milk yield } (A \text{ age}) = C - 0.583 \frac{D}{H} K + 0.583 \frac{D}{H} \text{ 7-day milk yield } (A \text{ age}) \tag{32}$$

where C = the curve showing the relation of the 365-day mean milk yield to age, or $C = 9432 + 2070A - 128.9A^2 + 1548 \log (A - 1.25)$

D = the curve showing the relation of the 365-day standard deviation of milk yield with age, or $D = 1682 + 652A - 44.6A^2$

H = the curve showing the relation of the 7-day standard deviation of milk yield with age, or $H = 53.01 + 2.626A - 2168A^2 + 18.82 \log (A - 1.25)$

K = the curve showing the relation of the 7-day mean milk yield with age, or $K = 328.7 + 3.41A - 0.830A^2 + 199.5 \log (A - 1.25)$

The solution of these equations for several ages is shown in the chapter on age in relation to milk secretion. The use of the equations may be illustrated as follows: A cow whose age is six years and three months makes a 7-day milk record of 600 pounds of milk, what will be her 365-day milk yield? From the chapter treating of age in relation to milk yield we obtain the following facts. The mean 7-day and 365-day milk records at six years and three months are 458 (K) and 18,414 (C) respectively. The standard deviations for milk yield at these points are 74 (H) and 4010 (D). From these data we find by substitution into the equation above

$$\text{365-day milk yield} = 3945 + 31.59 \text{ (7-day milk yield)}$$

The 7-day milk yield of 600 pounds indicates a probable 365-day milk yield of 22,900 pounds. The variation around this expected milk yield which would include 50 per cent of the cows of 600-pound 7-day milk yield may be found from the relation of the standard deviation of such a record.

$$\text{S.D.} = 4010 \sqrt{1 - 0.583^2}$$

The probable error, $0.67449 \times$ S. D., is equal to 2197 or 50 per cent of the cows with 600-pound 7-day records would be expected to have a milk yield between $22,900 \pm 2197$ or 20,703 to 25,097 pounds. In this same manner the predicted milk yield and the probable error of the same may be determined for any cow.

For those who prefer to avoid the labor of making this calculation a limited number of entries have been made to cover the most frequent ranges of 7-day milk yield and age. These are given in table 45.

The cow Daisy Queen Johanna at two years seven months of age gave 285 pounds of milk, it is desirable to know her 365-day lactation record. The 7-day milk yields are given above the table, 300 pounds at three years old, the second row, is equal to 12,480 pounds for the 365-day lactation record. The probable error of this record is given in the next to the last column or 12,480 $\pm$1773 and the range necessary to include 99 per cent of the cows is given in the last column or 12,480$\pm$6783. The actual production of the cow was 12,668 pounds, a much closer approximation to the expected record than would normally be found from such a comparison.

The problem for the 7-day butter-fat percentage is the same as that for the 365-day milk yield save for the fact that the butter-fat percentages have so little relation to age that age may be disregarded. Our data on 7-day butter-fat percentages are not so complete as they will be at a later date. For the mean and standard deviation of 7-day butter-fat percentage I take the records on 1387 individuals. The mean 7-day butter-fat percentage for these tests was 3.66 and the standard deviation was 0.524. From these data we find the generalized equation for the 365-day butter-fat percentage predicted from the 7-day butter-fat percentage to be:

$$\text{365-day butter-fat percentage} = 2.282 + 0.313\ (\text{7-day butter-fat percentage}) \tag{33}$$

For illustrating the use of the equation we might take record of a daughter of Hazelwood Bracelet Korndyke, Hazelwood Ormsby Korndyke. This cow at two years and one month of age had a record of 3.74 for 7-day butter-fat percentage. From the above equation we find the expected butter-fat percentage $(0.313 \times 3.74) + 2.282$ to be 3.45. The actual butter-fat percentage of this cow was 3.46 for the year test.

It must not be expected that as close an agreement between actual and expected will always be obtained. In another case the 7-day record was 4.70 per cent of butter-fat, the expected 365-day record was 3.75 per cent of butter-fat, the actual 365-day record was 3.47

TABLE 45

Probable 365-day milk yields as determined from the 7-day milk yields of the same lactation. The probable errors and 99 per cent range are also given

AGE	7-DAY MILK YIELD AT GIVEN AGE													
	200	250	300	350	400	450	500	550	600	650	700	750	Probable error	99 per cent range
	EXPECTED 365-DAY MILK YIELD AT GIVEN AGE													
years														
2	9,678	11,166	12,653	14,141	15,628	17,116	18,603	20,091					±1,539	±5,885
3		10,995	12,480	13,966	15,451	16,937	18,422	19,908	21,393	22,879			±1,773	±6,783
4		11,229	12,755	14,281	15,807	17,333	18,859	20,385	21,911	23,437	24,963		±1,960	±7,497
5		11,520	13,080	14,640	16,200	17,760	19,320	20,880	22,440	24,000	25,560	27,120	±2,097	±8,021
6		11,823	13,400	14,977	16,554	18,131	19,708	21,285	22,862	24,439	26,016	27,593	±2,185	±8,357
8		12,248	13.802	15,356	16,910	18,464	20,018	21,572	23,126	24.680	26,234		±2,214	±8,470
10		12,386	13,833	15,281	16,728	18,176	19,623	21,071	22,518	23,966	25,413		±2,049	±7,838
12		12,216	13,450	14,685	15,919	17,154	18,388	19,623	20,857				±1,688	±6,455

per cent of butter-fat, a difference 0.28 of a per cent. Some information must be available to indicate the variation found between expected and actual records. This variation is equal to:

$$0.67449\ (0.309\ \sqrt{1 - 0.531^2})\ \text{or}\ 0.67449\ (0.262) = 0.18$$

There is consequently a probable error of 0.18 to each expected 365-day record when calculated from the 7-day record.

This equation together with its probable error may also be used to indicate how trustworthy a given 7-day test may be. In the case of the 4.70 per cent cow presented above, the question might be asked whether the figures had not been manipulated to give a result higher than should have been obtained, since on the year test the butter-fat was actually 3.47 instead of the higher 3.75 per cent indicated by the 7-day test. This question may be answered by noting that the probable error of the 3.75 test was 0.18, or that only 50 out of every 100 cows would give between 3.57 and 3.93 per cent, the other 50 being outside this range. It would, consequently, not be at all unlikely that a cow testing 4.70 on the 7-day test would test 3.47 on the 365–day test.

For those who do not like to perform the necessary arithmetic to solve individual cases, table 46 has been compiled to give the probable 365-day record from the 7-day record.

The application of table 46 is very simple as it can be used for all lactations since the age of the cows has little effect on the butter-fat test. For illustration, we may assume that a cow gives a milk testing 4.4 per cent in the 7-day test. We find 4.4 per cent in the first column. Opposite this figure in the second column, is given the probable 365-day test, 3.66. In the third column is the range of butter-fat within which it would be equally probable that any given cow of 4.4 per cent butter-fat in the 7-day test would come in the 365-day record. The fourth column gives the range within which 99 out of 100 cows would test for their 365-day record when their 7-day record was 4.4 per cent.

The data may also be considered from this angle. For 100 cows testing 4.4 per cent on the 7-day test, the average butter-fat in the year test would be 3.66 per cent; 50 of the cows would have their butter-fat percentage between the 3.47 and the 3.84 per cent for the 365-day records; 99 of the cows would have their test between 2.98 and 4.33 per cent for the 365-day period.

TABLE 46

Probable 365-day butter-fat percentage record determined from 7-day record. Range necessary to include 50 and 99 per cent of the cows also given

BUTTER-FAT PERCENTAGE		RANGE OF BUTTER-FAT PERCENTAGE NECESSARY TO INCLUDE	
7-day	Probable 365-day	50 per cent of the cows	99 per cent of the cows
2.6	3.10	2.91–3.27	2.42–3.77
2.8	3.16	2.97–3.34	2.48–3.83
3.0	3.22	3.04–3.40	2.55–3.90
3.2	3.28	3.10–3.46	2.61–3.96
3.4	3.35	3.16–3.52	2.67–4.02
3.6	3.41	3.22–3.59	2.73–4.08
3.8	3.48	3.29–3.65	2.80–4.15
4.0	3.54	3.35–3.71	2.86–4.21
4.2	3.60	3.41–3.77	2.92–4.27
4.4	3.66	3.47–3.84	2.98–4.33
4.6	3.73	3.54–3.90	3.05–4.40
4.8	3.79	3.60–3.96	3.11–4.46
5.0	3.85	3.66–4.02	3.17–4.52
5.2	3.91	3.72–4.09	3.23–4.58
5.4	3.98	3.79–4.15	3.30–4.65
5.6	4.04	3.85–4.21	3.36–4.71
5.8	4.10	3.91–4.27	3.42–4.77

TABLE 47

Physical constants for the variation of the 7-day and 365-day milk yield where the 7-day record is made in a separate lactation from the 365-day milk yield

AGE		7-DAY MILK YIELD			365-DAY MILK YIELD		
7-day test	365-day test	Mean	Standard deviation	Coefficient of variation	Mean	Standard deviation	Coefficient of variation
years	*years*						
2–3	3– 4	341±5	61±4	17.9±1.1	15,869±335	3,881±237	24.5±1.5
2–3	4– 5	325±6	55±4	16.9±1.4	15,895±321	2,936±227	18.5±1.4
2–3	5– 6	329±8	64±6	19.4±1.8	17,630±464	3,571±328	20.3±1.9
2–3	6–10	305±6	50±4	16.4±1.3	17,784±437	3,940±309	22.2±1.8
3–4	4– 5	401±8	75±6	18.7±1.5	16,103±359	3,327±254	20.7±1.7
3–4	5– 6	437±12	90±9	20.5±2.1	18,480±621	4,605±439	24.9±2.5
3–4	6–10	364±11	72±8	19.7±2.3	18,211±539	3,488±382	19.2±2.2
4–5	5– 6	430±7	57±5	13.3±1.2	17,071±439	3,443±310	20.2±1.9
4–5	6–10	452±15	115±11	25.5±2.6	18,077±461	3,485±326	19.3±1.8
5–6	6–10	465±10	85±7	18.3±1.6	18,200±510	4,142±361	22.8±2.1

RELATION BETWEEN THE 7-DAY AND 365-DAY MILK YIELDS AND BUTTER-FAT PERCENTAGES IN DIFFERENT LACTATIONS

The problem presented by the 7-day records of one lactation as compared with the 365-day records of another is essentially similar to that of the 7-day record of one lactation with the 365-day record of the same lactation. The means, standard deviations, and coefficients of variation for the 7-day records and 365-day records for different lactations are given in table 47.

Similar data for the butter-fat percentage are presented in table 48.

The data contained in these two tables are in essential agreement with those presented in the other tables of this chapter. Our problem deals largely with the relative values of the 7-day test and the 365-day test as a measure of a cow's worth. The correlation coefficients to determine these relationships are given in table 49.

The relation between the milk yield of the 7-day test and the milk yield of the 365-day test when the 7-day test is in a separate lactation from the 365-day test is less than either of the two preceding correlations for 365-day milk yield with 365-day milk yield or 7-day milk yield with 365-day milk yield. From these considerations it follows that the 365-day test is a better indicator of a subsequent 365-day lactation's milk yield than is the 7-day test. A 7-day milk yield is a better indicator of the milk yield of a 365-day lactation of which it is a part than of the 365-day milk yield of another lactation. All of the correlation coefficients are high in comparison with those found heretofore for most economic quantitative characters. Such being the case it is well to give due consideration to any test for milk yield, however short it may be, in selecting dairy cows.

Essentially the same conclusion may be drawn from the relation of the 7-day butter-fat percentages and the 365-day butter-fat percentages. The yearly butter-fat percentage is a better measure of a subsequent yearly butter-fat percentage than a 7-day butter-fat percentage of a subsequent year butter-fat percentage. This holds true whether the 7-day test is a part of the 365-day test or is in a different lactation. The 7-day test is a better measure of the butter-fat percentage of the 365-day lactation record of which it is a part than of the 365-day lactation record of which it is not a part. All of the correlation coefficients are of good size showing that records as short

TABLE 48

Physical constants for the variation of the 7-day and 365-day butter-fat percentage when the 7-day record is made in a separate lactation from the 365-day record

AGE		7-DAY BUTTER-FAT PER CENT			365-DAY BUTTER-FAT PER CENT		
7-day test	365-day test	Mean	Standard deviation	Coefficient of variation	Mean	Standard deviation	Coefficient of variation
years	*years*						
2–3	3– 4	3.44±0.04	0.418±0.026	12.15±0.74	3.39±0.03	0.343±0.021	10.11±0.62
2–3	4– 5	3.65±0.05	0.451±0.035	12.37±0.94	3.51±0.03	0.286±0.022	8.16±0.62
2–3	5– 6	3.44±0.05	0.417±0.038	12.13±1.12	3.36±0.04	0.292±0.027	8.70±0.83
2–3	6–10	3.49±0.04	0.369±0.029	10.57±0.87	3.49±0.04	0.320±0.025	9.17±0.71
3–4	4– 5	3.59±0.05	0.436±0.033	12.15±0.93	3.42±0.03	0.282±0.022	8.25±0.61
3–4	5– 6	3.51±0.06	0.461±0.044	13.15±1.26	3.34±0.04	0.277±0.026	8.30±0.77
3–4	6–10	3.55±0.06	0.388±0.043	10.93±1.22	3.31±0.04	0.286±0.031	8.65±0.99
4–5	5– 6	3.57±0.04	0.343±0.031	9.61±0.91	3.46±0.04	0.338±0.030	9.77±0.91
4–5	6–10	3.61±0.08	0.601±0.056	16.65±1.64	3.47±0.05	0.395±0.037	11.38±1.04
5–6	6–10	3.58±0.05	0.446±0.039	12.46±1.06	3.46±0.04	0.332±0.029	9.60±0.88

as 7 days have a good deal of value in predicting what the probable butter-fat percentage of the cow will be under year-test conditions.

The variability of the correlation coefficients of table 49 seems to be due to chance. In view of this fact the prediction equations are based on the average correlation coefficients. The generalized equation for milk yield is

$$365\text{-day milk yield } (A \text{ Age}) = C - 0.475 \frac{D}{H} \text{K} + 0.475 \frac{D}{H} \text{ 7-day milk yield } (a \text{ age}) \qquad (34)$$

when the A age is the age at which the expected 365-day record is

TABLE 49

Coefficients measuring the relation between the 7-day record and the 365-day record for milk yield and butter-fat percentage, when the 7-day record is not a part of the 365-day record

AGE		MILK YIELD			BUTTER-FAT PERCENTAGE		
7-day test	365-day test	Correlation coefficient	Correlation ratio	$\eta^2 - r^2$	Correlation coefficient	Correlation ratio	$\eta^2 - r^2$
years	*years*						
2–3	3– 4	0.451±0.069	0.596	0.152±0.062	0.412±0.072	0.538	0.120±0.056
2–3	4– 5	0.521±0.080	0.561	0.043±0.044	0.355±0.096	0.499	0.123±0.072
2–3	5– 6	0.592±0.084	0.669	0.097±0.077	0.478±0.100	0.817	0.439±0.129
2–3	6–10	0.321±0.099	0.388	0.047±0.047	0.329±0.099	0.489	0.131±0.075
3–4	4– 5	0.318±0.097	0.455	0.106±0.067	0.429±0.088	0.609	0.187±0.084
3–4	5– 6	0.441±0.109	0.527	0.084±0.075	0.323±0.121	0.698	0.382±0.131
3–4	6–10	0.555±0.107	0.785	0.307±0.143	0.279±0.143	0.685	0.391±0.151
4–5	5– 6	0.478±0.098	0.738	0.315±0.118	0.465±0.100	0.757	0.357±0.122
4–5	6–10	0.636±0.079	0.666	0.039±0.052	0.630±0.080	0.841	0.310±0.122
5–6	6–10	0.441±0.099	0.587	0.150±0.088	0.534±0.088	0.826	0.397±0.120
Mean.....		0.475			0.423		

desired and a the age when the actual 7-day record was made. The A and a ages will not be the same for this equation is not applicable to the case where the 7-day lactation is a part of the 365-day lactation. The values for the letters are:

C = the curve showing the relation of the 365-day mean milk yield to age, or $C = 9432 + 2070A - 128.9A^2 + 1548 \log (A - 1.25)$

D = the curve showing the relation of the 365-day standard deviations of milk yield with age or $D = 1682 + 652A - 44.6A^2$

H = the curve showing the relation of the 7-day standard deviations of milk yield to age, or $H = 53.01 + 2.626a - 0.2168a^2 + 18.82 \log (a - 1.25)$

K = the curve showing the relation of the 7-day mean milk yield with age, or $K = 328.7 + 3.41a - 0.830a^2 + 199.5 \log (a - 1.25)$

The solution of these equations for age is shown in the chapter on age in relation to milk yield. For those who wish only an approximate answer, the table presented below will fill most practical wants.

Table 50 gives some appreciation of what 365-day record might be expected of a cow which had a previous 7-day record in another lactation. The first column gives the 7-day record. The next group of columns gives the probable 365-day milk yield depending on the age at which the 7-day record was made. The two lower rows give the probable errors and the range necessary to include 99 per cent of all the cows. The probable error is the range necessary to include 50 per cent of the cows. An illustration will indicate clearly the use of table 50. The cow, Yankee De Kol de Yong at five years and eight months of age had a 7-day record of 666 pounds of milk. What would be her probable 365-day production at seven years one month? In the field under "7-day milk yield at five years," and the column under eight years and in row 650 of the left hand column, the expected 365-day milk yield of 24,661 is given. For practical purposes this is about as close as we are likely to need the answer. Now in the next to the bottom row in this column we note that the probable error of this result, 24,661±2399 is 2399 pounds or it is equally likely that Yankee De Kol de Yong will have a production between 24,661 − 2399 = 22,262 and 24,661+2399 = 27,060 or on either side this range. From the bottom row of this column we find that the chances are 99 to 1 that Yankee De Kol de Yong will have as her production 24,661±9175 or from 15,486 to 33,836. Yankee De Kol de Yong's actual production at this age for the 365-day period was 24,704 pounds of milk.

The butter-fat percentage may be treated as one equation since the influence of age is negligible. The equation for the 365-day butter-fat percentage predicted from a 7-day record of an earlier lactation is:

$$\text{365-day butter-fat percentage} = 2.524 + 0.247\ (\text{7-day butter-fat percentage}) \qquad (35)$$

TABLE 50

Expected Holstein-Friesian 365-day milk yield determined from a record of 7 days where the 7-day record is in a different lactation from that of the 365-day record

7-DAY MILK YIELD	7-DAY MILK YIELD AT 2 YEARS (SEE FIRST COLUMN)						7-DAY MILK YIELD AT 3 YEARS (SEE FIRST COLUMN)				
	Expected 365-day milk yield at age indicated						Expected 365-day milk yield at age indicated				
	3 years	4 years	5 years	6 years	8 years	10 years	4 years	5 years	6 years	8 years	10 years
200	11,866	13,024	13,911	14,573	15,289	15,241					
250	13,264	14,568	15,563	16,294	17,034	16,856	12,851	13,726	14,380	15,093	15,060
300	14,661	16,112	17,215	18,016	18,779	18,470	14,188	15,157	15,871	16,604	16,459
350	16,058	17,656	18,867	19,738	20,524	20,085	15,526	16,588	17,362	18,116	17,857
400	17,456	19,201	20,519	21,459	22,269	21,699	16,863	18,019	18.854	19,627	19,256
450	18,853	20,745	22,171	23,181	24,014	23,314	18,201	19,450	20,345	21,139	20,654
500	20,251	22,289	23,824	24,903	25,759	24,929	19,538	20,881	21,836	22,650	22,053
550	21,648	23,833	25,476	26,624	27,504	26,543	20,876	22,312	23,327	24,161	23,451
600							22,213	23,743	24,818	25,673	24,849
650							23,551	25,174	26,310	27,184	26,248
700											
750											
Probable error ±....	1,921	2,123	2,271	2,366	2,399	2,219	2,123	2,271	2,366	2,399	2,219
99 per cent range ±..	7,347	8,119	8,686	9,052	9,175	8,489	8,119	8,686	9,052	9,175	8,489

7-DAY MILK YIELD	7-DAY MILK YIELD AT 4 YEARS (SEE FIRST COLUMN) Expected 365-day milk yield at age indicated				7-DAY MILK YIELD AT 5 YEARS (SEE FIRST COLUMN) Expected 365-day milk yield at age indicated			7-DAY MILK YIELD AT 6 YEARS (SEE FIRST COLUMN) Expected 365-day milk yield at age indicated		7-DAY MILK YIELD AT 8 YEARS (SEE FIRST COLUMN) Expected 365-day milk yield at age indicated
	5 years	6 years	8 years	10 years	6 years	8 years	10 years	8 years	10 years	10 years
200										
250	13,002	13,626	14,330	14,354	13,225	13,923	13,977	13,710	13,780	13,589
300	14,332	15,012	15,735	15,654	14,549	15,265	15,219	15,012	14,984	14,760
350	15,663	16,399	17,140	16,954	15,873	16,607	16,461	16,314	16,189	15,932
400	16,993	17,785	18,546	18,254	17,198	17,949	17,703	17,617	17,394	17,103
450	18,324	19,172	19,951	19,554	18,522	19,292	18,945	18,919	18,599	18,275
500	19,654	20,558	21,356	20,855	19,847	20,634	20,187	20,221	19,804	19,446
550	20,984	21,944	22,761	22,155	21,171	21,976	21,429	21,523	21,009	20,618
600	22,315	23,331	24,166	23,455	22,495	23,319	22,671	22,825	22,214	21,789
650	23,645	24,717	25,572	24,755	23,820	24,661	23,913	24,128	23,419	22,961
700	24,976	26,104	26,977	26,055	25,144	26,003	25,155	25,430	24,624	24,132
750	26,306	27,490	28,382	27,355	26,468	27,346	26,397			
Probable error ±.......	2,271	2,366	2,399	2,219	2,366	2,399	2,219	2,399	2,219	2,219
99 per cent range ±....	8,686	9,052	9,175	8,489	9,052	9,175	8,489	9,175	8,489	8,489

From this equation we may derive the probable 365-day butter-fat percentage of one lactation from the 7-day butter-fat percentage of another lactation. To facilitate the use of this information, table 51 is presented for a few 7-day butter-fat percentages.

From the five-year-old 7-day butter-fat percentage of Violet Hendrick 4th of 3.60 per cent, table 51 shows us that we should expect her 365-day record to be 3.41 per cent of butter-fat in another lactation. Her record would however have an equal chance of being between 3.22 to 3.60 per cent or below 3.22 or above 3.60, also the chances are 99 to 1 that her record would be between the limits 2.69 to 4.14 per cent. Violet Hendrick 4th's actual record for 321 days was 3.38 per cent of butter-fat.

TABLE 51

Probable 365-day butter-fat percentage derived from the 7-day butter-fat percentage of another lactation

7-DAY BUTTER-FAT PERCENTAGE	PROBABLE 365-DAY BUTTER-FAT PERCENTAGE OF ANOTHER LACTATION	RANGE NECESSARY TO INCLUDE	
		50 per cent of the cows	99 per cent of the cows
2.6	3.17	2.98–3.36	2.44–3.89
2.8	3.22	3.03–3.40	2.49–3.94
3.0	3.26	3.08–3.45	2.54–3.99
3.2	3.31	3.13–3.50	2.59–4.04
3.4	3.36	3.17–3.55	2.64–4.09
3.6	3.41	3.22–3.60	2.69–4.14
3.8	3.46	3.27–3.65	2.74–4.18
4.0	3.51	3.32–3.70	2.79–4.23
4.2	3.56	3.37–3.75	2.84–4.28
4.4	3.61	3.42–3.80	2.89–4.33
4.6	3.66	3.47–3.85	2.94–4.38
4.8	3.71	3.52–3.90	2.99–4.43
5.0	3.76	3.57–3.95	3.04–4.48
5.2	3.81	3.62–4.00	3.09–4.53
5.4	3.86	3.67–4.05	3.14–4.58
5.6	3.91	3.72–4.10	3.19–4.63
5.8	3.96	3.77–4.15	3.23–4.68

PERMANENCE OF MILK YIELD AND BUTTER-FAT PERCENTAGE VERSUS PERMANENCE OF OTHER ECONOMIC CHARACTERS

It is of interest to examine some of the other economic characters exhibited by domestic animals to see how accurately the production

of these products measures their production by the same animals at a subsequent date. Table 52 gives some of the published correlation coefficients on certain of the more important economic characters.

If a comparison is made between the 365-day tests of Guernsey and Holstein-Friesian cows, the reliability of these tests in predicting a subsequent 365-day retest is found to be practically the same. The average correlation for the Guernseys is 0.696 and for the Holstein-Friesian is 0.667. It takes but a glance at the rest of the table to show that these correlation coefficients are high. These tests are quite reliable in predicting the subsequent yield.

The Advanced Registry records appear to be more dependable than records taken for a whole herd as the correlation coefficients for the Jersey herd records are lower by about 0.1 as compared with the Advanced Registry records. This may be due to the better preparation given to cows put on Advanced Registry test.

The next highest coefficients are those found for the 7-day test in relation to the year tests. These coefficients are also high. The reliability of the 7-day test is not quite so great as that of the year test. The 7-day test has a much greater value for predicting the year test than has any point of conformation, in fact the 7-day test is nearly $2\frac{1}{2}$ times as good an indicator of year milk yield as any of the score points. It will be noted that the 365-day milk yield of both Guernsey and Holstein-Friesian cows has nearly the same predicting value for subsequent milk yield that the butter-fat percentage has for the subsequent butter-fat percentage. The 7-day milk yields have a slightly higher value in predicting the subsequent 365-day records than the 7-day records for butter-fat percentage have for predicting the subsequent 365-day butter-fat percentage. The eight-months butter-fat percentage record of the pure bred Jersey herd has a lower prediction value for the subsequent eight-months record than either the Guernsey or Holstein-Friesian year test have for the subsequent retest.

When compared with the other characters used to predict future production it is clear that both the 7-day and 365-day records have a relatively high value in predicting future production of dairy cattle, well up with those used for egg production and for wool clip.

TABLE 52

Correlation coefficients, between characters of economic importance

CHARACTER	BREED	CORRELATION	AUTHORITY
Milk yield and milk yield	Guernsey Advanced Registry	0.462–0.811	Gowen (1)
Milk yield and milk yield	Jersey purebred herd	0.214–0.731	Gowen (2)
Milk yield and milk yield (revised maximum)	British Holsteins and Shorthorns	0.394 0.762	Gavin (3)
365-day milk yield and 365-day milk yield	Holstein-Friesian	0.480–0.777	Gowen (4)
7-day milk yield and 365-day milk yield (same lactation)	Holstein-Friesian	−0.115–0.835	Gowen (4)
7-day milk yield and 365-day milk yield (different lactation)	Holstein-Friesian	0.318–0.636	Gowen (4)
Butter-fat percentage and butter-fat percentage	Guernsey Advanced Registry	0.637–0.893	Gowen (5)
Butter-fat percentage and butter-fat percentage	Jersey purebred herd	0.247–0.678	Gowen (6)
365-day butter-fat percentage and 365-day butter-fat percentage	Holstein-Friesian Advanced Registry	0.593–0.789	Gowen (4)
7-day butter-fat percentage and 365-day butter-fat percentage (same lactation)	Holstein-Friesian Advanced Registry	0.030–0.907	Gowen (4)
7-day butter-fat percentage and 365-day butter-fat percentage (different lactation)	Holstein-Friesian Advanced Registry	0.279–0.630	Gowen (4)
Wool clip of one season and wool clip of another season	Rambouillet	0.51	Hill (7)
Wool clip of one season and wool clip of another season	Grades, Rambouillet, Corriedales	0.044–0.985	Lush (8)

Lobe color and yearly egg production	White Leghorn	−0.527–−0.582	Harris, Blakeslee, Warner (9)
Monthly egg yield and other 11 months' production	White Leghorn	0.240-0.573	Harris, Blakeslee (10)
Milk yield and score (conformation)	Jersey Registry of Merit	−0.070–+0.194	Gowen (11)
Milk yield and size of body parts	Holstein-Friesian	0.009-0.425	

(1) Gowen, John W. 1923. The relation between the milk yield of one lactation with the milk yield of a subsequent lactation in Guernesy Advanced Registry cattle. Jour. Dairy Science, vol. vi, pp. 102–121.

(2) Gowen, John W. 1920. Studies in milk secretion. V. On the variations and correlations of milk secretion with age. Genetics, vol. 5, pp. 111–188.

(3) Gavin, William. 1913. Studies in milk records on the accuracy of estimating a cow's milking capacity by her first lactation yield. Jour. Agric. Soc., vol. 5, pp. 377–391.

(4) Gowen, Marie S., and Gowen, John W. 1922. Studies in milk secretion. XIII. Relation between milk yields and butter-fat percentages of the 7-day and 365-day tests of Holstein-Friesian Advanced Registry cattle. Annual report of the Maine Agricultural Experiment Station for 1922, Bulletin 306, pp. 21–60.

(5) Gowen, John W. 1923. Studies in milk secretion. XI. The relation between the butter-fat percentage of one lactation and the butter-fat percentage of a subsequent lactation in Guernsey Advanced Registry cattle. Jour. Dairy Science, vol. vi, pp. 330–346.

(6) Gowen, John W. 1920. Studies in milk secretion. VI. On the variations and correlations of butter-fat percentage with age in Jersey cattle. Genetics, vol. 5, pp. 249–324.

(7) Hill, J. A. 1921. Studies in the variation and correlation of fleeces from range sheep. Bulletin No. 127, Wyoming Agricultural Experiment Station, pp. 39–52.

(8) Lush, Jay L. 1923. The influence of age and individuality upon the yield of wool. Record of Proceedings of the American Society of Animal Production, December, 1922, pp. 105–109.

(9) Harris, J. Arthur, Blakeslee, A. F., and Warner, D. E. 1917. The correlation between body pigmentation and egg production in the domestic fowl. Genetics, vol. 2, pp. 36–81.

(10) Harris, J. Arthur, and Blakeslee, A. F. 1918. The correlation between egg production during various periods of the year in the domestic fowl. Genetics, vol 3, pp. 27–73.

(11) Gowen, John W. 1920. Conformation and its relation to milk producing capacity in Jersey cattle. Jour. Dairy Science, vol. iii, pp. 1–32.

SUMMARY

In this section data are presented to show the degree of permanence of milk yield and butter-fat percentage from one lactation to another and from the 7-day lactation to the 365-day lactation.

CHAPTER VII

Pedigree Promise and the Performance of Holstein-Friesian Advanced Registry Cows in Milk Yield and Butter-Fat Percentage

Generally speaking the purebred cow in America has one important advantage over the so-called grade or scub cow, namely that the pedigree of the purebred cow may be traced for at least a few generations. If the historical aspects of the live stock industry be examined it will be noted that in the beginning the cattle were without any form of registry or record. The first step toward the improvement of cattle lay in the establishment of a registry system. Soon the name purebred was attached to these registered cattle and the fact of registry began to be taken as an insurance card of true worth. Pedigree came to have a special significance even though it may have had few or no records in it. The Holstein-Friesian breed is in a better position than some of the other breeds in this regard because early in its history the Association was foresighted enough to establish a system of registry for milk records. Yet even in this breed, the pedigree and the animals it contained came to have a value based on the supposed worth of certain of the animals in it rather than on actual milk records. As this is the earliest practice used by the breeder (it is also extensively used today) it is of importance to examine the method carefully before embarking on an analysis of the more recent method of pedigree valuation based on cows with recorded milk production.

MATERIAL AND METHODS

The data for this comparison were derived thus. All Holstein-Friesian cows with 365-day Advanced Registry records up to volume 31, were tabulated. The cow's record nearest eight years old was chosen and corrected for the effect of age.[1] The corrected records of these different cows were arranged under their respective sires.

[1] See chapter on age and milk yield, Chapter IV.

The mean performance for milk yield and butter-fat percentage was tabulated for each sire's daughters. These records were arranged in order of the amount. This order shows the relative merit of the different sires on a progeny performance basis, the contribution of the dams being disregarded. The probable significance of these records is derived from their probable errors.

Forty-eight sires have daughters with sufficiently high milk records to make it probable that their records are significantly above the mean of the Advanced Registry cows. Twenty-nine sires have daughters with milk records so low as to make it probable that their records are significantly below the average of the Advanced Registry cows. Twenty-two sires have daughters with a sufficiently high average butter-fat percentage to make them significantly higher than the average of the Advanced Registry cows. Twenty-two sires have daughters with sufficiently low butter-fat percentage to make their records significantly lower than the average of the Advanced Registry cows. Thirty-three sires without Advanced Registry daughters form the fifth group. The first four groups are on the basis of known progeny performance. The performance of the progeny of the fifth group is an unknown quantity, the group being presented to show the pedigree composition of the Holstein-Friesian breed as a whole. These five groups form the basis of this study.

Inbreeding, and the fear of its supposed degenerating effects, has been and still is a force governing the matings in the cattle industry. In view of this fact it is important to examine these different groups to determine if the practices followed are different in this regard. For this purpose the method developed by Pearl[2] is used.

A study of the four generation pedigrees of these five groups shows average inbreeding and relationship coefficients as indicated in table 53.

The average inbreeding for any group is low compared with the possible inbreeding on the basis of a brother and sister mating. Individually considered the inbreeding in the fourth generation for single animals within the different groups may amount to 30 per cent. Table 53 shows that the inbreeding of the sires of high milk producing daughters is greater than the inbreeding of the sires of

[2] See Studies on inbreeding, I–VIII.

low milk producing daughters but is less than the sires who have no Advanced Registry daughters. Similarly the sires of daughters of high butter-fat percentage are slightly more inbred than the sires of low butter-fat percentage, but slightly less inbred than the sires which have no Advanced Registry daughters. The differences in all cases are small and, compared with their probable errors, are probably not significant. The results do show that the fear of inbreeding in so far as it is normally practiced in the Holstein-Friesian breed, within a sire's pedigree is not well grounded on fact.

The coefficients of relationship are likewise small in average amount. The relationship between the sire and dam of the sires

TABLE 53

Inbreeding and relationship in pedigrees of Holstein-Friesian sires. Pedigreed sires selected for their daughters' performance in milk yield and butter-fat percentage

	Inbreeding				Kinship		
	2d generation	3d generation	4th generation	Total	2d generation	3d generation	4th generation
Sires of high milking daughters	1.04	3.12	8.74	6.52	2.08	5.21	10.41
Sires of low milking daughters	0	1.29	5.40	3.04	0	1.72	4.74
Sires of high butter-fat percentage daughters	1.14	3.41	8.82	6.33	2.27	6.82	11.93
Sires of low butter-fat percentage daughters	0	2.27	6.95	3.86	0	2.27	6.25
Sires of no Advanced Registry daughters	2.27	6.82	9.86	9.82	4.54	7.58	9.85

of high milk producing daughters is greater than between the sire and dam of sires of low milking daughters. Likewise the relationship between the sire and dam of the sires of high butter-fat testing daughters is greater than between the sire and dam of the sires of low testing daughters. The random sample group of sires have a relationship slightly less than that for the high milk yield or high butter-fat percentage groups. The differences are all slight and as judged by their probable errors are not significant. These results indicate that some relationship between the sire and dam of a sire is not detrimental to the production of that sire's progeny, at least within the limits common to the Holstein-Friesian breed.

THE RELATION BETWEEN THE ANCESTORS AND THE MILK YIELDS OF THE COWS PEDIGREED

As mentioned above the feeling has become prevalent among many dairymen that the appearance of certain animals within a pedigree make this pedigree worth more than one where these animals are absent. In other words, certain animals are thought to have a diagnostic value for determining the probable milk yield or butter-fat percentage of their more distant progeny. It will be remembered that in these five pedigree groups there is a group of sires whose daughters excelled in milk production, another whose daughters were poor milk producers, a third whose daughters were high in their butter-fat percentage, a fourth whose daughters were low in their butter-fat percentage, and a fifth group of sires which had neither sons or daughters in the Advanced Registry.

An analysis of the pedigrees of these five groups of sires was made in order to determine which animals appear within each group. To better illustrate the results of such a study the animals reappearing most frequently in these five pedigree groups are given in table 54. The name of the bull or cow is given first. This is followed by the number of times it appeared in each pedigree group, the number of times it appeared on the sire's side of the pedigree being given first, followed by the number of times it appeared on the dam's side.

There are 48 sires whose daughters' average milk yields are sufficiently large to make them significantly above the average of the Advanced Registry cows. The hypothesis was once entertained that the study of the pedigrees of these sires would reveal the fact that these bulls as a group would go back to known ancestors, which were themselves probably excellent as producers, responsible for the worth of each of the 48 sires. If these ancestors, common to the 48 excellent sires, were responsible then it would be possible to get some idea of an unknown bull's probable worth by the study of his pedigree to determine if he could be traced back to these worthy animals. The hypothesis is very attractive.

Several facts should follow if this hypothesis is true or is going to be of any great practical benefit. Thus, if an equal group of 48 sires were selected to include those whose daughters were very low in milk yield, then it should follow that in the pedigrees of these

TABLE 54

Animals repeated eight or more times in the pedigrees of sires (1) whose daughters were significantly high or (2) low milk producers, (3) whose daughters were significantly high or (4) low in their butter-fat percentage or (5) a random sample group of sires without Advanced Registry daughters.

	NUMBER OF APPEARANCES IN PEDIGREES OF				
	(1) High milk yield	(2) Low milk yield	(3) High butter-fat per cent	(4) Low butter-fat per cent	(5) Random sample
Name of bull:					
Paul De Kol 14634	10–16	11–1	4–3	9–8	6–4
DeKol 2d's Butter Boy 21366	8–9	6–5	3–4	4–3	6–4
DeKol 2d's Paul DeKol 20735	8–7	7–1	6–2	4–6	5–4
Milla's Pietertje Netherland 7825	3–6	5–8	4–6	4–1	1–3
Sarcastic Lad 23971	9–7	3–0	2–2	7–1	1–0
Aaltje Salo 3d's Tritomia Netherland 19856	13–7		3–3	2–2	2–0
DeKol 2d's Netherland 11584	4–4	6–2	4–4	2–2	3–0
Manor DeKol 21226	5–6	6–2	3–3	1–2	1–1
Willem III 190 N.H.B.	5–3	6–1	5–3	5–0	1–1
Sir Abbekerk 19056	2–6	3–6	2–4	2–1	1–2
Hengerveld DeKol 23102	8–5	6–2	1–3	1–0	2–0
Aaggie Cornelia 5th's Clothilde Imperial 11822	6–9	3–1	1–0	3–4	
DeKol 2d's Butter Boy 3d 23260	6–4	3–2	3–1	1–1	2–2
Maurice Bonheur 22394	8–3	4–0	1–1	7–1	
DeKol 2d's Prince 2767	1–6	5–0	2–1	3–0	2–3
Pietertje Hengerveld's Paul DeKol 22128	6–2	2–1	3–1	2–2	1–2
Paul Mutual DeKol 18726	4–6	1–0	3–0	1–3	3–0
Johanna Rue 2d's Paul DeKol 21724	7–2	2–1	2–0	2–2	2–0
Paul DeKol Jr. 24762	10–0		2–1	2–2	1–1
Billy McKinley 23378	8–1	3–1	1–1	2–1	
Netherland Alban 4584 H. H. B.	1–2	4–2	3–5		1–0
Pontiac Korndyke 25982	9–1	4–0	2–0		2–0
Duke Netherland 1271	3–1	3–0	2–1	4–0	1–2
Homestead Jr. DeKol 28400	11–0	1–0	2–0	1–2	
Inka Princess Pietertje Netherland 13979	4–4	1–0	0–2	1–5	
King Segis 36168	8–1	3–1	1–0	2–1	
Mercedes Julip's Pietertje's Paul 29830	7–1	3–1	1–0	2–1	
Manor Josephine DeKol 22779	7–1	4–0	2–0		2–0
Aaggie Leila's Prince 4419 H. H. B.	3–0	3–0	5–1	2–0	1–0
Sir DeKol Mechthilde 21506	3–5		0–1	3–0	2–0
Tritomia's Netherland Carl 16406	5–2	0–1	3–1	1–1	
Uncle Hicks 6th 6154	3–1	1–2	2–2	1–0	2–0

TABLE 54—*Continued*

	NUMBER OF APPEARANCES IN PEDIGREES OF				
	(1) High milk yield	(2) Low milk yield	(3) High butter-fat per cent	(4) Low butter-fat per cent	(5) Random sample
Name of bull—continued:					
Johanna Aaggie's Sarcastic Lad 26935....	4–1	2–0		3–1	1–1
Mutual Friend 3d's Paul 23200...........	0–3	1–0	0–1	3–0	3–2
DeKol 2d's Alban 17064..................	2–2	1–3	1–2	1–0	0–1
Maurice Clothilde 17638.................	5–2	1–0		3–1	
Pietertje Hengerveld's Count De Kol 23224	2–3	2–1		2–1	1–0
Sarcastic 4729..........................	5–2	0–1		3–1	
Clothilde 4th's Imperial 1281............	1–4	0–2		0–1	3–1
Silvan Hartog 8161......................	4–0	3–1	2–0		2–0
Empress Josephine 3d's Sir Mechthilde 20255..............................	4–0	2–0	2–0	0–1	2–1
Mechthilde's Sir Henry of Maplewood 6296................................	3–0	0–1	3–1	0–1	2–1
DeKol Burke 22991......................	1–3	1–1	2–0	1–1	2–0
Piebe DeKol Burke 25368................	1–4	1–0	2–0	0–1	2–0
Paul Beets DeKol 22235.................	1–2	1–0		2–4	1–1
Sir Johanna 23446.......................	3–3			2–0	1–0
DeKol 2d's Mutual Paul 25700...........	0–2	1–1		3–0	2–0
Artis' Adriantum's Clothilde 15202.......	0–2	0–2		0–3	1–0
Gem Pietertje Hengerveld Paul DeKol 23300..............................	3–2	1–0	1–1		0–1
Aaggie Cornucopia Johanna Lad 32554...	3–1			2–0	1–1
Name of cow:					
DeKol 2d 734............................	11–17	19–5	12–10	9–4	4–6
Netherland Hengerveld 13106............	5–7	8–4	6–6	3–2	1–0
DeKol 6245 H. H. B.....................	5–3	6–1	5–3	5–0	1–1
Pauline Paul 2199 H. H. B...............	2–6	6–0	3–1	3–0	3–2
Pietertje Hengerveld 24137..............	5–3	3–2	3–1	3–1	3–1
Belle Sarcastic 23039....................	8–3	3–1	1–1	7–1	
Magadora 29237.........................	5–4	4–2	1–3	1–0	2–0
Belle Korndyke 13913....................	8–2	6–1	2–0		2–0
A & G Inka McKinley 55163.............	9–1	4–1	1–1	2–1	
Mercedes Julip's Pietertje 39480.........	8–2	2–1	2–0	2–1	
Segis Inka 36617........................	8–1	3–1	1–1	2–1	
Johanna Rue 2d 33788...................	5–4	0–1	2–0	1–1	3–0
Aaggie Cornucopia 21127.................	4–1	3–1	2–0	2–1	0–1
Sadie Vale Concordia 32259..............	5–2	0–1	0–1	2–2	1–1
Duchess Ormsby 16004...................	2–2	0–2	1–3	1–1	2–0

TABLE 54—*Concluded*

	NUMBER OF APPEARANCES IN PEDIGREES OF				
	(1) High milk yield	(2) Low milk yield	(3) High butter-fat per cent	(4) Low butter-fat per cent	(5) Random sample
Name of cow—continued:					
Homestead Heroine DeKol 46490	9–0		2–0	1–2	
Grace Fayne 2d 44124	9–0	1–0	2–0	1–0	
Johanna Aaggie 36477	4–1	3–0		3–0	1–1
Madame Hengerveld 1333	3–0	3–0	5–1	1–0	
Belvisia 2d 4553	5–2	0–1		3–1	
Grace Fayne 37840	9–0		2–0	1–0	
Kate Korndyke 238 D. F. H. B.	4–0	3–1	2–0		2–0
Rosa Bonheur 5th 11227	5–2	1–0		3–1	
Mutual Friend 3d 28389	0–3	1–1	0–1	4–0	2–0
Colantha 4th 35028	4–4		1–0	1–1	
Johanna Rue 21223	2–5	0–1	1–0	1–1	
Agnes DeKol's Ellen 30228	4–0	2–0	2–0		2–0
Colantha 6714 H. H. B.	2–4			0–1	2–1
DeKol 2d's Queen 6324	2–4	1–0	1–0	1–0	0–1
Johanna 344 H. H. B.	1–4	0–2		0–3	
Korndyke Queen DeKol 41934	4–1	3–1		1–0	
Piebe 2d 7402	2–0	0–2	1–2	1–0	2–0
Aaggie Cornucopia Pauline 48426	4–1		1–0	1–0	1–1
Helena Burke 22916	0–1	1–2	2–0	0–1	1–1
Johanna 4th 2129	3–0	3–0		3–0	
Lunde Beauty 34745	4–2	1–0	0–1		1–0
Mutual Friend 2d 10513	0–4	2–0		0–3	
Aaltje Salo 3d 7403	4–1		1–0	1–1	
Jessie Beets 8123	1–2			0–4	1–0
Johanna 5th 9343	0–4	0–1	0–1	0–2	
Mutual Friend 10139 H. H. B.	0–2	3–0	0–1	2–0	
Piebe Queen 29065	1–2	1–0	2–1		1–0
Pleasant Valley Maid's Pietertje 18314	2–2	1–1	0–1	1–0	
Pontiac Lunde Hengerveld 51585	4–2	1–0	0–1		

sires, there should be few or no ancestors who were frequent in the pedigrees of the sires whose daughters were high producing. Again, if a random selection of sires (to include only those who had neither son or daughter in the Advanced Registry) be made it should be true that the ancestors for this random group should not include those most frequent in the high milk producing group. For if the same ancestors were equally frequent in each group it would follow

that the high milk producing group only had its proportionate number of ancestors of any given family.

Study of table 54 makes it clear that those animals, both bulls and cows, which appear frequently in one group of pedigrees also appear in the other groups, whether these pedigrees be for high or low milk yield or for high or low butter-fat percentage or even for a random sample without records. It is a fact that a given sire repeatedly appearing in one group is almost certain to appear a number of times in some other group. Thus, Paul DeKol, appears 26 times in the pedigrees of the sires whose daughters are high in milk yield. He might well be considered a sire contributing largely to the high milk yield of this group. It is found, however, that he also appears 12 times in the pedigrees of the sires whose daughters' milk yields are low. Considering the fact that there are 48 pedigrees in the high milk producing group to 29 in the low milk producing group, it is evident that Paul DeKol's appearances in the two groups are in about the proportion they should be on the basis of random sampling.

If the comparison is made on a percentage basis it is found that the percentage of times which an ancestor repeats in the high milk producing group is closely similar to the percentage of times that the same ancestor repeats in the low milk producing group, but that either of these percentages (for the high milking group or for the low milking group) tends to be somewhat higher than that found for the random sample group. If the comparison is made for the butter-fat percentage similar facts are found, the high and low groups correspond closely and tend to be higher than the random sample groups. The finding of a celebrated bull or cow in a pedigree is then some indication of whether the animal pedigreed will be found in the Advanced Registry. The reason for this difference in percentage between the Advanced Registry groups and the random sample group is by no means, necessarily one of true worth. In fact, what it most probably means is the well known fact that Advanced Registry testing is carried on much more extensively by certain breeders, thus limiting the Advanced Registry animals to certain family lines. This fact is shown by the correlations between the number of appearances of a given ancestor in one group correlated with the number of appearances in another group for the same ancestor. These correlations are given in table 55.

The first noticeable feature of table 55 is that the bulls and cows have correlation coefficients which are closely similar to each other. Further, all correlation coefficients are plus and of good size. These facts make it clear that the male and female ancestors are governed by the same laws, the discussion of one being equally applicable to the other.

The size of the correlation coefficients brings out the point presented in the above discussion, namely, an animal appearing frequently in one pedigree group also appears relatively frequently in each of the other pedigree groups. The appearance of a worthy

TABLE 55

Correlation coefficients for the frequency of appearance of the bulls and cows within the pedigrees of the sires of the five groups indicated in table 54

GROUPS CORRELATED FOR NUMBER OF APPEARANCES	CORRELATION COEFFICIENT	
	Males	Females
High milk yield and low milk yield	0.595±0.033	0.687±0.027
High milk yield and high butter-fat percentage	0.574±0.033	0.659±0.027
High milk yield and low butter-fat percentage	0.735±0.025	0.645±0.027
High milk yield and random sample	0.474±0.036	0.461±0.036
Low milk yield and high butter-fat percentage	0.667±0.027	0.777±0.025
Low milk yield and low butter-fat percentage	0.496±0.034	0.535±0.033
Low milk yield and random sample	0.500±0.035	0.693±0.024
High butter-fat percentage and low butter-fat percentage	0.390±0.040	0.450±0.040
High butter-fat percentage and random sample	0.627±0.028	0.539±0.032
Low butter-fat percentage and random sample	0.459±0.036	0.392±0.039

animal in the third or fourth generation is of only doubtful diagnostic value for determining the worth of the animal pedigreed.

The cross correlations, those between milk yield and butter-fat percentage groups, are of special interest. A previous paper[3] has shown that the correlation between quantity of the milk yield and quality of the milk is so close to zero as to be scarcely significant. Such being the case, the appearance of the animals within the pedigrees of the cross groups, that is high milk yield and high butter-fat percentage, would be expected to be at random or the correlation

[3] Gowen, John W. 1919. Studies in milk secretion. IV. Variations and mode of secretion of milk solids. Jour. Agric. Research, vol. xvi, no. 3., pp. 79–102.

would be expected to be zero on the assumption that the animals frequently appearing in one group are responsible for that group's worth. Instead of being nearly zero these cross correlations average 0.618 for the males and 0.654 for the females. Again the hypothesis and facts do not agree.

Another factor undoubtedly influences the number of appearances of a given bull. This factor is the popularity of an animal due to advertising and not necessarily to real worth. If this factor is drawn into the discussion as a co-hypothesis to the previous one, the expectation would be quite different from that previously discussed. The cross correlations for the four variables, high milk yield with high butter-fat percentage, high milk yield with low butter-fat percentage, low milk yield with high butter-fat percentage, and low milk yield with low butter-fat percentage, would largely represent the correlation which would exist due to the popularity of the animals alone. The difference between the direct correlation coefficients for high milk yield and low milk yield, and high butter-fat percentage and low butter-fat percentage would represent to some degree the true worth of the animal.

The correlation coefficients for high milk yield and low milk yield are 0.595 for the males and 0.687 for the females. The average cross correlation coefficients are 0.618 for the males and 0.654 for the females. The correlation coefficients for high butter-fat percentage and the low butter-fat percentage groups are 0.390 for the males and 0.450 for the females.

The correlation coefficients for the milk yield clearly do not differ significantly from the cross correlation coefficients. Such being the case the following conclusions appear justified for milk yield.

a. The magnitude of the average cross correlation makes it clear that popularity of the bull or some similar cause rather than true worth plays a large part in causing the appearance of an animal within a pedigree, at least, in the third and fourth generations.

b. The great similarity of the correlation between the bulls appearing in the high milking group and the bulls appearing in the low milking group with the average cross correlation shows that the appearance of a bull in the pedigree of the high group is no necessary guide to that animal's worth.

c. The fact that a bull whose progeny performance is unknown, has a well known bull or bulls in his pedigree is unfortunately no

indication of that unknown bull's worth. The correlation coefficient for the high butter-fat percentage and low butter-fat percentage (0.390) is lower than the average of the cross correlations (0.618), by an amount which is, probably, significant.

The correlation coefficients for the number of appearances of the different animals within the random sample group compared with the appearances of these animals in the four Advanced Registry groups are closely similar to the other correlations previously obtained. Such being the case, the conclusion appears irresistible that the pedigree make-up of this random sample group is closely similar to that of the other group, although the frequency of any given popular ancestor may be somewhat less.

SUMMARY

The data relate to the value of the pedigree as an indication of productive capacity where knowledge of milk or butter-fat percentage records is assumed to be lacking. The analysis leads to certain conclusions important to the cattle breeding industry.

The inbreeding which exists in the pedigrees of Holstein-Friesian Advanced Registry animals is relatively small in average amount. This inbreeding shows no significant effect, good or bad, on the productivity of the offspring.

The relationship between the sire and dam of these same animals is also small in average amount. This amount of relationship shows little or no effect in the resulting milk yield or butter-fat percentages of the offspring.

Five groups of sires were available for pedigree study. The first group had daughters of significantly high milk yield; the second group had daughters of significantly low milk yield; the third group had daughters of significantly high butter-fat percentage; the fourth group had daughters of significantly low butter-fat percentage; and the fifth group had no record daughters or sons and was a random sample of the breed.

A study of these five groups was made to determine the frequency of the ancestors composing their pedigrees. It is clearly evident that the ancestors in the fourth generation had most chance of repetition while those in the second had least. The results derived from this study may be summarized as follows:

The nearly equal frequency with which any given bull or cow appears in the five groups makes it clear that the appearance of a supposedly worthy ancestor in a pedigree indicates little as to the real worth of the animal pedigreed. The magnitude of the cross correlations, ancestors in milk yield groups with ancestors in butter-fat per cent groups, shows that popularity of an ancestor, or some similar cause, rather than true worth plays a large part in causing the appearance of an animal within a pedigree, at least, in the third and fourth generation.

In the series of chapters which are to follow, the attempt will be made to show the relation of an ancestor's milk record to the future performance.

CHAPTER VIII

Analysis of the Stringency of Selection as Exerted by the Holstein-Friesian Advanced Registry Requirement

Evolution, or progress in animal breeding, rests on a logical base composed of two postulates. The first is that those individuals who are allowed to reproduce at any given time shall be superior in some quality, say milk yield or butter-fat percentage. Under this rule there is a selective elimination of the inferior individuals, or unfit, as measured by the character which is desired. The second postulate is that those individuals who are selected shall be capable of transmitting in some degree their superior quality to their offspring. The acceptance as a fact that these two principles are always in operation has given ground for the belief that the milking qualities of our present day cattle are due to the selection which has been practiced in the past by nature and more particularly by man. To further the progress by increasing the selection, the different breed associations have formed advanced registries wherein may be entered those animals who meet a minimum requirement. If it be granted that the above postulates of evolution apply to the Holstein-Friesian breed, then the conclusion is unavoidable that the milk yields of these cattle will be increased by a reproductive selection based on the Advanced Registry records. If, however, either of the two postulates is found inactive, the whole structure is weakened. The analysis of the problem on which we are about to embark is consequently an important one for at least three reasons. In the first place, it gives some insight on how much evolution there has been in dairy cattle and what the prospects are for the future. It furnishes concrete data on the general problem of selection from an angle important to the evolution problem not only for itself, but also because of its historical interest, in that the productivity of economically important animals has been cited as crucial evidence supporting natural selection. While these problems are important,

the writer's interest lies more particularly in the third problem, namely, if the Advanced Registry requirement is in truth stringently selective it is necessary before being able to use these records for inheritance studies to make suitable correction for this double selection, selection of offspring and selection of parent.

MATERIAL AND METHODS

Besides the Holstein-Friesian data already described and familiar to the reader the writer has made use of two other sets of published data for comparative purposes. The first of these for the Jerseys[1] gives all the 8 months milk yields of a purebred herd. The second of these gives a random sample of the mean weekly milk yield of all Scottish Ayrshire[2] cows. These last two data are subject to no selective influence like the Advanced Registry requirement. They represent all cows good, bad, or indifferent and should give a fair indication of what would be expected of a random sample of any breed.

In addition, the author has collected the opinion of some of the Holstein-Friesian breeders here in Maine on the questions: (1) The number of registered Holstein-Friesian cows then in their herd which were 3 years or more of age; (2) the number of these registered cows 3 years or more of age which the owner believed could make the Advanced Registry requirements.

THE STRINGENCY OF SELECTION EXERCISED ON MILK BY THE HOLSTEIN-FRIESIAN ADVANCED REGISTRY REQUIREMENT

Before presenting the data it may be well to consider the problem in some detail. The Advanced Registry requirement furnishes a sharp line cutting off all cows whose capacities are such that they cannot make more than the requirement for entry into the Advanced Registry. Were it possible to get a random sample of the milk yields of all the Holstein-Friesian cows we could form a frequency

[1] Gowen, John W. 1920. Studies in milk secretion. V. On the variations and correlations of milk secretion with age. Genetics, vol. 5, pp. 111–188.

[2] Pearl, Raymond, and Miner, John R. 1919. Variation of Ayrshire cows in the quantity and fat content of their milk. Jour. Agric. Research, vol. xvii, no. 6, pp. 285–321.

distribution to show the milk yields of these cows. If the Advanced Registry requirements were sufficiently low so that even the poorest cow of this random sample milked more than was required for entry, then the curve X to Y would remain complete as shown in figure 15. If, on the other hand, the Advanced Registry requirements were higher, cutting the random sample curve at the line A, we should have only the crosshatched section of the curve, A to Y, appearing in the Advanced Registry. The records appearing in the Advanced Registry would also be sharply curtailed showing the abrupt chopping off caused by the requirement for entry.

The requirement for entry into the Holstein-Friesian Advanced Register is based on the amount of butter-fat. Amount of butter-fat in any given milk is dependent on two variables, the amount of

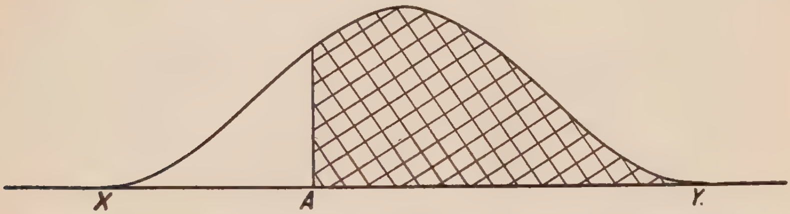

Fig. 15. Frequency Distribution Showing the Hypothetical Distribution of Milk Yield X to Y

Line A shows the cut which might result from an Advanced Registry requirement.

milk which the cow yields and the composition of this milk. In another section it has been shown that these variables are practically independent of each other in the Holstein-Friesian breed. In view of this fact the two basic variables, milk yield and butter-fat percentage, will receive most attention. This chapter will then be devoted to an analysis of the effect of the Advanced Registry requirement on them. Table 15 of Chapter IV shows the distribution of milk yield for each six months interval of age as compiled from 2586 Advanced Registry records.

As noted above, if the requirements are keeping a good proportion of cows from entry into the Advanced Registry there should be a sharp cutting off of the low producers in this table. No such cutting off of the low producers is noted. Unfortunately there is no good random sample Holstein-Friesian data for milk yield known

to the writer. Two sets of data, one on the Jersey[3] the other on the Ayrshire, are known and have been analyzed. These data show that random sample milk yield within any one breed and at any given age is slightly skew, generally in the plus direction, and with a fairly large range as compared with the average milk yield. The appearance of table 15 seems to bear out these same facts for the Holstein-Friesian Advanced Registry cattle. The problem is of so much importance that it appears wise to go further in the analysis and obtain the biometrical constants of the data. For this purpose the total populations are used. Using Sheppard's corrections we find the analytical constants for the milk yield of the Holstein-Friesian cows, the Jersey cows, and the Ayrshire cows to be those shown in table 56.

The means and standard deviations for the milk yield of the different groups are not comparable because of the different units used in the compilation of the data. Unfortunately, too, the mean ages of the different groups are not alike. The Holstein-Friesian Advanced Registry cattle are tested earlier than the Jerseys by one year and earlier than the Ayrshires by nearly two years. The Holstein-Friesian cattle are less scattered around the mean age than are either the Jerseys or Ayrshires.

Despite these differences the constants of the distributions are surprisingly alike in many particulars. The coefficients of variations are all large. The Holstein-Friesian Advanced Registry cows and Jersey random sample cows correspond closely in this particular. The Ayrshire cows have a slightly smaller coefficient of variation.

The moments have a general resemblance. The second moment is small compared with the rest. The third moment is plus in sign. The fourth moment is large compared with the rest.

The Holstein-Friesian and Jersey data correspond closely in the value of β_1. The Ayrshire data has a β_1 definitely smaller than that for the other two distributions.

The Holstein-Friesian and Jersey random sample data have β_2's which correspond closely in value. The Ayrshire data's β_2 is smaller in value.

The Holstein-Friesian and Jersey data have minus values of κ_1. The Ayrshire data has a plus κ_1. All of the values of κ_1 are subject to relatively large probable errors.

[3] Loc. cit.

The sign of κ_2 depends on the sign of κ_1. All of the values of κ_2 are subject to large probable errors. This fact shows that in each case, the type of the curve necessary to best fit the data is somewhat in doubt due to the fact that the critical point for two or more type curves is near.

The type of the curve called for by the criterion is the same for the Holstein-Friesian and for the Jersey data. The Ayrshire data calls for a curve of a different type.

TABLE 56

Analytical constants for milk yield

CONSTANTS	HOLSTEIN-FRIESIAN (365-DAY MILK YIELD)	JERSEY (EIGHT MONTHS MILK YIELD)	AYRSHIRE (AVERAGE WEEKLY MILK YIELD)
Mean	$16,234\pm54$	$4,888\pm20$	16.49 ± 0.03
Standard deviation	4039 ± 38	$1,250\pm14$	3.37 ± 0.02
Coefficient of variation	24.9 ± 0.4	25.6 ± 0.3	20.24 ± 0.17
μ_2	16.233	6.247	11.1345
μ_3	45.246	9.662	9.5595
μ_4	930.958	138.36	395.0781
β_1	0.4786 ± 0.0527	0.3829 ± 0.0613	0.0662 ± 0.0099
β_2	3.5330 ± 0.1484	3.5454 ± 0.1979	3.1867 ± 0.0608
β_2-3	0.5330	0.5454	0.1867
κ_1	-0.3698 ± 0.2073	-0.0581 ± 0.2582	0.1748 ± 0.1076
κ_2	-1.0876 ± 0.0152	$-5.4182\pm$*	0.2888 ± 0.2675
Skewness	0.3901 ± 0.0218	0.3150 ± 0.0222	0.1218 ± 0.0099
Type of curve	I	I	IV
Mean age	4.57	5.51	6.44
Standard deviation age	2.25	2.63	2.68

* Probable error very large.

Calculated by the writer from the data of Pearl and Miner.

All the distributions are significantly skew in the plus direction. The curves for the Holstein-Friesian and Jersey data correspond quite closely in their skewness. The Ayrshire curve is less skew. It will be noted that the skewness becomes less as the mean age of the different groups increases. The concentration of the records in the early ages would tend to make the data for the Holstein-Friesian more skew than the Jersey and the Jersey more skew than the Ayrshire.

The comparison shows that the curves describing the Holstein-Friesian Advanced Registry records for 365-day milk yield do not exhibit any marked differences from the whole milking population of dairy cattle within the Jersey or Ayrshire breeds. In fact the Jersey random sample differed fully as much from Ayrshire as did the Holstein-Friesian Advanced Registry cows from either one. We note further that an examination of the raw Holstein-Friesian data as given in table 15 (Chapter IV) does not show any noticeable evidence for the truncation of the records by the Advanced Registry requirement. In view of these facts, these Holstein-Friesian Advanced Registry records appear to be a fairly good random sample of the breed, at least practically speaking. This means that if all the purebred cows of the Holstein-Friesian breed were put on test, fed, and treated as cows on test for the Advanced Registry, practically all of the cows so tested would make the requirement or more.

Some data of Pearl's[4] further supports the general conclusion that the entry requirement for 365-day milk yield excludes few or no cows from the Advanced Registry. In this paper Pearl says,

> These records (Scottish Milk Record Society) correspond to American cow-test association records in this particular that the records of *all* cows in each herd good, bad, and indifferent, are included. During this past year we have been able. . . . to compare the American Advanced Registry Ayrshire records with these Scottish records. The results of such comparison, in part, are shown in table I.

TABLE I

Comparing mean weekly yields (in gallons) of (a) American Advanced Registry and (b) Scottish Milk Record Society Ayrshire cows

AGE OF COW	AMERICAN ADVANCED REGISTRY	SCOTTISH MILK RECORDS SOCIETY	DIFFERENCE
Two years	14.84±0.08	13.61±0.18	1.23
Three years	16.76±0.14	13.84±0.04	2.92
Four years	17.47±0.14	15.23±0.06	2.24
"Mature"	20.32±0.13	18.56±0.09*	1.76

* This figure is for nine-year-old cows.

[4] Pearl, Raymond. Report of progress on animal husbandry investigations in 1915. Me. Agric. Exper. Sta. Misc. Publ. 519, pp. 1–27, 1915.

From this table it will be seen that the American Advanced Registry Ayrshires outyield their Scottish sisters, on the average, from about $1\frac{1}{4}$ to 3 gallons per week, or roughly from 10 to 25 pounds. Looked at from a relative standpoint it appears that the American Advanced Registry animals give, as two-year-old heifers or as mature cows, about 9 per cent on the average more milk than the Scottish herds. For the three-year and four-year ages the percentage is higher.

From these data the necessary arithmetic shows that were the American Advanced Registry requirement to be applied to the Scottish Ayrshires there would remain in such a registry approximately 4972 cows out of the 6934 now having records. The percentage remaining would be 71.7. These Scottish cows are not subject to the previous fitting that cows coming to Advanced Registry test receive. They are not milked three times a day. In fact they receive just ordinary care. In view of this fact it is entirely fair to say that not more than 28.3 per cent of Ayrshire cows are excluded from Advanced Registry by the requirement for milk yield and in all probability with the care and feeding received by American cows on Advanced registry test, the percentage would be very much less.

THE STRINGENCY OF SELECTION EXERCISED ON THE BUTTER-FAT PERCENTAGE BY THE HOLSTEIN-FRIESIAN ADVANCED REGISTRY REQUIREMENT

The requirement for entry into the Holstein-Friesian Advanced Registry should have much the same influence on the butter-fat percentage as it does on the milk yield. The problem is of the same importance to the use of Advanced Registry records for any inheritance studies on butter-fat percentage. The data for the analysis of this problem in the Holstein-Friesian breed are given in table 24 (Chapter IV). Figure 15 illustrates how a truncating agent like the Advanced Registry requirement may affect the records. If the requirement for butter-fat percentage is so low that it falls below X then the whole of the curve will be represented in the Advanced Registry. If on the other hand, the butter-fat percentage requirement is higher cutting the curve at A, then only that portion of the curve contained between A and Y will be present.

The correlation surface of table 24 (Chapter IV) gives the individual distribution of butter-fat percentage for each 6 months

of age. The distributions appear to be fairly regular. No curtailing of the distributions is noticeable.

The frequency distribution for these Holstein-Friesian Advanced Registry butter-fat percentages has been analyzed for its analytical constants. The distribution taken for this analysis is that for the whole population irrespective of age. Jersey[5] random sample data, an analysis of which was published previously, and the Ayrshire data of Pearl and Miner are tabled for comparative purposes. These constants are found in table 57.

TABLE 57

Analytical constants for butter-fat percentage

CONSTANTS	HOLSTEIN-FRIESIAN (365-DAY BUTTER-FAT PER CENT)	JERSEY (EIGHT MONTHS BUTTER-FAT PER CENT)	AYRSHIRE (AVERAGE WEEKLY BUTTER-FAT PER CENT)
Mean	3.428±0.004	5.226±0.007	3.740±0.003
Standard deviation	0.309±0.003	0.449±0.005	0.349±0.002
Coefficient of variation	9.014±0.121	8.595±0.100	9.342±0.076
μ_2	9.4890	8.9668	3.0521
μ_3	11.2127	6.8443	2.8619
μ_4	311.6042	284.5584	45.3372
β_1	0.1471±0.0329	0.0650±0.0271	0.2881±0.0641
β_2	3.4607±0.1569	3.5391±0.2072	4.7018±0.8262
β_2-3	0.4607	0.5391	1.7018
κ_1	0.4799±0.2488	0.8833±0.3765	2.5394±1.5293
κ_2	0.2387±0.1193	0.0563±0.0279	0.0938±0.0478
Skewness	0.1670±0.0161	0.1003±0.0209	0.1617±0.0253
Type of curve	IV	IV	IV
Mean age	4.57	5.51	6.44
Standard deviation age	2.25	2.63	2.68

Certain innate differences appear in the data. The age of the Holstein-Friesian cows is about one year less than the age of the Jersey cows and about two years less than that of the Ayrshire cows. The breeds differ in their butter-fat percentage, the Holstein-Friesian having the least, the Ayrshire next, and the Jersey the most. There is another difference to be noted in the fact that the Holstein-

[5] Gowen, John W. 1920. Studies in milk secretion. VI. On the variations and correlations of butter-fat percentage with age in Jersey cattle. Genetics, vol. 5, pp. 249–324.

Friesian butter-fat percentage is not affected appreciably by the age of the cow, whereas the butter-fat percentages of the Jersey and Ayrshire cows are influenced by the age of the cow.

Despite these differences the three curves show certain common characteristics. The relative variation of the three curves is of about the same magnitude. It will be noted that the Holstein-Friesian Advanced Registry data is in between that of the random sample Jersey and the random sample Ayrshire in its variation.

The third moment is plus in every case. The β_1 for the Holstein-Friesian curve is in between that for the Jersey random sample and the Ayrshire random sample. The values of β_2 for the three curves are not significantly different from each other. The κ_1 is plus in every case. The actual values of κ_1 are not significantly different. The values of κ_2 are closely similar to each other.

The skewness of the three curves is in the same direction, plus. The values of the skewness in the three cases are nearly the same. The type of curve necessary to fit the data is Pearson's type IV.

In view of the close similarity between the curves it seems evident that the Holstein-Friesian Advanced Registry butter-fat percentage curve has not been seriously curtailed by the Advanced Registry requirement. On the other hand there is evidence which points to the probability that a selection has occurred. The examination of the 7-day records of the whole Advanced Registry and those of the cows which were continued to the completion of the 365-day test showed a difference in the average 7-day test of the two groups, favoring a higher 7-day record of cows continued to the 365-day records. Thus in a random sample of the 7-day record cows completing 365-day records it was found that these 7-day records were 31 pounds of milk above the average of all 7-day records. The butter-fat percentage of these cows was 0.16 per cent more than the average of all 7-day records. Some selection appears to have taken place in these records, the selection being such that the frequency curves remain comparable with those which might be expected of a random sample of the breed. Such a difference would correspond to a selection containing 80 per cent or more of the original population. There are, however, some difficulties in the way of considering this figure as the measure of the selection which has been practiced. The writer corresponded with breeders selected on the basis of their doing or of their having done some Advanced Regis-

try work. Eighteen breeders were in the list. Two hundred and seventy-five head, three years or more old, were represented. Of these cows, the breeders said that 271 could make the Advanced Registry. The four which were not indicated as able to make the requirements were all from one herd. The percentage of cows which could make the Advanced Registry was 98.5 per cent. This percentage bears out the other previous results.

These conclusions may be surprising to some. They are, however, reflected in the growing agitation among certain breeders for a more occlusive Advanced Registry requirement. These breeders believe that the Advanced Registry requirement as we now have it is too low, letting in almost all those cows which are tested. We need not go into the pros and cons of this argument as it is not germane to our subject, however important it may be. We need only notice that others, on what are probably purely observational grounds, have arrived at a conclusion similar to ours derived from the analysis of the data.

In using the records for the 7-day Holstein-Friesian cows, Rietz[6] has approached the problem in another way and with different results. Instead of attempting to determine the total population of Holstein-Friesian cows from the Advanced Registry tail portion either by the method of Pearson or by comparison of the characteristics of this curve with random sample data, he determines what proportion of the cows milking at a given time are in the Advanced Registry. His results for the year May 15, 1906, to May 15, 1907, are given in the table below.

	0 to 2.5 years	2.5 to 3.5 years	3.5 to 4.5 years	4.5 years and over	Total under 4.5 years
Cows in milk (N)	618	494	430	1,416	1,542
Number in Advanced Registry (n)	413	346	269	620	1,028
$\frac{n}{N}$	0.67	0.70	0.63	0.44	0.67

He then assumes that the proportions are the true proportions based on the capabilities of the Holstein-Friesian cows. That is, for those cows under 2.5 years old 67 per cent of them could if put

[6] Rietz, H. On inheritance of production in butter-fat. Biometrika, vol. 7, pp. 106–126, 1909.

on Advanced Registry test make the requirement and 33 per cent could not. It seems to the writer that this assumption is faulty. The great element in preventing Advanced Registry testing is not that the cow cannot make the Advanced Registry requirement, it is the actual cost of the supervisor and incidental expenses in making the test. Time and time again it has been demonstrated in the work that an inexperienced man, once convinced of the merits of Advanced Registry testing and placing his cows on test, has made some of the best records in the state. Not only this, but the cows put on test have in the majority of instances made the test with some to spare. In the case of those who did not make the requirement it has often been shown that this cow would make it under more favorable conditions. It seems to the writer, therefore, that the assumption involved on the method adopted by Rietz is questionable and may quite seriously alter the conclusions drawn from the data when they are handled in this way.

After balancing these considerations it was decided to make no corrections for what selection may have been practiced in these Advanced Registry records. Undoubtedly some correction should be made and the making of the correction would tend to increase the correlation coefficients between the daughter and her parents and grandparents, emphasizing the even greater importance of heredity than that portrayed by the results as they stand. On the other hand, the amount of correction to be applied is by no means clear. Also, if the results are used as they are they will apply to the Advanced Registry animals as the records are given.

SUMMARY

This paper presents an analysis of the Holstein-Friesian Advanced Registry 365-day records in comparison with other data on a random sample of the Jersey and Ayrshire breeds. The biometrical constants in general use for the analysis of such curves are presented. The comparisons indicate that the Holstein-Friesian Advanced Registry 365-day records for milk yield and butter-fat percentage are not far from being a random sample of the Holstein-Friesian breed supposing that all of the cattle were kept under conditions which were identical in feeding and care with those given the Holstein-Friesian cows which make the Advanced Registry. Other data indicate that some selection has taken place although the amount of this selection is not clear.

CHAPTER IX

On the Relation of the Milk Yields of a Sire's Daughters' Full Sisters

The spirit which must govern all work on cattle breeding in its search for truth is so well expressed by a letter received by the author that it is quoted:

> The dairy cattle industry has some uncertain and fragmentary information along the lines of the physiology and breeding of dairy cattle for milk production. This fragmentary information, often conflicting though it is, has governed the breeding operations of our industry. The attempt to present definite information along these hitherto uncertain lines is most encouraging.

It is a truth that the research worker in the cattle industry today is extremely unlikely to hit on anything of which someone may not say, "that idea was mine ten years ago," because if the writings of this industry be examined one will find almost all possible kinds of ideas on how to breed the big yielding cows. We can only hope to present concrete information in place of our now uncertain methods. This chapter is an attempt to present this information on just one phase of this subject and for only one breed. It is proposed to start with an analysis of the influence of the sire on his daughters. In other words to determine if possible whether milk yield is really inherited in our great breeds of cattle, and if it is inherited what we may expect when certain kinds of matings are made.

It is, of course, impossible to measure the milk yield of the daughter and compare it with the sire to determine any relation which may exist. Instead we have to resort to various artifices. If milk yield is truly inherited we should expect that the daughters of a sire would resemble each other more closely than would pairs of cows taken at random from the breed. If the daughters were full sisters we should expect this resemblance to be still greater, for here the influence of the dam is also exerted to make for a greater resemblance between the two daughters. We shall devote our attention to the degree of resemblance which may be expected of full sisters as a means of determining the sire's influence on milk yield.

MATERIAL AND METHODS

The data used for this study were the records for year tests of Holstein-Friesian cows contained in volumes 13 to 31 of the Advanced Register of the Holstein-Friesian Association.[1] The data and rules under which these tests are carried on are familiar to all. All records have been corrected for age by the methods previously indicated. The rules governing the selection of the cow's records were arbitrarily chosen to include the following items:

1. If a cow has one 365-day record that record is used.
2. If a cow has two or more 365-day records the record selected is that nearest eight years old.
3. The records of cows for less than 365-days are omitted from this study.

When a record is mentioned in subsequent sections of this book it is to be taken for granted that such a record is an age corrected record. These age corrected records are used without further correction for the reasons previously indicated.

The milk records of the daughters who are full sisters are correlated with each other to determine the relationship which exists between them, the ordinary biometrical methods being used. In the same manner the relationship which exists between the milk records of half sisters is also determined. These half sisters are, of course, those who have a common sire but different dam.

THE CORRELATION BETWEEN THE MILK YIELDS OF FULL SISTERS IN THE HOLSTEIN-FRIESIAN ADVANCED REGISTRY

One measure of a sire's worth is his ability to impress upon his offspring the qualities which are desired. For milk production this worth can be measured only through the milk yield of the sire's daughters or through the productive qualities of his sons' daughters. In this paper only the sire's daughters will be considered. These daughters are of two kinds, those which are full sisters and those which are half sisters.

The milk yields of a sire's daughters are dependent to a certain extent on the milk yields of their dams. It is unlikely that two cows will have an identical milk yield or that the inheritance for milk

[1] Holstein-Friesian Association of America. 1902–1920. Advanced Register Year Book, vol. 13–32.

production which they could give their daughters would be identical. Such being the case it is evident that two daughters of the same sire and from a given dam would be expected to have milk yields more

TABLE 58

Correlation surface showing the milk yields of full sisters in the Holstein-Friesian breed

MILK YIELD OF FULL SISTER	MILK YIELD OF FULL SISTER																						
	10,000–11,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000–32,000	
10,000–11,000					1																		1
11,000			2		2		1		1														6
12,000		2		1		2	1				1		1										8
13,000			1	2		2	2	1	1	2													11
14,000	1	2			6	4	2	1	3														19
15,000			2	2	4	2	4	3	1	2			1			1							22
16,000		1	1	2	2	4	6	2	2	4	1	2	1	2	2	1							33
17,000				1	1	3	2	2	2	2	4		1		1								19
18,000		1		1	3	1	2	2	2	3	3		1	3			1		2			1	26
19,000				2		2	4	2	3	2	3	5	2	4		1			1				31
20,000			1				1	4	3	3	4	5	2	1					1				25
21,000							2			5	5		5	1			1						19
22,000			1			1	1	1	1	2	2	5	6	2	1		1						24
23,000							2		3	4	1	1	2		2	3		1	1				20
24,000							2	1					1	2	2	2							10
25,000						1	1			1				3	2	2		1				1	12
26,000									1			1	1				2						5
27,000														1		1							2
28,000									2	1	1			1								1	6
29,000																							
30,000																							
31,000–32,000									1							1			1				3
	1	6	8	11	19	22	33	19	26	31	25	19	24	20	10	12	5	2	6			3	302

nearly the same than would another set of two daughters from the same sire but different dams. Study of these Holstein-Friesian records shows that actually there is such a difference. The milk yields of full sisters are more closely similar than are the milk yields of the half sisters when both sets are daughters of the same sire. There are approximately 2600 records available. Of this number

about 1700 are either full or half sisters. The number of sires involved is about 450. All records are age corrected and for the 365-day period. There are 302 full sisters within this body of records. The correlation surface showing the milk yield of the first and second sisters is given in table 58.

Table 59 gives the fundamental constants for the variation and correlation of the milk yields of the full sisters.

It will be noted that the milk yield of the full sisters is closely similar to that found for the whole breed. The standard deviation and coefficient of variation for the full sisters is likewise practically the same as that for all the records. Under these circumstances it seems clear that the full sisters as a group are closely similar to the whole Advanced Registry as a group.

TABLE 59

Variation and correlation of the milk yields of full sisters

Mean milk yield	19,348±159
Standard deviation of milk yield	4,102±113
Coefficient of variation of milk yield	21.2±0.9
Correlation between the milk yield of one full sister and that of another	0.548±0.027

The correlation coefficient representing the degree of relationship found between the milk yields of full sisters is high. The milk yields of full sisters resemble each other quite closely. This resemblance may be due to the common inheritance received from the sire or from the dam or both. It may also be due to a similarity in the environment under which the full sisters were tested. The relative influence of these two factors will be taken up later.

The data presented on table 59 assist us to answer these problems of the dairy breeding industry. In the light of our present knowledge of dairy husbandry the possible milk yield of the daughters resulting from any type of mating is an open question. The data just given will assist us to answer this question for full sisters of the Holstein-Friesian breed kept under conditions similar to those of the Advanced Registry animals. Concisely stated the first problem is then, what will the probable milk yield of a cow be, when her full sister has a given milk yield, e.g., 20,000 pounds of milk?

We know from experience that the milk yields of these sisters will vary, we desire to know within what limits these milk productions may vary. The second problem is, consequently, to determine within what range of milk production the subsequent milk production of other sisters will be when the milk production of one sister is known. This problem is really a double one. In the first place we desire to know the probable range of milk yield within which a sister

TABLE 60

Milk yield of other daughters when the milk yield of the first daughter is known. Full sisters

FIRST DAUGHTER'S MILK YIELD	EXPECTED OR AVERAGE MILK OF SECOND DAUGHTERS
10,000	14,229
11,000	14,777
12,000	15,324
13,000	15,872
14,000	16,419
15,000	16,967
16,000	17,514
17,000	18,062
18,000	18,609
19,000	19,157
20,000	19,704
21,000	20,252
22,000	20,799
23,000	21,347
24,000	21,894
25,000	22,442
26,000	22,989
27,000	23,537
28,000	24,084
29,000	24,632
30,000	25,179

will be found and in the second place, the probable maximum variation of the full sisters' milk yields.

For the first problem, we define as the probable range, the range to include on the average 50 per cent of all the full sisters. For the second, we define the limit of the range as that which will include approximately 99 per cent of all full sisters. This is considered to be the extreme limits within which the full sisters will be likely to vary.

These questions apply equally to the daughters that are full sisters and that are half sisters. Although of immediate practical importance the answers to these questions have a theoretical value as well, for the answers to them furnish critical evidence on the problem of whether or not milk yield is inherited. It must be remembered, however, that these data are Advanced Registry data. Such being the case, their actual numerical application is probably limited to those cows which are under the same conditions of feeding, milking, and handling as pertain to the conduct of Advanced Registry testing.

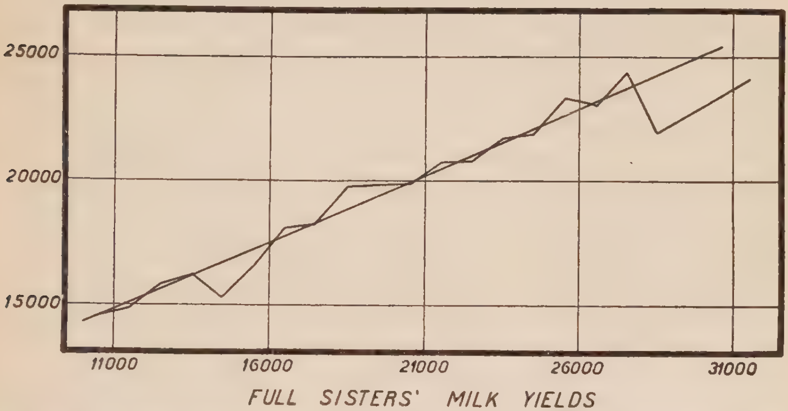

FIG. 16. THE REGRESSION OF THE MILK YIELD OF ONE FULL SISTER ON THE MILK YIELD OF THE OTHER FOR HOLSTEIN-FRIESIAN ADVANCED REGISTRY CATTLE, 365-DAY MILK RECORDS

PROBABLE MILK YIELD OF THE OTHER FULL SISTERS WHEN THE MILK YIELD OF THE FIRST FULL SISTER IS KNOWN

The knowledge of the milk yield of one sire's daughter enables us to predict the probable yield of another daughter. The expected milk yields for the second daughters where the milk yield of the first daughter is known are given in the table 60.

The equation of the curve used to make this prediction is found from the data presented in table 59. The equation is the ordinary linear regression equation:

$$Y = 8754 + 0.5475y \qquad (36)$$

where Y is the milk yield of the second full sister and y the known

milk yield of the first full sister. There are two assumptions in using this equation. The first is that the regression is in truth linear and homoscedastic. The nature of the regression lines is shown in figure 16.

Figure 16 shows that the relation between the milk yields of full sisters is a linear relation. The theoretical regression line follows the observational regression line nicely. The correlation table showing the distribution of the individual pairs appears to be sufficiently close to the Gaussian surface so that no grave error can be made in general conclusions from the results.

The examination of table 60 shows that where the first daughter is very low in production, say 10,000 pounds, the second daughter's production is also relatively low compared with the rest of the breed. The second daughter's production is, however, considerably larger than that of her 10,000-pound sister. If the production of the first daughter is extremely large, the second daughter's milk yield is also large compared with the rest of the breed. On the other hand the second daughter's production is less than the first daughter's by a significant amount. The average milk yield of all full sisters is 19,348 pounds for the year period. When a full sister has this production, the second full sister also has the same average production. Clearly, the lowest producing cows tend to have full sisters who are also low in their production, the medium producing cows tend to have medium producing sisters, and the high producing cows tend to have high producing sisters. This is, of course, the expectation if milk yield is inherited for the two sisters have much more chance of a common inheritance when they come from common parents than would two cows picked up at random within the breed.

VARIATIONS OF THE MILK YIELD BETWEEN WHICH 50 PER CENT OF THE SECOND FULL SISTERS ARE FOUND WHEN THE MILK YIELD OF THE FIRST FULL SISTER IS KNOWN

The correlation table for the different pairs of full sisters shows us that there is a rather wide variation of the milk yield of the second sisters even when the milk yield of the first sister is known. That is, while these milk yields will average 16,419 pounds for the second full sisters when the first full sister's milk yield is 14,000 pounds, there will be a variation of yield of the second full sisters so that some will be above this figure and some below it. It is of a

good deal of interest to know what this variation is and within what limit of milk production one can reasonably expect the milk yield of the second full sisters. If we consider the range within which one half the second full sisters should be found, as a reasonable limit, then the data of the next table furnishes an answer to this question. Table 59 gives the necessary data for this calculation. It may be

TABLE 61

Milk yields of first full sisters and the limits between which would lie the milk yields of 50 per cent of their second full sisters

FIRST FULL SISTER'S MILK PRODUCTION	RANGE OF MILK YIELDS OF 50 PER CENT OF THE SECOND FULL SISTERS
10,000	11,913–16,545
11,000	12,460–17,093
12,000	13,008–17,640
13,000	13,555–18,188
14,000	14,103–18,735
15,000	14,650–19,283
16,000	15,198–19,830
17,000	15,745–20,378
18,000	16,293–20,925
19,000	16,840–21,473
20,000	17,388–22,020
21,000	17,935–22,568
22,000	18,483–23,115
23,000	19,030–23,663
24,000	19,578–24,210
25,000	20,125–24,758
26,000	20,673–25,305
27,000	21,220–25,853
28,000	21,768–26,400
29,000	22,315–26,948
30,000	22,863–27,495

shown that the average standard deviation of the full sisters' milk yields about any average milk yield for the different classes is $4102 \sqrt{1-0.548^2}$ or is equal to 3434 pounds of milk. The limits of milk yield necessary to include 25 per cent of the second full sisters on either side of the mean, when their first sister's milk yield is known, are given by mean $\pm 0.67449 \times 3434$ assuming a Gaussian surface. These limits are given in table 61.

In common with most results that pertain to milk yield or butter-fat percentage, the milk yields of the second full sisters exhibit

quite a variation. Thus the full sisters to cows which produced 20,000 pounds of milk would have an even chance to produce between 17,388 to 22,020 pounds of milk or to produce less than 17,388 pounds or more than 22,020 pounds. Expressed concretely, of 100 full sisters of 20,000-pound cows we should expect, 25 to yield less than 17,388 pounds of milk, 50 to yield between 17,388 and 22,020 pounds, and 25 to yield more than 22,020 pounds.

TABLE 62

Milk yields of first sisters and the limits between which would lie the milk yields of 99 per cent of their full sisters

FIRST FULL SISTER'S MILK PRODUCTION	RANGE OF MILK YIELDS OF 99 PER CENT OF THE SECOND FULL SISTERS
10,000	5,369–23,090
11,000	5,917–23,638
12,000	6,464–24,185
13,000	7,012–24,733
14,000	7,559–25,280
15,000	8,107–25,828
16,000	8,654–26,375
17,000	9,202–26,923
18,000	9,749–27,470
19,000	10,297–28,018
20,000	10,844–28,565
21,000	11,392–29,113
22,000	11,939–29,660
23,000	12,487–30,208
24,000	13,034–30,755
25,000	13,582–31,303
26,000	14,129–31,850
27,000	14,677–32,398
28,000	15,224–32,945
29,000	15,772–33,493
30,000	16,319–34,040

RANGE OF MILK YIELD NECESSARY TO INCLUDE 99 PER CENT OF THE SECOND SISTERS WHEN THE MILK YIELD OF THE FIRST SISTER IS KNOWN

Another question suggested by the rather large range of milk yield for 50 per cent of the daughters is, within what range of milk yield may we expect most of the second sisters when the production of the first sister is known? The limits chosen to answer this question

are those which should include 99 per cent of all daughters, in other words 2.58 times the standard deviation on either side of the mean. The limits are shown in table 62.

Table 62 shows the limits between which would be found the milk yields of practically all full sisters, when one sister had a given milk yield. This range is quite large. However, it should be realized that even this range of productivity is sometimes exceeded. It is indeed enlightening to see how widely even full sisters may vary in yield. We should not lose sight of the fact, however, that if a sire has one daughter with a high milk yield it is probable that a second daughter, a full sister to the first, will also have a relatively high milk yield. Herein lies a key point to progress—the sire plainly influences the production of his daughters. They inherit from him a part of the stuff which makes them high or low milk yielders. It will be shown that the daughters inherit milk yield from the dam as well. Such being the case the old saying, that the bull is half the herd would better be changed to the saying that the sire and dam together are largely responsible for the milk yield of their daughters.

The data herein presented indicate that the sire and the dam jointly influence the milk yield of their daughters. In some experiments of the author's, now in progress, it has been shown that this conclusion is likewise true of crosses of Holstein with Jersey or Holstein with Angus. All 12 first generation crossbreds showed the influence of both parents. Crossbred 1 resembles her low producing parent 7.7 times as closely as she does the high producing parent. The other 11 crossbreds resemble the high producing line of milk production from 1.5 to 18 times as closely as they do the low line of milk production. Three levels of milk production are crossed in these experiments. The Aberdeen-Angus cattle constitute the lowest level; the Jersey, Guernsey, and Ayrshire cattle, averaging about the same in milk yield, constitute the intermediate level of production; and the Holstein-Friesian cattle, having the highest yield, represent the highest level of production. It is of some interest to compare the results of crossing the different levels. If we omit the result of crossbred 1, it is found that the Holstein-Friesian cows or bulls mated to the second group of cows or bulls (Jersey, Guernsey, or Ayrshire) produced 3 offspring who are 8.43 times as near the milk production of the high level on the average as they are the low line of production.

The only cross involving the Holstein-Friesian and Aberdeen-Angus, crossbred 44, is 2.2 times as close to the high line of production as she is to the low line of her parent's milk yield.

The crosses involving the second level of milk production (Jersey, Guernsey, and Ayrshire) mated to the third group, Aberdeen-Angus, have crossbred offspring resembling the high line 7.7 times as closely as they do the low line of production. This figure compares favorably with that of the Holstein-Friesian × Jersey crosses. For the other results occurring from this study the reader is referred to the original paper.[2] Suffice it to say that both sets of data strengthen the conclusion that the sire and dam are both responsible for the milk yield of their daughters.

SUMMARY

This chapter presents a study of the milk yields of those Holstein-Friesian Advanced Registry cows who are full sisters. The records are all for the 365-day period. All records are corrected for the effect of age on milk yield. No correction for double selection is made.

That the milk yield of a cow indicates the milk yield of her full sister is shown by the correlation coefficient of 0.548 ± 0.027 between the milk yields of such pairs. Tables are presented to show the probable milk yield of a cow when the milk yield of her full sister is known.

The variation of full sisters in their milk yields is shown to cover quite a wide range. Tables are presented to show the limits of this range for 50 and 99 per cent of the full sisters.

[2] Gowen, John W. 1920. Inheritance in crosses of dairy and beef breeds of cattle. II. On the transmission of milk yield to the first generation. Journal of Heredity, vol. xi (1920), no. 7, pp. 300–316.

CHAPTER X

The Milk Yields of Half Sisters as a Measure of the Influence of the Sire on Milk Production

In the preceding chapter of this series it was shown that there was a high degree of resemblance between the milk yields of full sisters. As pointed out in this same chapter this resemblance may be due to the influence of either sire or dam exerted separately or to their joint action. The relation which exists for the milk yields of half sisters forms a critical test of which of these hypotheses is correct, for there is no influence of the dam felt in the milk yields of half sisters when only the sire is common. In fact, taking them all in all the half sisters exhibit only a random effect of the dam. Similarly, the influence of the dam is the only common heredity factor in the resemblance of the milk yields of cows having common dams but different sires.

Besides this theoretical problem, other problems of more immediate practical importance are capable of being solved by data showing the relation which exists between milk yields of half sisters. These problems are: (1) What will be the probable milk yield of the additional daughters when one daughter has a given milk yield, e.g., 20,000 pounds of milk? (2) What will be the probable variation of this milk yield for the second sisters when the milk yield of the first half sister is known? (3) What are the limits of milk yield within which will be included 99 per cent of the half sisters' milk yields when the milk yield of the first half sister is known?

MATERIAL AND METHODS

The data for this study have been described. The tabulation of this data is given at the end of this chapter. The method of determining the constants measuring the relation which exists between the milk yields of sisters has been in use for sometime. For the proof the reader is referred to the original paper by Harris.[1]

[1] Harris, J. A. 1913. On the calculation of intra-class and inter-class coefficients of correlation from class moments when the number of possible combinations is large. Biometrika, vol. ix, pp. 391–446.

THE RELATION OF THE MILK YIELDS OF HALF SISTERS

Before considering the data it may be well to consider the problem a little more in detail. In the first place, let us suppose that the milk yield of one daughter in no way indicates the milk yield of another. Under these conditions, the second daughter of a sire which had one daughter with a 10,000-pound record would be likely to produce just as much milk as would the second daughter of a sire whose first daughter produced 30,000 pounds of milk. Put in another way, the half sister to the 10,000-pound cow would stand an equal chance with the half sister of the 30,000-pound cow to produce 30,000 pounds of milk. Or for that matter the half sister of the 30,000-pound cow and the half sister of the 10,000-pound cow would be equally likely to produce 10,000 pounds. Thus we come to the potent fact that if the above hypothesis were true the second cows' average milk yields would tend to equal the average for the breed, even though they had half sisters which had made 10,000 or 30,000 pounds.

On the other hand, if the milk yields of the second sisters were predicted exactly by the milk yield of the first sister, the milk yield for the second cow whose half sister had 10,000 pounds would be 10,000 pounds and the milk yield of the second cow whose half sister had 30,000, would be 30,000 pounds. This paragraph and the one above illustrate the point. If there is no relation between the milk yields of half sisters, the milk yield of the second sister will be on the average approximately equal to the milk yield of the rest of the Holstein-Friesian cows no matter what the milk yield of her sister may have been. But if the milk yields of the second sisters are determined by the milk yield of the first sister, then the milk yields of the second sisters will be 10,000 pounds or 30,000 pounds depending on whether the yield of the half sister is 10,000 or 30,000 pounds.

The physical constants measuring the relation of the milk yield of cows which have a common sire but different dams is shown in table 63.

As the derivation of these constants is somewhat complex it may be well to indicate the method. The moments are obtained for table 68 in the usual way. This gives us the moments for the calculation of the correlation of the milk yields of all Holstein-Friesian sisters whether they are full sisters or half sisters. To obtain the moments for the half sisters we subtract the moments found for the full sisters

as given in table 58 from these moments and proceed with the usual calculation using the reduced moments.

The average milk yield of the half sisters is 20,170 pounds. This average milk yield is nearly 900 pounds more than the average of the Advanced Registry. This increased milk yield is probably to be accounted for by the fact that the sires which have daughters of more than average yield appear to have a slight tendency to have more daughters tested. This selection of sires of superior merit is not, however, nearly so stringent as we are sometimes led to believe. The standard deviation is significantly less than for the whole of the Advanced Registry cows. The coefficient of variation is also significantly less than that found for all of the cows.

TABLE 63

The milk yields of half sisters

Mean milk yield	20,170±68
Standard deviation	3,799±48
Coefficient of variation	18.8±0.2
Correlation coefficient for the milk yield of one half sister with that of another	0.362±0.015

The correlation coefficient shows that there is a fairly well-marked relation between the milk yields of half sisters. As already pointed out this relation shows the influence of the sire on the milk yields of his daughters since these daughters are from different dams with variable hereditary factors for milk yield. It is of interest to note these differences more closely. The correlation coefficient of the full sister's milk yield is 0.548. The correlation coefficient for half sisters' is 0.362. It may be shown that the degree of control over the variation of any character by another is equal to S.D. $-$ S.D. $\sqrt{1-r^2}$. The values of r^2 for the full sisters is 0.3003 and for the half sisters is 0.1303. The value of the standard deviation for the full sisters is 4102 while it is only 3799 for the half sisters. As noted above the standard deviation of the milk yields of the full sisters is larger than the standard deviation of the half sisters. While it does not seem probable that this difference has any true relation to the fact that one group are full sisters and the other half sisters, it is a fact that the difference exists and affects the result. The standard deviation remaining after the elimination of the variation due to full sisters is

3434. The standard deviation remaining after eliminating the variation due to half sisters is 3541. The variation in pounds of milk controlled by the full sisters is 668 pounds in contrast to 258 pounds for the half sisters, or the relationship of the milk yields of full sisters, because of their having common parents, is twice as great as the relationship between the milk yields of half sisters with only common sires. On this fact hinges a nice practical point in dairy breeding as it furnishes a guide to the proper matings to make. This point is taken up in the next section.

THE MILK YIELD OF THE SECOND HALF SISTER WHEN THE MILK YIELD OF THE FIRST HALF SISTER IS KNOWN

We have found that the correlation coefficient for the milk yields of full sisters is larger than the correlation coefficient for the milk yields of half sisters. We should expect, therefore, the milk yields of the second daughters to be more nearly like the milk yield of the first daughters, in the case of full sisters, than in the case of half sisters, especially when the milk yield of the first daughters is extremely high or low. It is possible to find the average milk yield of second half sisters for any given grade of first half sister's milk yield. The data for this calculation are given in table 63. Thus with linear regression the second half sister's average milk yield is equal to:

$$\text{Second half sisters' milk yield} = (\text{Mean}_{\text{H.S.}} - r_{\text{H.S.}}\ \text{Mean}_{\text{H.S.}}) + r_{\text{H.S.}} \times \text{milk yield of first half sister}$$

The Mean $_{\text{HS}}$ is equal to the average milk yield of half sisters and the r_{HS} is the correlation of the milk yields of half sisters. The necessary arithmetic shows the equation to be:

$$\text{Second half sisters' milk yield} = 12{,}879 + 0.362 \text{ first half sister's milk yield} \tag{37}$$

From this equation we obtain the expected average milk yields of second half sisters as given in table 64. I repeat the table for the full sisters for comparative purposes.

Table 64 shows that the milk yields of half sisters are partly interdependent. Thus the average milk yield of the second half sisters is 16,494 where the average milk yield of the first half sister is 10,000 pounds. The average milk yield of the second half sister increases as the milk yield of the first half sister increases. Thus where the

first half sister is a 10,000-pound cow the average milk yield of the second half sisters is 16,494 pounds, and where the first half sister is a 30,000-pound cow the average milk yield of the second half sisters is 23,724. This milk yield is over 7000 pounds greater than the average milk yield of the cows with half sisters of 10,000 pounds. These facts show us that if a sire's daughters, half sisters, average high in

TABLE 64

Milk yields of the second daughters where the milk yield of the first daughter is known. Half sisters and full sisters

FIRST DAUGHTER'S MILK YIELD	AVERAGE MILK YIELD OF SECOND DAUGHTERS. HALF SISTERS	AVERAGE MILK YIELD OF SECOND DAUGHTERS. FULL SISTERS
10,000	16,494	14,229
11,000	16,856	14,777
12,000	17,217	15,324
13,000	17,579	15,872
14,000	17,940	16,419
15,000	18,302	16,967
16,000	18,663	17,514
17,000	19,025	18,062
18,000	19,386	18,609
19,000	19,748	19,157
20,000	20,109	19,704
21,000	20,471	20,252
22,000	20,832	20,799
23,000	21,194	21,347
24,000	21,555	21,894
25,000	21,917	22,442
26,000	22,278	22,989
27,000	22,640	23,537
28,000	23,001	24,084
29,000	23,363	24,632
30,000	23,724	25,179

milk production it is probable that the subsequent daughters to come from this sire will be satisfactory producers.

The milk production of full sisters is more closely similar than the average production of half sisters. This may be seen from a comparison of columns two and one, and three and one of the above table. Thus where the first sister's production was 10,000 pounds the average production of her half sisters would be 16,494 pounds and of her full sisters 14,229. Similarly for the 30,000-pound sister, her half

sisters' average production would be 23,724 pounds and her full sisters' would be 25,179 pounds. The full sisters' production is much closer to the production of the first sister than is the production of the half sisters.

The reader will probably note one fact, that at a production of about 20,000 pounds, or at approximately the average of the Advanced Registry as a whole, the milk yields of the half or full sisters are all practically the same. These differences in milk production are nowhere near as great as they are at the extremes of production, 10,000 or 30,000 pounds. The regression of the milk yields of second sisters is more pronounced for half sisters than for full sisters.

The comparison of these milk yields of full sisters offers us the first criterion by which we may judge a bull's worth. It may be noted that the full sisters' milk yields resemble each other closely. In view of this fact it is reasonable to assume that, if a given bull is mated to a cow and produces a daughter of low production, that same bull mated to the same cow will have another daughter of low production. In other words, in the slang of cattle breeding a "nick" was not made. On the other hand, if this cow is mated to another sire the production of a resulting half sister will be considerably increased over the milk yield of the full sister. In fact, where the first cow's milk yield is 10,000 pounds, the average milk yield of a half sister is 2200 pounds more than the milk yield of a full sister. Here then is a real basis in fact for the practice of changing a sire if under good treatment his first daughters do not turn out to be good producers. If the first daughter of a sire is a medium producer, table 64 shows that the continued breeding of the sire to this same cow will tend to develop medium producing daughters. The changing of the sires will not materially change the average production of the daughters although, as will be shown in the next table, the daughters themselves will vary more in their milk yield. Where the daughter of a sire is a high producer should the mating be changed in the hope of getting a higher producer or should the cow be mated to the same sire again? A study of table 64 for the cases where the first daughters are high producers shows that the chance is better of maintaining this high average of production by breeding the cow to this same sire than by breeding her to another sire.

VARIATION OF THE MILK YIELDS. THE MILK YIELDS BETWEEN WHICH 50 PER CENT OF THE SECOND HALF SISTERS ARE FOUND WHEN THE MILK YIELD OF THE FIRST HALF SISTER IS KNOWN

Individually considered, the milk productions of the half sisters will vary. This variation will be around the average shown in table 64. It is of a good deal of practical importance to know what this

TABLE 65

Milk yields of first daughters and the limits between which would be found the milk yields of 50 per cent of their half sisters

FIRST DAUGHTER'S MILK PRODUCTION	RANGE OF MILK YIELD OF 50 PER CENT OF THE SECOND DAUGHTERS. HALF SISTERS
10,000	14,105–18,883
11,000	14,467–19,245
12,000	14,828–19,606
13,000	15,190–19,968
14,000	15,551–20,329
15,000	15,912–20,691
16,000	16,274–21,052
17,000	16,636–21,414
18,000	16,997–21,775
19,000	17,359–22,137
20,000	17,720–22,498
21,000	18,082–22,860
22,000	18,443–23,221
23,000	18,805–23,583
24,000	19,166–23,944
25,000	19,528–24,306
26,000	19,889–24,667
27,000	20,251–25,029
28,000	20,612–25,390
29,000	20,974–25,752
30,000	21,335–26,113

variation in production is likely to be, for if a sire has a daughter which produces 14,000 pounds it is desirable to know what chance this sire has of bettering this production in a subsequent daughter. The range of milk yield within which we should expect to find 50 per cent of all second half sisters for a known production of the first half sister is given in table 65.

There is quite a wide range of production which may be expected of half sisters. If the first sister were to produce 20,000 pounds of

milk, table 65 shows that it would be an even chance that her half sisters' production would be somewhere between 17,720 pounds of milk and 22,498 pounds of milk. For each grade of milk yield of the first half sister the corresponding range of milk yield to include 50 per cent of the second half sisters moves up a little in its milk yield.

TABLE 66

Milk yields of first daughters and the limits between which would lie the milk yields of 99 per cent of their half sisters

FIRST DAUGHTER'S MILK PRODUCTION	RANGE OF MILK YIELD TO INCLUDE 99 PER CENT OF THE SECOND DAUGHTERS. HALF SISTERS
10,000	7,357–25,631
11,000	7,719–25,993
12,000	8,080–26,354
13,000	8,442–26,716
14,000	8,803–27,077
15,000	9,165–27,439
16,000	9,526–27,800
17,000	9,888–28,162
18,000	10,249–28,523
19,000	10,611–28,885
20,000	10,972–29,246
21,000	11,334–29,608
22,000	11,695–29,969
23,000	12,057–30,331
24,000	12,418–30,692
25,000	12,780–31,054
26,000	13,141–31,415
27,000	13,503–31,777
28,000	13,864–32,138
29,000	14,226–32,500
30,000	14,587–32,861

THE MILK YIELDS BETWEEN WHICH 99 PER CENT OF THE SECOND HALF SISTERS ARE FOUND WHEN THE MILK YIELD OF THE FIRST HALF SISTER IS KNOWN

Of perhaps equal importance to the knowldge of the probable range of milk yield is the knowledge of the limits of the range expected for the milk yield of second daughters where their half sister's milk yield is known. Table 66 gives this information for the range necessary to include 99 per cent of the second half sisters' milk yields when the first half sister's milk yield is known.

Practically speaking the limits within which the milk yield of any second half sister will be found are given in the above table. The range covered by these limits is quite wide. It should, however, be remembered that even this range is sometimes exceeded.

THE DIRECT EFFECT OF THE SIRE ON THE MILK YIELD OF HIS DAUGHTERS

The direct effect of parents on their offspring is generally determined by a comparison of the character in the parent with this character in the offspring. Unfortunately, the bull does not give milk so that the comparison cannot be made between the milk yields of the sire and his daughters in the same way that the comparison is made between the milk yields of the dam and her daughters. To measure the effect of the sire some more roundabout method must be adopted. Information has just been presented to show that the sire undoubtedly plays his part in determining the milk yield of his daughters. This influence is measured by a comparison of the degree of resemblance which exists between the milk yields of full sisters and half sisters. It is now proposed to approach the problem in another way.

The somatic possibilities of a dam, based on her genetic complex, if fully expressed can give rise to only one grade of milk yield. Her daughters, on the other hand, will have a range of milk yield. In other words, the daughters will form a portion of one and of only one array of a correlation table. This same reasoning holds for a sire, although the somatic expression of his genetic complex for milk yield is impossible. If, then, we take the distribution of one array of the correlation surface for the determination of the relation of the milk yields of a sire it is noted that we have the distribution of one array of the correlation surface for the determining of the relation of the milk yields of the sire and his daughters.

It may be shown that in a normal correlation surface the standard deviations of the arrays bear a relation to the correlation coefficient and the standard deviation of the whole table. This relation is equal to

$$\Sigma \text{ array} = \Sigma \text{ table } \sqrt{1 - r^2}$$

From this relation, given a normal correlation surface it is possible to determine the direct relation of the sire to the milk yields of his

daughters. This method is very simple and beautiful but there are some difficulties in it that should be noted before applying it. In the first place, the correlation surface should be normal. Unfortunately, we cannot determine this point. The array standard deviations are calculated around the rough observational means. If these means do not lie close to a straight line corresponding to the regression line then the correlation coefficient found by this method is larger than the product moment r. Again, in determining the standard deviation of an array it should be noted that numbers of the individuals count for they influence the size of the standard deviation to some extent. Furthermore it takes only the influence of a relatively trivial cause to make a pronounced effect on the correlation coefficient calculated in this manner. The difficulty, and it appears to be a real one, is that the correlation coefficient calculated in this way is too large when compared with that obtained by the direct method of calculation. Table 67 gives the sires' numbers and the standard deviations of the milk yields of the sires' daughters.

The standard deviations of the milk productions presented in table 67 are those for all sires which have 5 or more daughters. The average standard deviation for these sires' daughters (when obtained from the squared S.D. weighted for N) is 2951 pounds. The average standard deviation for all daughters is 3812 pounds. From this by the calculation indicated above the correlation coefficient is found to be 0.63. That this coefficient is probably too large is shown by the following results.

The value, 2951 pounds for the average standard deviation of the individual sire's daughters, is less than it should be because of the fact that these standard deviations are based on relatively small numbers for each case. Student, Fisher, and others[2] have worked out what correction should be made based on the number of individuals from which the standard deviations are calculated. Applying these correction factors we note that there are 70 sires in this list of table 67. These sires have daughters distributed as follows: 17 sires have 6 daughters; 10 have 7; 12 have 8; 5 have 9; 4 have 10; 3

[2] See Editorial. 1915. On the distribution of the standard deviations of small samples. Appendix 1. To papers by "Student" and R. A. Fisher. Biometrika, vol. 10, pp. 522–529.

Young, Andrew W. 1916. Note on the standard deviation of samples of two and three. Biometrika, vol. 11, pp. 277–280.

TABLE 67

Standard deviation of the milk yields of the daughters of those sires having 5 or more daughters

SIRE'S NUMBER	STANDARD DEVIATION OF DAUGHTERS' MILK YIELDS	SIRE'S NUMBER	STANDARD DEVIATION OF DAUGHTERS' MILK YIELDS
22,991	3,700	41,266	3,840
23,260	3,444	42,033	2,928
23,450	2,539	42,147	3,172
23,538	3,912	42,940	2,848
24,954	1,278	43,697	2,363
25,467	1,790	44,293	2,324
25,982	4,366	44,554	1,854
26,025	3,775	44,658	3,365
26,940	2,285	45,224	2,733
29,588	1,772	45,671	2,748
29,600	3,590	47,843	1,996
30,550	1,161	48,328	2,494
30,624	2,674	48,663	4,981
30,674	1,491	49,288	2,267
31,212	2,987	49,722	495
32,422	2,357	50,290	3,770
32,481	3,621	50,672	2,753
32,492	2,482	50,999	2,848
32,558	3,005	52,927	2,494
34,944	1,979	53,257	2,976
35,227	4,121	53,418	3,315
35,269	2,326	53,825	1,893
36,974	3,972	55,993	2,667
37,314	2,439	59,682	1,528
38,214	2,267	60,344	2,222
38,291	2,379	60,574	2,730
38,401	3,024	60,751	4,460
38,446	2,934	60,966	3,018
39,037	2,958	62,924	3,325
39,357	2,204	74,219	3,342
39,972	2,921	81,663	3,184
40,338	2,837	82,505	1,379
40,358	3,551	93,909	3,544
40,534	2,749	94,217	2,259
41,206	3,892	94,882	2,055

have 11; 6 have 12; 4 have 13; 3 have 14; 1 has 17; 1 has 18; 1 has 19; 2 have 20; and 1 has 37. The average weighted correction factors for the standard deviation on the basis of these numbers of daughters is 0.9074. If we apply this correction, the average raw standard

deviation of the arrays, 2951.3, corrected for the effect of numbers becomes 3252.4 and the corrected correlation coefficient equals 0.52. The size of the correlation between the milk yield of daughter and sire shows that the sire has a pronounced influence on the productivity of the daughter. By getting ahead of our story a little it may be noted that the correlation between the milk yields of daughter and dam is 0.50. This agrees rather well with the results just given for the sire considering the defects in the method of arriving at the result. This agreement in the results shows that for the first generation the effect of the sire on the milk yield of his daughters is practically the same as the effect of the dam on the milk yield of her daughters. As already noted this result is in harmony with the results coming from a careful analysis by controlled mating on the necessarily limited data of crosses between different grades of milk yield.[3]

SUMMARY

This chapter presents a study of the milk yield of those Holstein-Friesian Advanced Registry cows who are half sisters. Conclusions of considerable importance have been brought to light by this study, to wit:

1. The milk yield of a cow indicates the probable milk yield of her half sister, the correlation coefficient for half sisters being 0.362 ± 0.015.

2. The probable variation of the milk yields of the half sisters is presented both as a whole and for different grades of milk production for one of the half sisters.

3. The relation of the milk yields of full sisters is nearly twice as great as the relation of the milk yields of half sisters.

4. The direct effect of the sire on the milk yields of his daughters is analyzed and shown to be equal to a correlation coefficient of about 0.5. The effect of the sire is then approximately the same as the effect of the dam.

[3] Gowen, John W. 1920. Inheritance in crosses of dairy and beef breeds of cattle. II. On the transmission of milk yield to the first generation. Journal of Heredity, vol. xi, no. 7, pp. 300–316.

TABLE 68

Milk yields of sires' daughters. Half sisters

SIRE'S NUMBER	9,000–10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000	34,000	35,000–36,000
20,994					1					1																	
21,724				2	1		1	1																			
22,233					2						1	1															
22,235												1	1							1							
22,499						1		1																			
22,699						1	1			1																	
22,894										1					1												
22,991							1		3	1	2									1							
23,102									1		2	1			1												
23,260							1				2	1	1			1	1	1									
23,366							1				1																
23,450						1	1	3	1			2		1													
23,538							1			1	1	5		2							2						
24,091		1					1																				
24,777				1	1																						
24,819								1	1																		
24,937						2			1																		
24,954			1	1	1	3	1																				
25,049									1	1																	
25,166										1		1	1														
25,467								2	3	4	2	2			1												
25,700													1		1												
25,796					1			1																			
25,865			2	1									1														
25,982				1			1		1				1	1	1		1										
26,025								1	2		2	1			1			1	1								
26,167					1		1																				
26,345										1		1						1									
26,734							1								1												
26,937										1		1	1														
26,940						1	1				3	1															
26,939								1		1				1					1			1					
26,980							1					1															
27,141										1				1													
27,282												1				1											
27,284						1			1																		
27,796				1		1																					
27,868										1		1								1	1						
27,930							1		1																		

TABLE 68—*Continued*

SIRE'S NUMBER	9,000–10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000	34,000	35,000–36,000
28,243							1			1																	
28,296												1	1	1			1										
28,301												2					1		1								
28,340									1	1				1	1												
28,400								1	1		1																
28,430								2											1								
28,504									2		1																
28,633					1							1															
28,835										1		1		1	1												
28,982										1				1													
29,027				1						1	2	1															
29,214													1	1													
29,236										1					1												
29,303													1			1	1	1			1						
29,328				1		2				1																	
29,358							1			1		1															
29,463												1		1													
29,500							1			1		1	1														
29,548										1				1													
29,588					1		2	1		2																	
29,600				1		1		1				2		1													
29,737							1			1																	
29,813							1				1																
29,876							1		1																		
29,882						1		1																			
30,139							1				1																
30,190							1			1						1											
30,266									1					1													
30,369									1		1	1	1														
30,413								1		1																	
30,550								5		1	1																
30,551											1						1										
30,624							1		1	1	3	1	3	1	1			1									
30,634							1	1	1																		
30,674							2	1	1	1	1																
30,830											1	1															
31,168							1													1							
31,208							1		1																		
31,211						1		2				1															
31,212	1				2		3	1	3		3		1														

TABLE 68—*Continued*

SIRE'S NUMBER	9,000–10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000	34,000	35,000–36,000
31,215									1		1																
31,338								1		1			1														
31,387							1				1																
31,438						1	1					1		1	1												
31,789						1		1																			
31,846								1		1																	
31,957							2																				
32,059									1		1			1													
32,110				1								1															
32,254					1		1			1	1	1															
32,260						1	1																				
32,322						1	1				1		1		1												
32,386										1	1																
32,422						1		1	1	1		1	1														
32,481										1	1				1		2		1	2							
32,492								2	1	2	2	2					1										
32,510				1										1													
32,554								2															1				
32,558								1			2	1	1	2	4	1	2	1	1	1							
32,655			1								1																
32,731							2													1							
33,289							1	1																			
33,437						1		1				1		1													
33,642									1		1																
33,661							1										1										
33,894				1	1	1			1																		
33,957								1			1																
33,979																1	1										
34,205									1	1																	
34,467										1	3		1														
34,503										1		1	1					1									
34,944								1		1		2	1	1													
34,989									1			1															
34,990									1	1	1	1															
35,074							1									1											
35,119																		1	1								
35,165									2			1		1													
35,188					1				1																		
35,226					1		2																				
35,227							1		3	1			1			1				1							

TABLE 68—*Continued*

SIRE'S NUMBER	9,000–10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000	34,000	35,000–36,000
35,250										1			1														
35,269							1			1	3	3		2	1	1											
35,431				1	1		1																				
35,469								1				1															
35,639					2		1																				
35,887							1		1	1																	
35,900							1					1															
35,903							1		1	1		1															
35,911										1			1														
35,975						1						1															
36,158								1	1						2	1											
36,220						1							1	1													
36,380													2			1				1							
36,632									1				1														
36,636												1										1					
36,664								1													1	1					
36,819						1					2																
36,903							1	1																			
36,974								1			1			2	1	2		1					1				
37,094							1			1	1		1														
37,200								1		1																	
37,222						1	1																				
37,226				1								1					1										
37,302														1	1		1										
37,314					2		3			2	1	1															
37,409									1				1														
37,689						1					1																
37,770					1	1		1										1	1								
37,807			1					1																			
37,852								1			1																
38,079							1	2	1	1																	
38,214								3	1				2														
38,243									1			1			1												
38,291									1		1	1		3	5	1	1		1								
38,401					1				2	2	1					1											
38,446							1		1		2	1		2			1										
38,460							1		2	1																	
38,496													1					1									
38,505														1	1												
38,639											2	1															

TABLE 68—*Continued*

SIRE'S NUMBER	9,000–10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000	34,000	35,000–36,000
38,652						1	1	1																			
38,883					1				1																		
38,977									1		1																
39,037							1	1	3		3	2	6	4	10	2	2	1	1		1						
39,074									1	1	1	1															
39,156					2																						
39,314											1			1													
39,357					1		1	1	1	1	2	1															
39,426						1			2		1																
39,566							1		1		1																
39,849										2																	
39,904					1		1																				
39,972						2		1	1	1			1	1													
40,207									1		2																
40,214								1							1												
40,248									1						1												
40,338										2	1	2	1			2	1	1									
40,358										1	1		1	1		1		2			1						
40,368						1			1																		
40,386									1							1											
40,492											1	2			1												
40,534								3		1			1		1												
40,547							1		1																		
40,663						1				1																	
40,718						1					1																
40,871							1			1																	
41,009								1	1																		
41,038									1	1																	
41,114												1										1					
41,206			1	1	2	3	2	1	1	2	1	2	1		1	2											
41,220								1		1				1													
41,266					1	1		4	2			1				1		1									
41,416					1					1	2			1													
41,484						2																					
41,562						1	1				1																
41,660						1								1													
41,751						1	1		1		1			1													
41,757														1	1												
41,803																	1		1								
41,866												1			1												

TABLE 68—*Continued*

SIRE'S NUMBER	9,000–10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000	34,000	35,000–36,000
41,895							1	1																			
41,980						1			1	1	1		1														
42,033						1	2	2					1	1													
42,147								1	1	2	1		3	2	3	2		1	2								
42,183							2			1		1															
42,235						2			1				1														
42,381									2																		
42,448							2	1									1										
42,451					1								1														
42,682							1				2																
42,729									1					1													
42,940							2	2	1			1	1		1												
43,075			1			1																					
43,113				1						1																	
43,496											2		1														
43,666									1		1																
43,697				1	3		2	2	1	2		1															
43,964												1					1										
44,034												1	1			1											
44,085											1	1															
44,293						1			2	1		3	2	1													
44,367															1				1								
44,444							3			1					1												
44,465									1										1								
44,543							2	1																			
44,554								1	1		2	2	1	1													
44,658					1	1			1		1	1		2													
44,742																2											
44,758							1				1		1														
44,781							1			2																	
44,916						2																					
45,074										2		1															
45,103			2				1			1																	
45,224		1				2	1	1	1	3	1	1															
45,502				1	1		1				1																
45,652									1		1																
45,671									2	1	1		1	1			1										
45,674															1	1				1		1					
35,805													1		1												
45,807				1	1		1			1																	

TABLE 68—*Continued*

SIRE'S NUMBER	9,000-10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000	34,000	35,000-36,000
45,998						1		1																			
46,212					1			1																			
46,301						2	1	2																			
46,514												1		1													
46,593		1					1					1															
46,595								1		1																	
46,763								1	1																		
46,977										1			1			1											
47,010											1				1							1					
47,135						1	1								1												
47,282						1				1	1		1														
47,306													1	1													
47,341								1	1																		
47,808					1			1																			
47,843				1	1	3	1	1			1																
48,121				1		1																					
48,328				1			1		1	1	2																
48,358												1		1	2												
48,498							1			1			1					1									
48,557					1							1															
48,663				2								1		1	1	1											
48,696													1		1							1					
48,702																			2	1							
48,832					1		2			1																	
48,846						2																					
49,207									1									1									
49,221						1		1				1															
49,288		1			1	1	1	1	1																		
49,722						4	3																				
49,795									1		1																
49,910			1	1				1																			
50,290						2	1	1	1	1	1		1	1	2	1	1										
50,347													1							1							
50,361									1		1	1															
50,672					1		2	1	1	2	1	2	1		1												
50,758										1										1							
50,865							1								1												
50,920					1			1																			
50,999								1		6	2		2	2	2	2	1	1									

TABLE 68—*Continued*

SIRE'S NUMBER	9,000–10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000	34,000	35,000–36,000
51,002										1			1														
51,067				1			1																				
51,078									1		1							1									
51,148														2													
51,217								1	2		1																
51,505										2																	
51,510							1				2																
51,511								1	1																		
51,523						2	1																				
52,145				1					1		2	1															
52,163						1	1																				
52,297						1		1																			
52,410											2				1												
52,527						1	1																				
52,791														2	1												
52,927						1			1	1	1	1		1													
52,983									1			2															
53,055												1											1				
53,059							2		1					1													
53,247				1								1															
53,257									1	1		3		3	2			1		1							
53,303										2	1	1		1													
53,309										1		1															
53,399											1	1															
53,418							1		1	1		2	3	3	5	1			1	1	1						
53,618									1				1														
53,825		1			2	1	1	1																			
53,918											1			1													
54,001					1					1																	
54,120															1		1						1				
54,424										1	2																
54,909						1		1																			
55,237											1				1		1										
55,732								2	1	1	1																
55,736									2																		
55,819			1					1																			
55,993							2	1		1	3	1				1											
56,435												3				1											
56,635															1							1	1				
57,188										1						1											

TABLE 68—*Continued*

SIRE'S NUMBER	9,000–10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000	34,000	35,000–36,000
57,256													1				1										
57,476			1	1					1																		
57,908								1			1		1														
57,926								1			1																
58,022							1	1																			
58,041						1				1		2		1													
58,219									1	1	1	1			1												
58,313										1	1	1		1													
58,314								2																			
58,769							1				1	1															
59,156											1					1											
59,219									1					1													
59,272					1		1																				
59,660															1				1								
59,682					1		1		4																		
59,755						1		1																			
59,959											1								1								
60,047						1	1		2			1															
60,278								1	1																		
60,344						1			2	1	2	1		1													
60,360		1									1																
60,529						1			1					2													
60,574					2				3	2	2	2	1		1												
60,577														2	3												
60,710						1	1																				
60,751									1		1	1				1		1				1					
60,966										1			1		1	1	2		1	1							
61,435									2			1															
62,049								1					1														
62,178												2		1	1		1										
62,239											1			1				1									
62,483												1			1												
62,924							2		1		1	1	3	1	2				1								
63,578											1																1
63,637							3																				
63,895													2														
63,994									2		1																
64,143								1		1				2													
65,016									2																		
65,750								1			1		1														

TABLE 68—*Continued*

SIRE'S NUMBER	9,000–10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000	34,000	35,000–36,000
65,981							1					1															
66,092								1	1																		
66,417						1						1															
67,203					1			1																			
67,211						1	2		1																		
68,544				1	2						1		1														
68,946										1	1							1	1								
69,686												1			1	1	1										
70,753			1					2																			
70,857								1				2				1											
71.013								1	1																		
71,071							1		1																		
71,158									1				1	1	1												
71,527								2																			
72,013											1				1												
72,099																1						1					
72,287										1		1															
73,416									1	1						1											
74,219							1	1		2	1	2	1	2	1	1				1							
74,982							1			1																	
75,012											1					2	1			1							
75,034				1	1																						
75,304						1			1				2														
75,361			1		1	2				1																	
75,627											2																
76,663			1			1						1															
76,665			1					1																			
76,861						1		1																			
77,572												2															
78,027																		1	1								
78,123											1		1														
79,602														1		1											
80,551									1	1																	
80,596																1	2	1									
80,916		1			2		1																				
81,142															1			1									
81,405										2	1																
81,497										1			1														
81,630							1			1	2		1														
81,663										1		2		1	1					1							

TABLE 68—*Concluded*

SIRE'S NUMBER	9,000–10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000	34,000	35,000–36,000
82,505					2	2	2	3	2																		
83,354												1								1							
84,099																	2	1	1								
84,472								1	1																		
85,556																		2									
85,968											1	1	1														
86,166								1	1		1																
86,837								1	1	1				1		1											
88,318										1		2	1		1												
91,668										2																	
93,149							1			1																	
93,229						1		1			1	1	1														
93,371													1		1												
93,909																2	3		1				1	2	1		
94,156											2																
94,217										2	1	1		1	1	1											
94,292						1				1	1	1	1														
94,882					1	2	1		1		1																
94,984														1						1							
96,076									1						1												
97,208				1	1	1			1																		
97,224						1		1																			
97,403									1			1															
97,472											1			1		1	2										
98,063												1			1												
98,762							1			1																	
102,469												1					1		1	1							
103,899															1		1	2	1								
104,002											2		1														
105,106											1	1															
107,790						1		1																			
107,891							1				1			1													
110,157															1					1							
110,474																1						2					
112,077													2														
113,938									1	1																	
114,797												1											1				
121,546							1	1																			
128,445											1	1															

CHAPTER XI

The Butter-fat Percentages of Full Sisters

The period of usefulness of a sire is surprisingly short. The problem of the replacement of the herd bull is a problem which every breeder is constantly facing. Several schemes to determine the true worth of the sire are in vogue among breeders. The idea behind these schemes is to get a good bull to head the herd and at the same time to reduce the chance of getting a poor one, since it may take from two to five years to recognize it as such. Among these schemes may be mentioned two which have something in their favor. The first consists simply in trying the bull out by breeding him to enough cows to get one or two daughters. The bull is then saved until the daughters come in milk, when, if the daughers prove worthy, he is used to head the herd. The other plan is that followed by the larger breeder. The breeder has two or more bulls to try out. He breeds them to the same cow, if possible, and when the daughters come in milk the breeder selects the herd bull on the basis of the production of these daughters.

Given the second method of selecting the bull to head a herd on what would you base your choice of the bull? The reader will undoubtedly say, choose the bull with the daughter which has the highest butter-fat percentage and in general the reader is right, although not always.

The discussion of the problem may be taken up from this angle. In selecting a bull on the above basis we want to know what chance there is that one is really better than the other. In other words we want to know if the second bull's daughter is better than the first bull's daughter by a difference sufficiently great to make it unlikely that such a difference would exist if the cows were full sisters. As a first approach to this problem we wish to know what the probable butter-fat percentage of a second full sister would be when the butter-fat percentage of the first sister is known. It is known further that these full sisters will not milk exactly the same amount of milk

nor will their milk test exactly the same. They will vary. We must know the probable limits of this variation as well as the extreme limits of it. There are these three problems on which information is desired: (1) What will be the probable butter-fat percentage of a sire's daughter when the butter-fat percentage of another daughter, full sister, is known? (2) What will be the limits within which the daughters will normally vary? These limits are those within which 50 per cent of the daughters will be found. (3) What will be the extreme limits of butter-fat percentage within which will be found most if not all of the daughters of a given sire?

THE RELATION WHICH EXISTS BETWEEN THE BUTTER-FAT PERCENTAGES OF FULL SISTERS IN THE HOLSTEIN-FRIESIAN ADVANCED REGISTRY

Our problem is to determine on what basis we shall proceed in selecting a bull to lead the herd. In previous discussions of this problem free use is made of such terms as the "prepotent bull" and the "nick" without any particular attempt to get behind these empty phrases and find out what they mean in terms of breeding dairy stock and the control of the milk production of this stock. In this chapter and the one to follow the attempt will be made to determine on what basis the "prepotent" bull may be selected early enough in his life to be of most service to the breeder. To determine this point involves the careful study of the poor producers as well as those animals which are noted for their production.

Wright[1] has defined the "prepotent" sire as the sire which carries in homozygous condition those dominant factors which make for the highest possible expression of the desired character. The highest expression of the desired character must also depend on these dominant factors and not on recessive factors. This definition expresses nicely the probable meaning of prepotency in the light of our recent knowledge of the mechanism behind observed inheritance. It may now be asked how such an analysis can be applied to the transmission through inheritance of butter-fat percentage.

[1] Wright, Sewall. 1920. Principles of live stock breeding. Dept. of Agric. Bulletin 905, p. 36.

Several difficulties of more or less weight present themselves. What can be done is to ascertain for any given mating what chance there is to obtain a like "nick" when this same mating is repeated. As a first step in this direction we must know what relation exists between the butter-fat percentages of full sisters.

TABLE 69

Correlation surface showing the butter-fat percentages of full sisters

SECOND SISTER'S BUTTER-FAT PERCENTAGE	FIRST SISTER'S BUTTER-FAT PERCENTAGE																			TOTAL
	2.6–2.7	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4–4.5	
2.6–2.7				1			1													2
2.7				1		2														3
2.8				1				2	4	1				1						9
2.9	1	1	1		2	2		1	1	1										10
3.0				2	6	1	4	4	4											21
3.1		2		2	1	4	4	3	2	3			1							22
3.2	1				4	4	4	5	4	6		1							1	30
3.3			2	1	4	3	5	6	7	3	9	2								42
3.4			4	1	4	2	4	7	10	4	6	1	3	2				1		49
3.5			1	1		3	6	3	4	14	1	2	2							37
3.6								9	6	1	4	5		1	1	1				28
3.7							1	2	1	2	5		3	3						17
3.8						1			3	2		3				1				10
3.9			1						2		1	3		2	2	2				13
4.0											1			2						3
4.1											1		1	2						4
4.2																				
4.3									1											1
4.4–4.5							1													1
Total...	2	3	9	10	21	22	30	42	49	37	28	17	10	13	3	4		1	1	302

There are 302 full sisters who have 365-day butter-fat percentages. The correlation surface showing the butter-fat percentage for the first and second sisters is given in table 69.

Table 70 gives the fundamental constants for the variation and correlation of the butter-fat percentage of full sisters.

The mean butter-fat percentage of these full sisters is seen to be closely similar to that for the whole Advanced Registry. The standard deviation of these butter-fat percentages is slightly, although not significantly, less than the standard deviation of the whole Holstein-

Friesian Advanced Registry. The coefficient of variation is also slightly less than that of all the available Holstein-Friesian Advanced Registry data. These facts indicate that the group of records is a fair sample of all the records of the Advanced Registry.

The correlation coefficient measuring the degree of resemblance between the butter-fat percentage of the full sisters is probably representative of the breed. This coefficient is fairly high. The butter-fat percentages of full sisters resemble each other fairly closely. This resemblance is due to at least two common causes, the common inheritance received from the sire and from the dam, and the common environment under which the test was probably made.

The degree of resemblance for the butter-fat percentage of full sisters is not quite so great as for their milk yields. The correlation

TABLE 70

Physical constants of variation and correlation for the butter-fat percentages of full sisters

Mean butter-fat percentage	3.418±0.012
Standard deviation of butter-fat percentage	0.303±0.009
Coefficient of variation of butter-fat percentage	8.87 ±0.26
Correlation coefficient between the butter-fat percentage of one full sister with that of another	0.464±0.032

coefficient for the relation of the milk yields of full sisters was 0.548±0.027 while the correlation coefficient for the relation of the butter-fat percentages of full sisters is 0.464±0.032. The difference is 0.084±0.041 or the difference is only a little over twice the probable error. Such a difference may come solely from chance. The conclusion is consequently justified that the relation of the butter-fat percentages of full sisters is the same within the errors of random sampling as the relation of the milk yields of the same full sisters.

The data presented in table 70 assist in obtaining the answer to the problem of how to select a desirable sire. There are several points to be considered in such a selection. Among the first on which information is necessary, is this relation of the butter-fat percentages of full sisters. Thus if a sire is bred to a cow and produces a daughter whose test is 2.6 per cent and another sire bred to the same cow produces a daughter whose test is 3 per cent, is

this difference any necessary guarantee of the second sire's worth? The answer to the question obviously hangs on the probable butter-fat percentage of a second daughter of the first sire when this daughter is out of the same dam. For if this second daughter would be likely to have a butter-fat percentage even higher than the 3 per cent of the second sire's daughter obviously the first sire stands at least an even chance of being as good or better than the second sire. The data given in the next section gives the answer to this question.

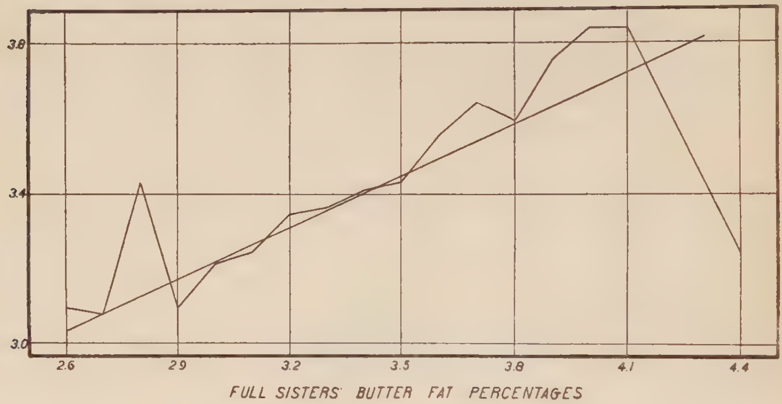

FIG. 17. THE REGRESSION OF THE BUTTER-FAT PERCENTAGE OF ONE FULL SISTER ON THE BUTTER-FAT PERCENTAGE OF THE OTHERS FOR HOLSTEIN-FRIESIAN ADVANCED REGISTRY 365-DAY RECORDS

THE AVERAGE BUTTER-FAT PERCENTAGES OF SECOND FULL SISTERS WHEN THE BUTTER-FAT PERCENTAGE OF THE FIRST FULL SISTER IS KNOWN

By the use of data contained in table 70 it is possible to predict the butter-fat percentage of a second full sister when the butter-fat percentage of the first full sister is known. This prediction depends on the ordinary linear equation. It gives the average butter-fat percentage of the additional full sisters, when the first full sister's butter-fat percentage is of a given grade, for the data of table 69.

The equation for this curve is the ordinary linear regression equation:

$$\text{B.F. per cent} = 1.833 + 0.464 \text{ b. f. per cent} \tag{38}$$

where the *B. F. per cent* is the butter-fat percentage of the second full sisters and the *b. f. per cent* is the known butter-fat percentage of the first full sister. The assumption is, of course, that the raw regression line is a linear function. The accuracy of this assumption is shown in figure 17.

TABLE 71

Butter-fat percentage of the second daughters when the butter-fat percentage of the first daughter is known. Full sisters

FIRST DAUGHTER'S BUTTER-FAT PERCENTAGE	EXPECTED BUTTER-FAT PERCENTAGE OF SECOND DAUGHTERS
2.6	3.04
2.7	3.09
2.8	3.13
2.9	3.18
3.0	3.22
3.1	3.27
3.2	3.32
3.3	3.36
3.4	3.41
3.5	3.46
3.6	3.50
3.7	3.55
3.8	3.60
3.9	3.64
4.0	3.69
4.1	3.73
4.2	3.78
4.3	3.83
4.4	3.87
4.5	3.92
4.6	3.97

Figure 17 shows a highly irregular regression line for the raw data. In large measure this irregularity is due to the few individuals, especially in the classes beyond the 4 per cent. The reader will, I think, be convinced of this fact by a study of table 69. The rise in the third observation is also probably due to the few numbers although here these data are more numerous than at the right of the curve. Taking these facts into consideration it is believed to be true that the raw regression curve is a linear function whose equation is that of the straight line drawn through the observational

curve. Study of table 69 shows further that the data approach the Gaussian surface sufficiently closely to justify the use of this surface.

Table 71 shows that a sire which has a low butter-fat percentage daughter, for example, a daughter whose milk tests 2.6 per cent, will probably have his second daughter from this same dam test quite a little higher in her butter-fat percentage. Thus when the butter-fat test of the first sister was 2.6 per cent, the average butter-fat test of the second sister was 3.0 per cent, or the increase was 0.4 per cent. Clearly, then, if full sisters can be expected to have a difference like this, we cannot assert that a similar difference between the offspring of two bulls means that one is better than the other.

If the first daughter has an average test of say 3.4 per cent it will be noted that her full sister on the average also has this test. Here any difference between the daughters of different bulls would be more often due to a real difference between the sires. On the other hand, should the first bull have one daughter with an extreme butter-fat test of 4.6 per cent, a second daughter would be expected to have a butter-fat test of 4 per cent, or a difference of 0.6 per cent. Under such circumstances, this difference could not be attributed to a real difference in two different sires for a similar difference would be expected should the same bull be the sire of both cows.

VARIATION OF THE BUTTER-FAT PERCENTAGE. THE BUTTER-FAT PERCENTAGE BETWEEN WHICH 50 PER CENT OF THE SECOND FULL SISTERS ARE FOUND WHEN THE BUTTER-FAT PERCENTAGE OF THE FIRST FULL SISTER IS KNOWN

What differences can be considered real? This problem may be approached in the following way. Suppose we determine the limits of butter-fat percentage between which it will be equally probable to find the second full sister when the butter-fat percentage of the first full sister is known. If the second daughter of a given dam falls considerably outside these limits as determined from the first daughter's butter-fat percentage then it is probable that the second sire is a better or a worse sire for butter-fat percentage than the first one. The further away from these limits the second sire's

daughter is the more probable it becomes that there is a difference between these sires. The data showing the probable limits of butter-fat percentage between which would be found the butter-fat percentage of 99 out of 100 second full sisters when the first sister had a butter-fat percentage shown in the first column, are given in table 73. Table 70 gives the necessary data to determine these

TABLE 72

Butter-fat percentages of first daughters and the limits between which would lie the butter-fat percentages of 50 per cent of their full sisters

FIRST DAUGHTER'S BUTTER-FAT PERCENTAGE	RANGE OF BUTTER-FAT PERCENTAGE OF 50 PER CENT OF THE DAUGHTERS. FULL SISTERS
2.6	2.8–3.2
2.7	2.9–3.3
2.8	3.0–3.3
2.9	3.0–3.4
3.0	3.0–3.4
3.1	3.1–3.5
3.2	3.1–3.5
3.3	3.2–3.5
3.4	3.2–3.6
3.5	3.3–3.6
3.6	3.3–3.7
3.7	3.4–3.7
3.8	3.4–3.8
3.9	3.5–3.8
4.0	3.5–3.9
4.1	3.6–3.9
4.2	3.6–4.0
4.3	3.6–4.0
4.4	3.7–4.1
4.5	3.7–4.1
4.6	3.8–4.1

limits. It may be shown that the standard deviation of the full sisters' butter-fat percentage for the different classes is equal to $0.303\sqrt{1-0.464^2}$ or is equal to 0.269. The limits of butter-fat percentage necessary to include 25 per cent of the second full sisters on either side of the mean when their first sisters' butter-fat percentage is known are given by mean $\pm\ 0.67449 \times 0.269$, assuming the Gaussian surface. These limits are given in table 72.

Table 72 gives the range of butter-fat percentage between which 50 out of 100 second full sisters would be found were the butter-fat percentage of the first daughter to be that shown in the first

column. Concretely stated, if the first daughter's butter-fat percentage were 3.0 per cent there should be 50 out of every 100 of her full sisters with a butter-fat test more than 3 per cent and less than 3.4 per cent.

To return to our problem, the choice of a suitable sire. If the first sire's daughter has a butter-fat test of 2.8 we note that 50 per cent of her full sisters will have a butter-fat test between 3 and 3.3 on the average. About 25 per cent of the time the full sisters' butter-fat percentages would be under 3 per cent and about 25 per cent of the time would be above 3.3 per cent. Now if 50 per cent of the time a full sister of the first sire's 2.8 per cent daughter would have a butter-fat test between 3 and 3.3 per cent, obviously, the second sire's daughter must be beyond this range to make his worth decisive. On the other hand, if the second sire has a daughter whose butter-fat percentage is considerably higher than the high point of the range (3.3 per cent), say 4.00 per cent, it is markedly in his favor for it is indicative that the second sire is truly better than the first sire in his capacity to transmit butter-fat percentage. But if this second sire has a daughter with a butter-fat percentage significantly less than the low point of the range (3 per cent), then it is probable that the first sire is the better. By a similar use of table 72 it is possible to determine something of any sire's worth without too great outlay of time.

RANGE OF BUTTER-FAT PERCENTAGE NECESSARY TO INCLUDE 99 PER CENT OF THE SECOND SISTERS WHEN THE BUTTER-FAT PERCENTAGE OF THE FIRST SISTER IS KNOWN

There is, however, one other question, namely how can the really exceptional sire, the one in 100 kind, be chosen from the rest. From the above discussion it is obvious that the exceptional sire in such a test as the above would be the one whose daughter tested the higher per cent of butter-fat and was furthest away from the high end of the range of butter-fat given in the above table. Thus suppose that the first sire had a 2.6 per cent butter-fat testing daughter from a cow named Beauty. The range within which it would be equiprobable that another full sister would lie is shown in the above table to be 2.8 to 3.2 per cent. Now if a second sire's daughter from Beauty tested as much as 3.8 per cent

it would be probable that this sire had an inheritance for butter-fat percentage that would make him an exceptional sire.

Table 73 gives the limits of butter-fat percentage between which 99 per cent of the second full sisters should be found if the first full sisters have a butter-fat test as given in the first column. This table will give some appreciation of the limits beyond which a

TABLE 73

Butter-fat percentages of first daughters and the limits between which would lie the butter fat percentages of 99 per cent of their full sisters

FIRST DAUGHTER'S BUTTER-FAT PERCENTAGE	RANGE OF BUTTER-FAT PERCENTAGE OF 99 PER CENT OF THE REMAINING DAUGHTERS. FULL SISTERS
2.6	2.3–3.7
2.7	2.4–3.8
2.8	2.4–3.8
2.9	2.5–3.9
3.0	2.5–3.9
3.1	2.6–4.0
3.2	2.6–4.0
3.3	2.7–4.1
3.4	2.7–4.1
3.5	2.8–4.1
3.6	2.8–4.2
3.7	2.9–4.2
3.8	2.9–4.3
3.9	2.9–4.3
4.0	3.0–4.4
4.1	3.0–4.4
4.2	3.1–4.5
4.3	3.1–4.5
4.4	3.2–4.6
4.5	3.2–4.6
4.6	3.3–4.7

sire's daughters must produce to make this sire worthy of careful consideration. A sire whose daughters do exceed these limits probably has a valuable inheritance for butter-fat percentage.

The use to which table 73 may be put has been illustrated in the discussion given above. It is important to note the range of butter-fat percentage between which even full sisters may vary, for here lies the solution to many a perplexing problem which has puzzled, and sometimes discouraged, the breeder.

SUMMARY

This section presents a study of the butter-fat percentages of those Holstein-Friesian Advanced Registry cows that are full sisters. Study of the records shows that there is a fairly high relation between the butter-fat percentages of full sisters. The correlation coefficient representing this relationship is 0.464 ± 0.032. The regression lines are shown to be best represented by linear functions. Tables are presented to show the probable butter-fat percentage of a cow when the butter-fat percentage of her full sister is known.

The variation of the butter-fat percentage for full sisters is shown to cover a wide range. Tables are presented to show the limits of the range of this butter-fat percentage for 50 and 99 per cent of the full sisters.

CHAPTER XII

The Correlations and Variations in the Butter-fat Percentage of a Sire's Daughters. Half Sisters

The problem of "prepotency" and the breeding for a "nick" may be analyzed further than it was carried in the previous chapter. In chapter IX data was presented to show the mode of selection of a sire on the basis of the performance of full sisters. Thus, let us suppose two young sires are to be tested for their worth. One is bred to a cow preferably of known performance, and gets a daughter. The other bull is bred to the same cow to get another daughter. By comparing the records of the resulting daughters the sire is selected. The application of this method is limited for it is necessary to get two daughters from the same cow.

Of more universal appreciation is the method of breeding one of the bulls to one cow and the other to another cow and selecting the bulls on the basis of the resulting progeny records. This method is of wider application for it enables us to test a larger number of bulls. There are some points concerning it, often overlooked, which deserve careful consideration. Let us consider the problem.

The selection of the sire is based on the butter-fat producing qualities of his daughter. Suppose the first bull (A) of our illustration had a first daughter with 3.4 per cent of butter-fat and the second bull (B) has a daughter which produced 3.6 per cent of butter-fat. Should we be justified in concluding that one of these sires was actually better than the other and would get better offspring than the other? The answer to this question depends on the probable butter-fat percentage of a second daughter of sire A. Thus, if such a second daughter of sire A could be expected to produce a percentage of butter-fat greater than 3.6 per cent we would not be justified in concluding that the daughter of sire B was better than a second daughter of sire A. Therefore, we could not conclude that sire B was better than sire A as a breeder. The data for the solution of this problem in its different ramifications are given in the paragraphs to follow.

THE VARIATIONS AND CORRELATION OF THE BUTTER-FAT PERCENTAGE OF A SIRE'S DAUGHTERS. HALF SISTERS

The constants showing the variation and correlation of a sire's daughters, which are from different dams, are given in table 74. These constants are derived from the raw data given in table 79. The moments are obtained from these data in the ordinary way. These moments represent those for all of the sire's daughters whether they are full or half sisters. The moments for the half sisters are obtained by subtracting the moments for the full sisters, as obtained from table 69 of the preceding chapter from those obtained from table 79 of this chapter.

The mean butter-fat percentage of these half sisters is slightly higher than the mean butter-fat percentage of the whole Advanced

TABLE 74

Physical constants for the butter-fat percentages of a sire's daughters

These daughters are from different dams

Mean butter-fat percentage	3.493 ± 0.006
Standard deviation	0.332 ± 0.004
Coefficient of variation	9.50 ± 0.12
Correlation coefficient for the butter-fat percentage of one half sister with that of the others	0.374 ± 0.015

Registry. The standard deviation and the coefficient of variation are slightly, although not significantly, more than those of the Advanced Registry taken as a whole. All differences are so slight in absolute amount as to make little real difference in the conclusions to be drawn from these data.

The correlation coefficient shows that the butter-fat percentages of half sisters are to some extent dependent on each other. If we compare this interdependence of half sisters with that of full sisters as given in the previous chapter, it is noted that the full sisters' butter-fat percentages are more dependent on each other than are the butter-fat percentages of half sisters. The full sisters exhibit the influence of a common dam. The half sisters on the other hand have different dams with diverse heredity for butter-fat percentage. This diverse heredity makes for variation and less relation between the butter-fat percentages of half sisters than is found for full sisters. Here we have a measure of the influence of the dam. The influence

of the sire and dam may be scrutinized more closely. The correlation coefficient for full sisters' butter-fat percentages is 0.464 ± 0.032. The correlation coefficient for half sisters is 0.374 ± 0.015. The degree of control over the variation of the butter-fat percentage by any character is proportional to S.D. $\sqrt{1 - r^2}$. The value of r^2 for full sisters is 0.2153 and for the half sisters is 0.1399. The value of the standard deviation for the full sisters is 0.303 and for half sisters is 0.332. The standard deviation of butter-fat percentage remaining after eliminating the variation due to full sisters is 0.269. The standard deviation of butter-fat percentage remaining after eliminating the variation due to half sisters is 0.308. The variation in butter-fat percentage due to full sisters is 0.031. The variation in butter-fat percentage due to half sisters is 0.024. The degree of control over the butter-fat percentage of full sisters is not far different from the degree of control of the half sisters. This is chiefly due to the fact that the standard deviation of the half sisters is larger than that for the full sisters in this limited series of data.

The correlation coefficient for the milk yield of half sisters is 0.362 ± 0.015. The correlation coefficient for the butter-fat percentage of half sisters is 0.374 ± 0.015. The difference between these correlation coefficients is clearly not significant. Such being the case the milk yield of one half sister predicts the milk yield of another to nearly the same degree of accuracy that the butter-fat percentage of one half sister predicts the butter-fat percentage of the other.

THE BUTTER-FAT PERCENTAGES OF THE ADDITIONAL HALF SISTERS WHEN THE BUTTER-FAT PERCENTAGE OF THE FIRST HALF SISTER IS KNOWN

From the data now in hand it is possible to determine the average butter-fat percentage of the other half sisters when the butter-fat percentage of the first half sister is known. These data for different grades of first half sister's butter-fat percentage are given below. The average butter-fat percentages of the second half sisters are calculated by the use of the ordinary linear regression formula:

$$\text{B. F. per cent} = \left(\text{Mean}_{\text{B. F. per cent}} - r_{xy}\,\frac{\sigma\,\text{B. F. per cent}}{\sigma\,\text{b. f. per cent}}\,\text{Mean}_{\text{b. f. per cent}}\right)$$

$$+\, r_{xy}\,\frac{\sigma\,\text{B. F. per cent}}{\sigma\,\text{b. f. per cent}}\,\text{b. f. per cent}$$

In this formula the standard deviations of B. F. per cent and b. f. per cent are identical since the table is symmetrical. The means are also identical. When the data of table 74 are substituted into this formula we have:

$$\text{B. F. per cent second half sisters} = 2.1853 + 0.3744 \text{ b. f. per cent first half sister} \tag{39}$$

This is the equation used in the formation of table 75.

TABLE 75

Butter-fat percentage of second daughters when the butter-fat percentage of the first daughter is known. Half sisters

FIRST DAUGHTER'S BUTTER-FAT PERCENTAGE	EXPECTED BUTTER-FAT PERCENTAGE SECOND DAUGHTERS. HALF SISTERS	EXPECTED BUTTER-FAT PERCENTAGE OF SECOND DAUGHTERS. FULL SISTERS
2.6	3.16	3.04
2.7	3.20	3.09
2.8	3.23	3.13
2.9	3.27	3.18
3.0	3.31	3.22
3.1	3.35	3.27
3.2	3.38	3.32
3.3	3.42	3.36
3.4	3.46	3.41
3.5	3.50	3.46
3.6	3.53	3.50
3.7	3.57	3.55
3.8	3.61	3.60
3.9	3.65	3.64
4.0	3.68	3.69
4.1	3.72	3.73
4.2	3.76	3.78
4.3	3.79	3.83
4.4	3.83	3.87
4.5	3.87	3.92
4.6	3.91	3.97

From this table it will be noted that if the first daughter of a given sire has a butter-fat test of 2.6 per cent then the second daughter of this same sire but from a different dam would be expected to have a butter-fat test of 3.2 per cent, on the average. Similarly, if a sire's first daughter has a butter-fat test of 4.6 per cent it would be expected that the second daughter will have a butter-fat test of 3.9

per cent. It will be noted that the second daughter's average butter-fat test tends to approach the average of the whole breed, 3.4 per cent, when the first daughter's butter-fat percentage is at the extremes of the range of butter-fat percentage, 2.6 or 4.6 per cent. When the first daughter's butter-fat percentage is average, 3.4 per cent, then the second daughter is also average.

These facts furnish the data necessary for the comparison which we desire to make in determining the worth of the two sires. Thus in our illustration if the first sire (A) has a daughter whose butter-fat percentage is 2.6 per cent and a second sire (B) has a daughter with a butter-fat test of 3.2 per cent we could not conclude that sire B was better than sire A for the reason that another daughter of sire A might be expected to be identical in butter-fat test (3.2 per cent) with that of the daughter of sire B. That is, sire B's daughter is no better than we should expect a second daughter of sire A to be.

A little thought shows that the sire of merit is the one whose daughter produces not 3.2 per cent of butter-fat but 4 per cent or more. In fact the larger the daughter's butter-fat test the better it is for the sire. The sire whose daughter tests 3.2 per cent would stand a good chance of being the same quality sire as one with a 2.6 per cent daughter.

The expected average butter-fat percentages of second full sisters are repeated for each grade of first full sisters as well as the expected butter-fat percentages of the half sisters. Study of these records indicates rules of breeding which are of some importance. It will be noted first that if a first daughter of a bull has a low butter-fat percentage, for instance 2.8, a second full sister to this cow will probably have a butter-fat percentage of 3.13 whereas a half sister to this cow will have a butter-fat test of 3.23 or about 0.10 per cent higher than the full sister. From such comparison the following rule may be drawn,—if the first daughter of a sire is poor in her butter-fat percentage do not breed this sire to this cow again. Instead breed him to other cows. For if he is bred to the first cow twice he stands a much greater chance of producing another poor butter-fat testing daughter than he does if bred to other cows. In other words do not follow up an unfortunate "nick" by repeating the mating.

Should it happen that the first daughter is of average butter-fat percentage, 3.4, there is little choice as to what system of mating

should be followed since an average butter-fat percentage daughter is likely to result in either case.

For the sire whose first daughter is a high butter-fat testing daughter, that is 4 per cent, the mating should be repeated, for table 75 shows us that in this way the chances are greatest for the production of another high testing daughter. In other words when once found,

TABLE 76

Butter-fat percentage of first daughters and the butter-fat percentage limits between which 50 per cent of their half sisters would be found

FIRST DAUGHTER'S BUTTER-FAT PERCENTAGE	RANGE OF BUTTER-FAT PERCENTAGE OF 50 PER CENT OF THE SECOND DAUGHTERS. HALF SISTERS
2.6	3.0–3.4
2.7	3.0–3.4
2.8	3.0–3.4
2.9	3.1–3.5
3.0	3.1–3.5
3.1	3.1–3.6
3.2	3.2–3.6
3.3	3.2–3.6
3.4	3.3–3.7
3.5	3.3–3.7
3.6	3.3–3.7
3.7	3.4–3.8
3.8	3.4–3.8
3.9	3.4–3.9
4.0	3.5–3.9
4.1	3.5–3.9
4.2	3.5–4.0
4.3	3.6–4.0
4.4	3.6–4.0
4.5	3.7–4.1
4.6	3.7–4.1

the favorable "nick" should be followed up by other identical matings rather than by changing the sire to other cows in the hopes of producing something better.

VARIATION IN THE BUTTER-FAT PERCENTAGES. THE BUTTER-FAT PERCENTAGES BETWEEN WHICH 50 PER CENT OF THE SECOND HALF SISTERS ARE FOUND WHEN THE BUTTER-FAT PERCENTAGE OF THE FIRST SISTER IS KNOWN

To determine properly a sire's worth it is necessary to know something of the amount by which a second sire's daughter should exceed

the butter-fat percentage of a first sire's daughter for the former sire to be considered as worthy. There is, of course, a range of butter-fat per cent around which the daughters from a given sire will vary. Table 76 gives the range within which should be included 50 per cent of the second daughters when the first half sister had a given butter-fat test. The limits of the 50 per cent range are given by mean butter-fat percentage $\pm$ 0.67449 S.D. $\sqrt{1 - r^2}$, when S.D. is the standard deviation.

From table 76 we see that if a sire had one daughter that tested 2.6 per cent it would be equally probable that this sire would have another daughter which tested between 3 and 3.4 per cent. If two sires are being compared it is only reasonable to expect one of them to have a daughter which shall be above the highest butter-fat per cent of this range if we are to consider that sire to be much more valuable than the first one as a breeder. Thus, if the first sire's daughter had a butter-fat percentage of 2.6 the daughter of the second sire should have a butter-fat percentage above 3.4 per cent if this second sire is to be considered as of probable superior merit. If a sire's daughter is just over 3.4 per cent he could be considered as only slightly better than the first sire with a 2.6 butter-fat percentage daughter. On the other hand, if this second sire's daughter is distinctly over the 3.4 per cent line, say 4 per cent, it would be much more likely that this sire truly has an inheritance which he could transmit to his daughters to make them high testing cows. It is of importance to note some of the limits beyond which would be found the really exceptional sire.

THE BUTTER-FAT PERCENTAGES BETWEEN WHICH 99 PER CENT OF THE SECOND HALF SISTERS ARE FOUND WHEN THE BUTTER-FAT PERCENTAGE OF THE FIRST HALF SISTER IS KNOWN

A sire can be considered as exceptionally worthy of merit if his daughter's butter-fat percentage is so high that only once in 100 times would the second daughter of another sire have had such a butter-fat percentage. This gives us a measure by which we may select the sire of merit. The limits of butter-fat percentage for such a case are given in table 77.

The limits of the butter-fat percentage necessary to include 99 per cent of the second full sisters are the mean butter-fat percentage $\pm$2.58 times the standard deviation. If the Gaussian surface be

assured (and this assumption appears justified) 2.58 times the standard deviation of the array on either side of the mean gives the limits of this 99 per cent range.

In our illustration sire (A) has a daughter with a butter-fat percentage of 2.6 per cent. We note on the table that for such a sire 99 per cent of his daughters should lie within the range of 2.4 to 4 per

TABLE 77

Butter-fat percentage of first daughters and the limits between which would be found the butter-fat percentage of 99 per cent of their half sisters

FIRST DAUGHTER'S BUTTER-FAT PERCENTAGE	RANGE OF BUTTER-FAT PERCENTAGE OF 99 PER CENT OF THE SECOND DAUGHTERS. HALF SISTERS
2.6	2.4–4.0
2.7	2.4–4.0
2.8	2.4–4.0
2.9	2.5–4.1
3.0	2.5–4.1
3.1	2.6–4.1
3.2	2.6–4.2
3.3	2.6–4.2
3.4	2.7–4.3
3.5	2.7–4.3
3.6	2.7–4.3
3.7	2.8–4.4
3.8	2.8–4.4
3.9	2.9–4.4
4.0	2.9–4.5
4.1	2.9–4.5
4.2	3.0–4.6
4.3	3.0–4.6
4.4	3.0–4.6
4.5	3.1–4.7
4.6	3.1–4.7

cent of butter-fat. Clearly the exceptional daughter would be the one that produced over 4 per cent of fat. If therefore we have a second bull which has such a daughter, producing milk testing 4.1 per cent it is probable that this bull is much superior in his ability to transmit butter-fat percentage than the bull whose daughter's butter-fat test was 2.6 per cent. In the choice of a bull if the choice lay between these two, it is very probable indeed that the bull with the 4.1 per cent daughter would be the one of most merit in transmitting to his daughters those qualities which would make them big

producers. This case holds strictly for the bulls which have only one daughter each, for of course if the sire has twenty daughters his chance of getting a 4 per cent cow increases greatly. In those cases where a breeder can buy a bull with a large number of recorded daughters, the butter-fat percentage of each and every one should be taken into consideration as well as the range of this butter-fat percentage.

THE DIRECT INFLUENCE OF THE SIRE ON THE BUTTER-FAT PERCENTAGE OF HIS DAUGHTERS

As indicated in an earlier chapter it is possible to find the direct influence of the sire on the butter-fat percentage of his daughters even though the bull does not himself express the character, butter-fat percentage. Were it possible to determine the butter-fat percentage of a sire, this butter-fat percentage could fall in but one class of the correlation table. The daughters of the sire would form part of the array of this table. Other sires of like butter-fat percentage would also fall in this class and their daughters would, combined with those of the first to form the complete array. It is a fair assumption that two daughters of any one sire form a reasonable random sample of this array. Thus if the standard deviation of the butter-fat percentage of any one sire be determined, this standard deviation may, with justice, be considered the standard deviation of the array. In a normal correlation surface this standard deviation of the array bears the following relation to the correlation coefficient and the standard deviation of the whole table:

$$S.D._{\text{array}} = S.D._{\text{table}} \sqrt{1 - r^2}$$

The difficulties inherent in the method have been discussed elsewhere. Table 78 gives the sires' numbers and the standard deviations of the milk yields of the sires' daughters.

The standard deviations of butter-fat percentage presented in table 78 are those for all sires which have five or more daughters. The average weighted standard deviation for these sires' daughters is 0.256 per cent of butter-fat. The weighting is based on the number of daughters which a sire had recorded and on the squared standard deviations of the sires' daughters' butter-fat percentage. The standard deviation for the butter-fat percentage of all the

TABLE 78

Standard deviations of the butter-fat percentage of the daughters of those sires having five or more daughters

SIRE'S NUMBER	STANDARD DEVIATION OF DAUGHTER'S BUTTER-FAT PERCENTAGE	SIRE'S NUMBER	STANDARD DEVIATION OF DAUGHTER'S BUTTER-FAT PERCENTAGE
22,991	0.192	41,266	0.231
23,260	0.235	42,033	0.288
23,450	0.220	42,147	0.314
23,538	0.212	42,940	0.267
24,954	0.136	43,697	0.357
25,467	0.190	44,293	0.218
25,982	0.200	44,554	0.343
26,025	0.157	44,658	0.362
26,940	0.180	45,224	0.191
29,588	0.203	45,671	0.136
29,600	0.224	47,843	0.391
30,550	0.192	48,328	0.107
30,624	0.164	48,663	0.177
30,674	0.160	49,288	0.426
31,212	0.361	49,722	0.287
32,422	0.309	50,290	0.374
32,481	0.412	50,672	0.162
32,492	0.273	50,999	0.154
32,558	0.152	52,927	0.287
34,944	0.107	53,257	0.271
35,227	0.233	53,418	0.265
35,269	0.161	53,825	0.281
36,974	0.270	55,993	0.278
37,314	0.254	59,682	0.115
38,214	0.161	60,344	0.289
38,291	0.373	60,574	0.186
38,401	0.295	60,751	0.197
38,446	0.402	60,966	0.250
39,037	0.240	62,924	0.186
39,357	0.212	74,219	0.173
39,972	0.504	81,663	0.197
40,338	0.279	82,505	0.302
40,358	0.171	93,909	0.310
40,534	0.157	94,217	0.148
41,206	0.215	94,882	0.227

sires' daughters is 0.313 per cent. Because of the fact that the individual sire's daughters' standard deviations for butter-fat percentage are based on few individuals it is necessary to correct

for this influence.[1] This correction factor is equal to 0.9074. The standard deviation corrected for numbers of observations becomes 0.2818 per cent of butter-fat. From these data the correlation coefficient between the butter-fat percentage of the daughters and the sire is found to be 0.53.

The reader will recall that the correlation between the daughters' milk yields and the sire was found to be 0.52. The comparison of this coefficient with that for butter-fat percentage, 0.53, shows that the sire has an almost identical effect on the milk yield or butter-fat percentage of his daughters. By stating the conclusion to evidence not yet presented, the influence of the dam on the butter-fat percentages of her daughters is found to be measured by a correlation of 0.41 or the dam had approximately the same effect as the sire on the butter-fat percentage of their daughters.

These results are a confirmation of some results previously presented by the writer[2] involving wider crosses of butter-fat percentage. In crosses of Holstein-Friesian with Guernsey, Jersey, or Aberdeen-Angus it was found that either way the cross was made the butter-fat percentage of the resulting offspring was intermediate between that expected for the two parents. In other words both the sire and the dam influenced the butter-fat percentage of their offspring.

In a similar experiment analyzed by Castle,[3] Holstein-Friesian and Guernsey cows being used, the crossbred cows had an average butter-fat percentage of 4.08. The butter-fat percentages assumed for the parents were 3.3 per cent for the Holstein-Friesian and 5 per cent for the Guernsey. Here again the influence of both sire and dam is felt in the butter-fat percentage of the offspring.

Other miscellaneous data furnish experimental evidence for the same conclusion. The cows resulting from crosses of Holstein-Friesian bulls to scrub cows of rather high butter-fat percentage in Kildee's and McCandlish's[4] experiments show that the butter-

[1] See Chapter X.

[2] Gowen, John W. 1920. Inheritance in crosses of dairy and beef breeds of cattle. III. Transmission of butter-fat percentage to the first generation. Jour. Heredity, vol. xi, no. 8, pp. 365–376.

[3] Castle, W. E. 1919. Inheritance of quantity and quality of milk production in dairy cattle. Proc. Nat. Acad., vol. 5, pp. 428–434.

[4] Kildee, H. H., and McCandlish, A. C. 1916. Influence of environment and breeding in increasing dairy production. Bul. 165, Iowa Agric. Exper. Sta., pp. 383–402.

fat percentage for these F_1 cows is intermediate between the two parents approaching if anything the butter-fat percentage of the lower testing Holstein-Friesian sires.

SUMMARY

The relation of the butter-fat percentages of Holstein-Friesian Advanced Registry half sisters is presented in this paper. Throughout the study only 365-day butter-fat percentages were used. No correction of any kind was made for the raw data. Briefly stated the conclusions resulting from this study are:

1. The butter-fat percentage of a Holstein-Friesian Advanced Registry cow indicates the probable butter-fat percentages of her half sisters, the correlation coefficient being 0.374 ± 0.015.

2. The probable variation of the butter-fat percentage for the half sisters is presented, both as a whole and for the different grades of butter-fat percentage for one of the half sisters.

3. Half sisters are shown to resemble each other less in their butter-fat percentages than full sisters.

4. The direct influence of the sire on the butter-fat percentage of his daughters is analyzed and shown to be equal to a correlation coefficient of about 0.5. From this result the conclusion may be drawn, that the effect of the sire and dam on the butter-fat percentage of their offspring is approximately equal.

TABLE 79

Butter-fat percentages of sires' daughters. Half sisters

SIRE'S NUMBER	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6–4.7
20,994										2												
21,724			1				2		2													
22,233						1		2										1				
22,235						1			1		1											
22,499						1					1											
22,699								1			1	1										
22,894							1		1													
22,991						2	2		2		2											
23,102							1		1					1	1		1					
23,260								1	1	2	2		1			1						
23,366										1		1										
23,450									1	2	2	2			1	1						
23,538				1	2			3	3	2	1											
24,091											2											
24,777								2														
24,819								1										1				
24,937						2							1									
24,954									1	3		2	1									
25,049									1					1								
25,166									1	1		1										
25,467						1	1	3	4	2	2			1								
25,700								1	1													
25,796																	1			1		
25,865											1			1	1					1		
25,982									1		2	2	1			1						
26,025						1			4	2	1	1										
26,167							1		1													
26,345												1		1		1						
26,734				1					1													
26,937			1			1				1												
26,940							1			2	2		1									
26,939									2		1			1			1					
26,980									1	1												
27,141					1				1													
27,282							1				1											
27,284							1								1							
27,796							1			1												
27,868									1				2		1							
27,930												2										
28,243								1		1												
28,296						1		1	1		1											

TABLE 79—*Continued*

SIRE'S NUMBER	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6–4.7
28,301							1		2		1											
28,340							2	1	1													
28,400						1				1			1									
28,430									1	1			1									
28,504												1	1			1						
28,633								1		1												
28,835			1				1	1	1													
28,982									1						1							
29,027								3	2													
29,214					1			1														
29,236										1			1									
29,303												1		1	2					1		
29,328												1	1	1			1					
29,358						1			1				1									
29,463										1		1										
29,500						1			1		2											
29,548									2													
29,588						2		2	1			1										
29,600									1		3				2							
29,737													1	1								
29,813										1	1											
29,876							1		1													
29,882						1			1													
30,139									1					1								
30,190								1			1	1										
30,266					1		1															
30,369									1	3												
30,413						1			1													
30,550						2			4			1										
30,551			1							1												
30,624						1	3	3	2	1	3											
30,634								1			2											
30,674						1	1	1	2		1											
30,830				1							1											
31,168							1							1								
31,208								1			1											
31,211											2	1	1									
31,212					1	1		2	3	1		2	1			2	1					
31,215									1		1											
31,338					1					1	1											
31,387									1									1				
31,438										3	1	1										

TABLE 79—*Continued*

SIRE'S NUMBER	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6–4.7
31,789					1		1															
31,846											2											
31,957						1			1													
32,059								1	1		1											
32,110												1	1									
32,254							1	1		1	1	1										
32,260										2												
32,322						1					3	1										
32,386								1						1								
32,422							2		1				1	2								
32,481		1				1	1	2			2						1					
32,492						1	1					4	2	1	1							
32,510								1									1					
32,554							1	2														
32,558							2	1	4	6	2	1	1									
32,655								1			1											
32,731					1			1		1												
33,289									1				1									
33,437											3	1										
33,642									1				1									
33,661							2															
33,894								1	1	1	1											
33,957							1				1											
33,979							1		1													
34,205								1			1											
34,467							1		1		2		1									
34,503								2	1		1											
34,944							1	1	2	2												
34,989					1	1																
34,990									1		2	1										
35,074							1				1											
35,119										1						1						
35,165						1	2			1												
35,188									2													
35,226									1	1	1											
35,227					1	1	2	1		1	1	1										
35,250								1		1												
35,269						1				4	3	4										
35,431								1			1				1							
35,469									1			1										
35,639					3																	
35,887									1			2										

TABLE 79—*Continued*

SIRE'S NUMBER	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6–4.7
35,900											1		1									
35,903										1	3											
35,911					1	1																
35,975											1	1										
36,158										2	1			1	1							
36,220				1					1	1												
36,380										2	2											
36,632					1		1															
36,636			1				1															
36,664				1	1	1																
36,819					1	1		1														
36,903									1			1										
36,974	1		2			1	3	1		1												
37,094					1		1	1			1											
37,200											1				1							
37,222								1		1												
37,226							2	1														
37,302					1	2																
37,314						1	2		2		1	1	2									
37,409							1		1													
37,689							1			1												
37,770						1		1		1	1	1										
37,807												2										
37,852										1				1								
38,079					1	2		1		1												
38,214						1		3		1	1											
38,243						1	1	1														
38,291								1	1			2	1	1	4	1			1		2	
38,401								1	1		1	1	1		1		1					
38,446							1			1	2			1	1				2			
38,460					1	2		1														
38,496											1	1										
38,505										1	1											
38,639											1			2								
38,652			1			1				1												
38,883						1					1											
38,977							1		1													
39,037								1	1	3	3	6	5	7	5	3	1	1	1			
39,074									1			1	2									
39,156				1					1													
39,314									1		1											
39,357						1		1	1	1	2	1	1									

TABLE 79—*Continued*

SIRE'S NUMBER	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6–4.7
39,426											1	1	1		1							
39,566							1			1					1							
39,849										1		1										
39,904										1		1										
39,972						3		1	1						1					1		
40,207										1		1	1									
40,214							1			1												
40,248									1			1										
40,338						2	2	2		1	1	1			1							
40,358							1	1	2	1	1	2										
40,368											1	1										
40,386					1	1																
40,492					1		1	1	1													
40,534								2			3	1										
40,547					1		1															
40,663							1	1														
40,718					1			1														
40,871						1									1							
41,009								1		1												
41,038												1			1							
41,114						1	1															
41,206			1	1	1	6	1	2	4	3	1											
41,220				1	1				1													
41,266								1	1	3	1	1	1	1	2							
41,416								1			1	1			1		1					
41,484								1		1												
41,562							2					1										
41,660								1				1										
41,751									2	1				1			1					
41,757				1		1																
41,803								1	1													
41,866							1		1													
41,895								1						1								
41,980								1				3				1						
42,033			1			1	2				3											
42,147			3	2	1	2	2	2	1	4					1							
42,183								1	2	1												
42,235									1	1				1			1					
42,381						1	1															
42,448				2	1					1												
42,451				1				1														
42,682								1				1		1								

TABLE 79—*Continued*

SIRE'S NUMBER	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6–4.7
42,729								1			1											
42,940						1				1	1		3	1	1							
43,075											1			1								
43,113		1							1													
43,496							1		1		1											
43,666										2												
43,697							1	1	3	2		1		2	1					1		
43,964											1				1							
44,034								1				1					1					
44,085										1	1											
44,293										2		2	1	3		1	1					
44,367									1	1												
44,444								1	1		3											
44,465					1			1														
44,543							1		1		1											
44,554		1				2	1				2	2										
44,658							1	1			3		1						1			
44,742									2													
44,758							1		1	1												
44,781										1			1				1					
44,916												1		1								
45,074							1		1	1												
45,103									1		1			1		1						
45,224								2	2	1	1	2	3									
45,502								1		1			2									
45,652					1	1																
45,671								1	2		3	1										
45,674						1				1	1				1							
45,805								2														
45,807									1		1	1				1						
45,998							1	1														
46,212					1					1												
46,301								1		3						1						
46,514								1			1											
46,593										1	1						1					
46,595							1		1													
46,763					1		1															
46,977								2				1										
47,010							1		1										1			
47,135										1	1			1								
47,282							2				1	1										
47,306							1							1								

TABLE 79—*Continued*

SIRE'S NUMBER	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6–4.7
47,341					1				1													
47,808							2															
47,843							1	2		1	1			1	1				1			
48,121										2												
48,328											1	1	2	2								
48,358									1	2	1											
48,498							2	1	1													
48,557					1			1														
48,663						2		1	2		1											
48,696					1							1		1								
48,702					1			1			1											
48,832													1			2	1					
48,846								1			1											
49,207	1										1											
49,221							1							1				1				
49,288								2	1			1		1						1		
49,722				1		1	1		1	1	1		1									
49,795							1		1													
49,910									1			2										
50,290									1	2			1	2		1	1	2	2		1	
50,347										1		1										
50,361									1	1		1										
50,672					1		3	4	2		2											
50,758																1	1					
50,865									1			1										
50,920											1			1								
50,999							1	2	5	5	2	3	1									
51,002											1		1									
51,067												2										
51,078						1					1					1						
51,148						2																
51,217							1	1	2													
51,505														1						1		
51,510							1	1	1													
51,511							1					1										
51,523								1		1	1											
52,145									1	2	1	1										
52,163							1	1														
52,297								1				1										
52,410							1	1	1													
52,527								1	1													
52,791								1			2											

TABLE 79—*Continued*

SIRE'S NUMBER	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6–4.7
52,927							1		2		1	1				1						
52,983								2	1													
53,055						1				1												
53,059							1	2			1											
53,247					1								1									
53,257									2	3	3	1		1		1		1				
53,303																	3	2				
53,309						1							1									
53,399							1		1													
53,418					1		2	1	2	5	2	1	3	1	2							
53,618						1	1															
53,825				1			1	1		2			1									
53,918										1			1									
54,001								1			1											
54,120							1	1	1													
54,424						1				1							1					
54,909										1				1								
55,237												2		1								
55,732						1		2		2												
55,736						1					1											
55,819							1			1												
55,993						1	2		1			2	3									
56,435							1	1				1		1								
56,635																			1		1	1
57,188									2													
57,256						1			1													
57,476										1			1		1							
57,908									1			2										
57,926													1	1								
58,022										1	1											
58,041							1		1	1		2										
58,219							2	1		1				1								
58,313							1		1		1			1								
58,314										1		1										
58,769									1	1			1									
59,156			1					1														
59,219			1					1														
59,272						1				1												
59,660								1	1													
59.682							3	1	1	1												
59,755					1				1													
59,959							1			1												

TABLE 79—*Continued*

SIRE'S NUMBER	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6–4.7
60,047								2		3												
60,278									1			1										
60,344									4	1	1	1						1				
60,360									1	1												
60,529		1							1	1	1											
60,574							1	2	1	3	2	2	2									
60,577						1	2	1				1										
60,710						1			1													
60,751						1		1	1	1	1	1										
60,966					1	1			1	2	2		1									
61,435								2										1				
62,049														2								
62,178											1		1		1	1			1			
62,239				1					2													
62,483							1		1													
62,924					1		2	2	2	3	1	1										
63,578					2																	
63,637									1				1		1							
63,895										1		1										
63,994										1		2										
64,143						1		1	2													
65,016										1					1							
65,750						1	2															
65,981						1		1														
66,092					1		1															
66,417						1		1														
67,203										1						1						
67,211								1		1			1	1								
68,544							1				1	1		1				1				
68,946			1		2		1															
69,686						1			2		1											
70,753							1		1		1											
70,857		1	1			1		1														
71,013							1	1														
71,071							1				1											
71,158							1				2		1									
71,527									1		1											
72,013									1			1										
72,099								2														
72,287							1		1													
73,416						1					1		1									
74,219					1		3	4	3		1	1										

TABLE 79—*Continued*

SIRE'S NUMBER	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6–4.7
74,982										1			1									
75,012							3		1				1									
75,034								1				1										
75,304				1	1		1	1														
75,361							1			4												
75,627				1		1																
76,663								1	1		1											
76,665									1			1										
76,861											1				1							
77,572					1				1													
78,027							1				1											
78,123							1		1													
79,602									1	1												
80,551					1		1															
80,596			2		1	1																
80,916										1	2		1									
81,142											1			1								
81,405					1			2														
81,497									2													
81,630						1			1			1	2									
81,663						1		1	1	1	1	1										
82,505					1		1	1	4	1	1		1				1					
83,354						1		1														
84,099											3		1									
84,472									1				1									
85,556					1	1																
85,968								1		1		1										
86,166											2					1						
86,837										1	1		2			1						
88,319									1	1	2		1									
91,668							1		1													
93,149											1		1									
93,229								2	1		1		1									
93,371			1	1																		
93,909				1				1	3	2	1			1		1						
94,156								1					1									
94,217										2			4	1								
94,292				1	1		1				1				1							
94,884										1	1	1	1	1			1					
94,984						1			1													
96,076						1	1															
97,208						1		1	1		1											

TABLE 79—*Concluded*

SIRE'S NUMBER	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6–4.7
97,224								1	1													
97,403								1	1													
97,472							1	1	1			1				1						
98,063								1		1												
98,762										1	1											
102,469										2			2									
103,899								1		2	1	1										
104,002				1	1	1																
105,106						1		1														
107,790									2													
107,891							2					1										
110,157									1								1					
110,474					1	1			1													
112,077									1		1											
113,938									1				1									
114,797													1	1								
121,546								1		1												
128,445									1		1											

CHAPTER XIII

On the Relation of the Milk Yields of Mother and Daughter in the Holstein-Friesian Advanced Registry

That the milk yield of the mother is transmitted to the daughter is an assertion frequently found in the popular literature on the inheritance of milk yield. The proof cited to substantiate this hypothesis for it is not more than that, is based almost entirely on reasoning which appeals to its audience because it is reasonable, and it is reasonable to suppose that milk yield is inherited from mother to daughter. It would be equally reasonable to suppose that egg production would be transmitted directly from a 200-egg mother to her daughter. Yet when the test was made many years ago by Pearl and Surface[1] it was found that the offspring of the 200-egg mothers were inferior in egg production to the offspring of mothers whose egg production was between 150 and 200 eggs, or who were below the 200-egg class. The pitfalls in an argument depending for its validity on its reasonableness are indeed many. And yet milk production from mother to daughter is inherited. The daughters of 20,000 pound mothers are better milk producers than are the daughters of mothers whose milk yield is less than 20,000 taking them on the average. This statement is made, not as an assertion, but as a proposition subject to proof.

THE RELATION OF THE MILK YIELDS OF MOTHER AND DAUGHTERS IN THE HOLSTEIN-FRIESIAN ADVANCED REGISTRY

If milk yield is inherited, one measure of this inheritance would be the relation of the milk yields of mother and daughter. Up to and including volume 30 of the Holstein-Friesian Advanced Registry there are 611 daughters whose dams have 365-day records. These

[1] Pearl, Raymond and Surface, Frank M. Studies on the physiology of reproduction in the domestic fowl. II. Data on the inheritance of fecundity of the daughters of "200-egg" hens. Annual report of the Maine Agricultural Experiment Station for 1909, Bulletin 166, pp. 48–84.

pairs of age-corrected milk records are arranged in table 80 to show the relation of the milk yields of mother and daughter. From this table the constants measuring this relationship may be determined in the usual way.

TABLE 80

Correlation surface showing the relation of the milk yield of mother and daughter in the Holstein-Friesian Advanced Registry

DAM'S CORRECTED MILK YIELD	DAUGHTER'S CORRECTED MILK YIELD																								
	10,000–11,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000–34,000	
10,000–11,000			1		2		2																		5
11,000	1		1	1	1	1	2							1											8
12,000		1		3	2	2	2	1	3		1														15
13,000	1	2	3	3	3	7	4	3	5		1	1		1		1		1							36
14,000	1		1	6	5	3		5	7	6	3			2	2				1						42
15,000			2	4	9	12	5	7	4	6	3	2	2	2	5	1		1	2						67
16,000			2	2	3	3	3	10	3	3	1	7	1	1			1								40
17,000			1	1	2	2	9	13	7	5	5	3	3	2	3		2	1							59
18,000		2	1		2	4	6	7	11	9	10	4	1	2	2	2	1	1					1		66
19,000				2		3	4	5	6	9	7	6	3	6		1			4						56
20,000					1	2	3	6	2	8	7	7	7	3	4	1									51
21,000					2		2	3	3	4	3	2	3	4		2	4		1			1			34
22,000		1		3	1	2	4	3	2	3	5	4	2	3	3	5	1		1						43
23,000							1	2	1	2	2	1	4	4	3	2	1	3			2				28
24,000						1		1		2	1	1	1	1	1	3	2	1							15
25,000									1	3	2	2		1	1			1	1	1		1			14
26,000										1	1	1	2		1		2	2							10
27,000												1	1	1	1	1		1						1	7
28,000										1	1	1	1	1			1				1				7
29,000							1			1				1	1	1			1						6
30,000													1												1
31,000–32,000																			1						1
	3	6	12	25	33	42	48	66	55	63	53	43	32	36	27	20	15	12	12	1	3	2	1	1	611

Attention may be directed to the fact that table 80 has the ordinary appearance of a Gaussian correlation surface when the correlation coefficient is about 0.5. The fundamental constants showing the variation and correlation of milk yields for mother and daughter as drawn from this table are given in table 81.

Attention may be called to the fact that the milk yields of the daughters are over 300 pounds more than the rest of the breed as determined from all the Advanced Registry records. While this difference is not significant in view of its probable error, it may still be indicative in the direction of a slight improvement in the milk yield of the cows of today. The difference of the milk yields of the mother and daughters is also noticeable. The daughters are 801 ± 157 pounds of milk more than the dam. This difference is over 5 times the probable error and consequently significant. There are, however, some difficulties in the way of this direct comparison. A number of dams have two or more daughters and thus appear in the dam column twice. If the comparison is made

TABLE 81

The variation and correlation of the milk yields of mother and daughter in the Holstein-Friesian Advanced Registry

Mean milk yield, daughter	19,604 ± 113
Mean milk yield, mother	18,803 ± 109
Standard deviation of milk yield, daughter	4,132 ± 80
Standard deviation of milk yield, mother	3,986 ± 77
Coefficient of variation of milk yield, daughter	21.1 ± 0.4
Coefficient of variation of milk yield, mother	21.2 ± 0.4
Correlation coefficient between the milk yields of mother and daughter	0.497 ± 0.021

between the daughters and dams entering the dams only once the difference of the milk yields becomes 773 ± 167.3 or the difference is 4.6 times its probable error. Again the difference is significant. The interpretation of the cause of this significant difference is obscured by the fact that there are two schools each of which will claim credit for it. The first will say that better breeding or a better appreciation of the laws of heredity is the responsible cause. The other school will claim that the credit is due to a better understanding of the rules of nutrition. The attempt will be made to analyze the relative influence of these factors on milk yield in a later chapter.

The standard deviation of the milk yields of daughters is somewhat larger than the standard deviation of the dam's milk yields. The difference is not significant, however. The relative variations of the milk yields of mother and daughter are practically the same.

The correlation coefficient representing the degree of relationship found between the milk yields of the daughter and her dam is fairly high. In other words the milk yields of the daughters are governed to a certain degree by the milk yields of their dams. This resemblance in the milk yields of daughter and mother may be due in part to environment although it seems more probable that heredity is the main factor.

If heredity plays this part in the milk yield of the daughters it is evident that progress in the cattle-breeding industry is to be made by a careful consideration of the milk records of the cows which are bred. The problem of the small breeder often hinges on this point. It is a question of whether he can improve his herd more by spending his money for a high-priced bull, or for cows known to be better producers than those within his herd. The question is complex and the argument is many-sided. As has been previously indicated the sire and the dam probably have approximately equal influence on the milk yield of the daughter, the correlation for each being about 0.5. Unfortunately it is not possible to measure the sire's worth directly by a milk record. The only criteria which we have of his milk yield are the milk yields of his progeny or the milk yield of his mother. The question before the small breeder of limited funds is often the question of the purchase of an untried bull with perhaps a milk record of his mother. As will be shown later the milk yield of a bull's mother is only about half as good a measure of the milk production of the bull's daughters as the record of the dam is of her daughters. The gamble on a bull's worth is then much greater than on the daughter's worth when the daughter comes from good producing cows. Such being the case it seems probable that greater progress in increasing the quantity of the milk yield in a man's herd will be made if he devotes more money to the female side of the herd, especially if he can associate himself with other breeders in the purchase and use of a tested bull.

THE RELATION OF THE MILK YIELDS OF MATURE DAUGHTERS AND MATURE MOTHERS IN COMPARISON WITH THE MILK YIELDS OF IMMATURE DAUGHTERS AND THEIR IMMATURE MOTHERS

In his interesting contribution to the problems of the inheritance of butter-fat yield Rietz[2] came to the conclusion that inheritance in

[2] Rietz, H. L. 1909. On inheritance in the production of butter-fat. Biometrika, vol. vii, pp. 106–126.

production is more pronounced in mature than in immature cows. The evidence on which this conclusion is based is cited in some detail as the point has quite a marked bearing on all research in this field. The correlation coefficients between the 7-day butter-fat yields of mother and daughter were 0.344 ± 0.023 for the cows under four years old at time of test and 0.284 ± 0.025 for the cows over four years old at time of test. It is then shown that the correlation for the under four years group is probably too large. The basis of the argument is the fact that the correlation coefficient between the butter-fat of mother and daughter for those cows under 2.25 years old is 0.145 ± 0.045. By regarding the correlation 0.344 as arising from the combining of two sub-groups it is then

TABLE 82

	0 to 2.5 years	2.5 to 3.5 years	3.5 to 4.5 years	4.5 and over	Total under 4.5 years
Cows in milk (N)	618	494	430	1,416	1,542
Number in Advance Registry (n)	413	346	269	620	1,028
n/N	0.67	0.70	0.63	0.44	0.67

shown that it is equal to or less than 0.145. Those results are then corrected for the selection automatically practiced by the Advanced Registry requirement. The basis of determining the amount of correction to be applied is as follows. The proportion of the cows milking at a given time to the proportion in the Advanced Registry are determined. The results for the year May 15, 1906 to May 15, 1907, are given in table 82.

The proportions are then assumed to be those proportions based on the capabilities of the Holstein-Friesian cows. That is, for those cows under 2.5 years old 67 per cent of them could if put on Advanced Registry test make the requirement and 33 per cent could not. From these results the final correlation coefficients for the butter-fat yields of mother and daughter are, mature cows, $r = 0.63$ from $R = 0.284$ of selected group; cows under 2.25 years old, $r = 0.25$ from $R = 0.145$; and for cows under five years old without regard to age in selecting pairs, $r = 0.18$ from $R = 0.132$ for selected groups. From these results the conclusion is drawn that the inheritance of milk yield is greater in mature cows, cows over four years old, than it is in immature cows.

As indicated in Chapter VIII it seems to the writer that this conclusion is faulty because of certain assumptions made in the calculations. In the first place the original correlation coefficients for butter-fat yield of mother and daughter at mature age do not differ significantly from each other. The correlation coefficient for mature cows is 0.284 ± 0.025. That for cows under 2.25 years of age is 0.145 ± 0.045. The difference is 0.139 ± 0.051 or their difference is 2.7 times the probable error. This difference is not significant. Now, while the probability may be strengthened that 0.145 is the true coefficient by the method of proof adopted by Rietz, still it does not seem to the writer that we can be justified in disregarding the probable error.

In the method of correction it is questionable if the correction factors adopted are the right ones. In the writer's experience with the Advanced Registry it has been found that the great elements preventing Advanced Registry testing are the actual cost of the test and the indisposition of purebred cattle owners to take up Advanced Registry work. Time and again our work has demonstrated that a man, once convinced of the merits of Advanced Registry testing and placing his cows on test, has sent in some of the best records in the state. Furthermore, the cows put on test have made the requirement with some to spare.

The question of whether or not there is a difference in mature vs. immature cows also hinges on the validity of the percentage 44 as representing the true percentage of mature cows which can make the Advanced Registry while 67 represents the true percentage of cows under 2.25 years which can make the Advanced Registry. The writer believes both percentages are too low. For the arguments and data supporting this conclusion the reader is referred to the chapter on the stringency of selection exercised by the Advanced Registry requirement. The difference in the number of cows tested at four years and over and those tested at four years and under comes, in the writer's experience, not from any difference in the ability of the cows to make the Advanced Registry but rather from the whim of the breeders. We find that those breeders who make a business of testing their cattle plan to test the cows on their first lactation at two years old or the second lactation at three years old. If a cow has once made a record those breeders seldom test her again. The reason for this procedure is that Advanced Registry

cattle command a higher price than unrecorded cows, so the breeder naturally wants his animals to have records. On the other hand, the breeder who rarely tests is much more likely to test his cows at older ages than is the breeder who makes a business of testing all the cows he has. These smaller breeders are in the minority as compared with those who regularly do Advanced Registry work. There are also those who do not test any of their cows. In view of these conditions it is not surprising that the numbers of cows tested under four years old are a greater proportion of the population than are those which are tested after four years of age. The difference in these percentages collected by Rietz need not have and in fact probably do not have any relation to the relative abilities of the cows to make the Advanced Registries.

Another line of evidence bears out this contention. If the mean butter-fat yields of Holstein-Friesian Advanced Registry cows are at all indicative, the writer showed in the 1920 Animal Husbandry report[3] that the average butter-fat yield of cows over four years was from 50 to 100 pounds more in excess of the Advanced Registry requirement than the average butter-fat yield of cows under four years old. That is, it would be easier for the cows over four years old to make the requirement than for cows under four years old to make it. These records are for 365-day records and not for 7-day.

Again if the mature cows' milk yields are more representative of their probable inheritance, cows at a mature age should have a larger correlation coefficient between their milk yields in different lactations than would immature cows. This would be true because of the fact that the character, milk yield, would not be fully expressed in the immature cows. This problem has been analyzed from several angles in several papers[4] from this laboratory. The outcome of these results seems to show no such difference between mature and immature cows.

[3] Gowen, John W. 1921. Report of progress on animal husbandry investigations in 1920. Annual report of the Maine Agricultural Experiment Station for 1921, Bulletin 299, pp. 85–120.

[4] Gowen, John W. 1920. Studies in milk secretion. V. On the variations and correlations of milk secretion with age. Genetics, vol. 5, pp. 111–188.

Gowen, Marie S., and Gowen, John W., 1922. Studies in milk secretion. XIII. Relation between milk yield and butter-fat percentage of the 7-day and 365-day tests of Holstein-Friesian Advanced Registry cattle. In press.

If the arguments just presented are sound we are not justified in applying a greater correction for Advanced Registry selection in the case of mature cows than in that of the younger cows. If we do apply the same correction, in other words call $\mu = 0.85$, the r is found to be 0.38 for the corrected correlation between the butter-fat yield of daughter and dam. Considering the probable errors attached to these coefficients it seems likely that the mature cows would not differ significantly from the others. If this coefficient of correlation, 0.38, is the correct one the results of Rietz would agree with those of the writer save that the coefficient of correlation between the mothers and daughters in the 7-day test is somewhat less than that for the 365-day test. This conclusion is supported by the fact that the 7-day test is not quite so reliable as the 365-day

TABLE 83

Correlation coefficients for the milk yields of mother and daughter in certain age groups

AGE OF MOTHER	AGE OF DAUGHTER	CORRELATION COEFFICIENT FOR MILK YIELD
Over 4 years	Over 4 years	0.490 ± 0.039
Under 4 years	Over 4 years	0.550 ± 0.088
Over 4 years	Under 4 years	0.479 ± 0.029
Under 4 years	Under 4 years	0.516 ± 0.053

test for determining the milking qualities or butter-fat percentage of a cow.

Turning now to the age-corrected 365-day records of mother and daughter the correlation coefficients are noted for different age divisions in table 83.

These correlation coefficients clearly do not differ significantly from each other. Under these circumstances these data for the 365-day age-corrected records support the conclusion that the inheritance of milk yield is as pronounced in immature animals as it is in mature animals. If these records under four years are still further sub-divided we find that for those cows under three years the correlation between the milk yields of mother and daughter is 0.459 ± 0.088 and for those cows with ages three to four years the correlation is 0.496 ± 0.098. These correlation coefficients are not significantly different from the rest. Unfortunately the

numbers on which they are based are small. The conclusion, that the inheritance in mature and immature animals is approximately the same, is, however, supported by all the results.

THE MILK YIELD OF THE DAUGHTER WHEN THE MILK YIELD OF THE DAM IS KNOWN

There are 216 daughters who came from dams whose milk yield was over 20,000 pounds. These daughter's average production was 21,778 pounds of milk for the 365-day period. The 395 daughters who came from dams whose milk yield was less than 20,000 pounds had an average production of 18,393 pounds. In other words, the daughters from cows whose production was greater than 20,000 pounds had a milk yield 3385 pounds greater than the daughters that came from dams whose milk yield was less than 20,000 pounds. It is scarcely necessary to draw the conclusion that the selection of dams of 20,000 pound milk yield would cause an increase in the milk yield of resulting offspring.

Table 81 furnishes the necessary data to determine the probable milk yields of the daughter when the milk yields of her mother are known, subject, of course, to the conditions of feeding and handling of the Advanced Registry and the age correction of the records. The equation for this relation is:

$$\text{Daughter's milk yield} = 9957 + 0.512 \text{ dam's milk yield} \qquad (40)$$

The tabulation of the expected milk yields of the daughters for different grades of dams' milk production is given in table 84. This is the average expected milk production of the daughters and is not necessarily the milk yield that a single daughter will give.

From table 84 it is clear that the milk yields of the daughters are dependent on the milk yields of the dams. If the average milk yields of the daughters for each grade of dam are compared with the milk yields of second full sisters for each grade of milk yield of first full sister, as shown in table 60 it will be noticed that the full sisters resemble each other slightly more in their milk yield than do the mothers and daughters. If the comparison is made for mothers and daughters with half sisters (table 64), the mothers' and daughters' milk yields are found to resemble each other the more closely.

When the milk yield of a dam is very low the milk yield of the daughter is, on the average, a good deal larger. Thus if a poor

producing dam is bred it is reasonable to expect a higher milk-producing daughter. Such a condition of affairs shows us that we should not attribute too good qualities to a bull who makes such a mating and has the expected milk-yielding daughter. The "nick" may not be his fault. On the other hand when a bull is bred to a high producing cow he is likely to produce a lower producing

TABLE 84

Milk yields of dams and the average yield of their daughters. Holstein-Friesian Advanced Registry

MILK YIELD OF DAM	EXPECTED AVERAGE MILK YIELD OF DAUGHTERS
pounds	*pounds*
10,000	15,081
11,000	15,593
12,000	16,106
13,000	16,618
14,000	17,131
15,000	17,643
16,000	18,155
17,000	18,667
18,000	19,180
19,000	19,693
20,000	20,205
21,000	20,717
22,000	21,230
23,000	21,742
24,000	22,255
25,000	22,767
26,000	23,279
27,000	23,792
28,000	24,304
29,000	24,817
30,000	25,329

daughter. Here again we should not condemn the bull too severely for the cow may be milking more than she has hereditary qualities to pass on to her offspring. The action of "prepotency" and "nick" will be discussed further in the following section.

In using these equations to predict the average milk yield of the daughter from the actual milk yield of the dam the assumption is made that the regression lines are linear. The validity of this assumption may be determined by examining the raw regression

line in comparison with the fitted regression line as given in figure 18. This chart shows that the mean milk yields of the daughters for each grade of milk yield of the dam form a straight ascending line well fitted by the theoretical regression line. The assumption of linear regression lines is consequently justified.

VARIATION OF MILK YIELD. THE MILK YIELDS BETWEEN WHICH 50 PER CENT OF THE DAUGHTERS ARE FOUND WHEN THE MILK YIELD OF THE DAM IS KNOWN

Those who are familiar with milk yields of dairy cattle have cognizance of the fact that all the daughters of 20,000 pound dams will not have milk yields of 20.205 pounds. This will be their average, if

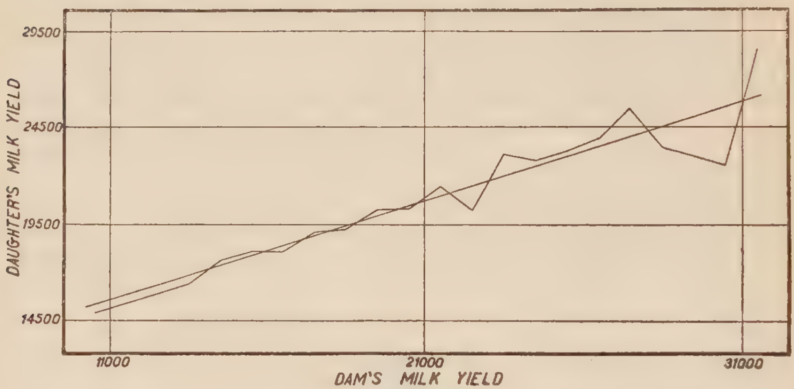

FIG. 18. MILK YIELDS OF MOTHER AND DAUGHTER FOR THE 365-DAY RECORDS

there are enough of them, but individually they will vary. What we want to know is how much will they vary from the average of 20,205 pounds. Suppose we say that it shall be an even chance that any one daughter will lie inside or outside a certain range of milk yield. Thus if we had 100 daughters which came from dams producing just 20,000 pounds of milk, the milk yield of 50 of these daughters would be expected to lie within 17,787 pounds as the low limit and 22,623 pounds as the high limit of production. The other 50 would be expected to be either under or over this limit of milk production.

Table 81 gives necessary data to determine this range. The average standard deviation of the daughters' milk yield about any average milk yield of the dam is $4132\sqrt{1 - 0.497^2} = 3585$

pounds of milk. The limits of milk yield necessary to include 25 per cent of the daughters on either side of the mean when the dam's milk yield is known are given by mean $\pm$ 0.67449 $\times$ 3585. These limits are given in table 85.

It will be noted that there is quite a range of production to include even 50 per cent of the daughters of a given grade of dam's milk

TABLE 85

Milk yields of dams and the limits between which would lie the milk yields of 50 per cent of their daughters

DAM'S MILK PRODUCTION	RANGE OF MILK YIELD OF 50 PER CENT OF THE DAUGHTERS
pounds	*pounds*
10,000	12,663–17,500
11,000	13,175–18,011
12,000	13,688–18,524
13,000	14,200–19,036
14,000	14,713–19,549
15,000	15,225–20,061
16,000	15,737–20,573
17,000	16,250–21,086
18,000	16,762–21,598
19,000	17,275–22,111
20,000	17,787–22,623
21,000	18,299–23,135
22,000	18,812–23,648
23,000	19,324–24,160
24,000	19,837–24,673
25,000	20,349–25,185
26,000	20,861–25,697
27,000	21,374–26,210
28,000	21,886–26,722
29,000	22,399–27,235
30,000	22,911–27,747

yield. Thus 100 daughters from dams whose milk yield was 20,000 pounds would be expected to have a certain range of milk yield. Fifty of these daughters would be expected to have a range limited on the low side by 17,787 pounds of milk and on the high side by 22,623 pounds of milk. It will be noted that as the milk yields of the dams increase the milk yields of the daughters also increase.

Table 85 furnishes a measure of what constitutes true worth in a bull. Thus if it is an even chance that a bull's daughters will produce

between 17,787 and 22,623 pounds of milk when the dam's milk yield is 20,000 pounds it is no great credit to this bull to have his daughters produce milk yields within this range. The bull of worth would be the one whose daughters produced more than 22,223 pounds of milk and the bull to be regarded with suspicion is the one whose daughters

TABLE 86

Range of milk production within which would be expected 99 per cent of the daughters for a given grade of dam

DAMS'S GRADE OF MILK YIELD	RANGE OF MILK YIELD OF 99 PER CENT OF DAUGHTERS
pounds of milk	*pounds of milk*
10,000	5,832–24,330
11,000	6,344–24,842
12,000	6,857–25,355
13,000	7,369–25,867
14,000	7,882–26,380
15,000	8,394–26,892
16,000	8,906–27,404
17,000	9,419–27,917
18,000	9,931–28,429
19,000	10,444–28,942
20,000	10,956–29,454
21,000	11,468–29,966
22,000	11,981–30,479
23,000	12,493–30,991
24,000	13,006–31,504
25,000	13,518–32,016
26,000	14,030–32,528
27,000	14,543–33,041
28,000	15,055–33,553
29,000	15,568–34,066
30,000	18,080–34,578

produce less than 17,787 pounds when his dam was one from 20,000 pound cows. The ranges of daughters' milk yield for other grades of dams are given in table 85. Here then is a table available to show what may be reasonably expected of a sire to prove him worth while.

RANGE OF MILK YIELD NECESSARY TO INCLUDE 99 PER CENT OF THE DAUGHTERS WHEN THE MILK YIELD OF THE DAM IS KNOWN

A table to show the milk yields which a sire's daughters from dams of a given grade of milk yield must produce to make him one of the

exceptional sires has a place in any such study as the one we are now making. To indicate this milk yield we shall determine the range of milk yield necessary to include 99 per cent of the daughters from dams of a given grade of milk yield. This range is determined from table 81. The limits of it are mean $\pm 2.58 \times 3585$. When so defined, the limits of production wherein we would expect to find 99 per cent of the daughters are given in table 86.

This table shows the range of milk yield of 99 per cent of the daughters from any given grade of dam. This range is quite large. However, when it is realized that even this range of productivity is now and then exceeded it shows how extreme the limits of milk yield occasionally may be. It would be doubtful indeed if the owner did not recognize as exceptional the bull whose daughters produced 25,000 pounds of milk when the dam's production was only 12,000 pounds. The lower limit is equally important. The sire whose daughters produce only 14,000 pounds when their dams produce 25,000 is clearly not to be desired or perpetuated by continued breeding.

SUMMARY

This chapter presents a study of the relation which exists between the milk records of a dam and her daughters. The Holstein-Friesian 365-day age-corrected records are used throughout the study. Broadly speaking the results of this study furnish another link in the chain of proof necessary to establish the theory that the milk yield of the cow is dependent on the inheritance derived from the sire and dam. Specifically the conclusions show:

1. That the milk yield of the cow is dependent on the milk yield of her dam, the correlation coefficient being 0.497 ± 0.021.

2. That the milk yields of cows from dams of a given grade of milk yield are highly variable. The ranges of milk production to include 50 and 99 per cent of the daughters are presented.

3. The evidence analyzed indicates no difference in the inheritance of milk yield in mature and immature cows.

CHAPTER XIV

The Relation Between the Milk Yields of Cows from the Same Dam but by Different Sires. Half Sisters

The question of keeping any cow in the herd depends on her worth as a milk producer and the ability of her progeny to produce milk. The quicker these two points can be determined, the more likely is the owner to succeed. It has previously been shown that the milk yield of the dam predicts the milk yield of the daughter with a fairly high degree of accuracy. The accuracy with which the milk yield of a dam's daughter predicts the milk yield of her half sister has not been determined. The object of this section is the analysis of this problem.

TABLE 87

Variation and correlation of half sisters' milk yields in the Holstein-Friesian Advanced Registry

Mean milk yield	19486±158
Standard deviation of milk yield	4135±111
Coefficient of variation of milk yield	21.2±0.6
Correlation coefficient for the relation of half sisters' milk yields	0.381±0.033

The age-corrected records for the milk yield of all half sisters from a common dam were collected. Table 92 gives the necessary data.

THE RELATION BETWEEN THE MILK YIELDS OF A DAM'S DAUGHTERS. HALF SISTERS

The variations and correlations of the milk yields of these half sisters are given in table 87.

The average of the milk yields of these half sisters is slightly although not significantly more than that for the whole Advanced Registry. The standard deviation is practically the same as that for the rest of the Advanced Registry animals. The coefficient of

variation is likewise practically the same as that of the rest of the breed.

The correlation coefficient showing the relation of the milk yields of half sisters indicates clearly that the milk yield of one half sister determines to a fair degree of accuracy the milk yield of the other half sister. This resemblance is due to two causes, common heredity of the two half sisters from the mother and the common environment under which the two half sisters may have been brought up.

It is of some interest to note that the correlation coefficient between the milk yields of half sisters when the sire is common to each is the same, within the limits of random sampling, as the correlation coefficient for the milk yield of half sisters when the dam is common to each. Thus for the sire's daughters, the correlation coefficient for the milk yield is 0.362 ± 0.015 and for the dam's daughters, 0.381 ± 0.033. The difference is 0.020 ± 0.036. This difference is less than the probable error. The common sire has the same chance to govern the milk yields of his daughters that the common dam has to govern her daughters' milk yields. The results above show that the degree of influence that each exert is the same. Such a result furnishes further proof of the fact that the sire and dam exert a like influence on the milk yield of their daughters.

The correlation coefficient for the relation of the milk yields of full sisters is 0.548 ± 0.027. The correlation coefficient for the milk yields of half sisters, the dam being common, is 0.381 ± 0.033. The difference of the correlation coefficients is 0.167 ± 0.043 or the difference is 3.9 times its probable error. Full sisters are consequently more highly correlated in their milk yield than half sisters. These differences between the full sisters and half sisters may be analyzed further. The degree of control over the variation of any character by another has been shown to be equal to S.D. − S.D. $\sqrt{1 = r^2}$. The value of r^2 for the full sisters is 0.3003 and for the half sisters, the dam being common, is 0.1452. The value of the standard deviation for the full sisters is 4102 and for the half sisters is 4135. The standard deviation remaining after the elimination of the variation due to full sisters is 3434. The variation of milk yield due to half sisters is 312 pounds and the variation due to full sisters is 668 pounds. As the only difference in full sisters and half sisters is in the fact that the full sisters have a common sire, it seems justifiable to attribute these

differences to the influence of the sire. This demonstrates in still another way the influence of the sire and of the dam on the milk yield of the progeny.

THE MILK YIELD OF THE SECOND HALF SISTERS WHEN THE MILK YIELD OF THE FIRST HALF SISTER IS KNOWN. THE HALF SISTERS HAVE A COMMON DAM

From the data given in table 87 we obtain the average milk yield of one half sister when the milk yield of the other is known. This determination depends on the assumption of linear regression lines and on the surface being approximately Gaussian. These assumptions are probably at least approximately correct in view of the evidence already presented for the relation of the milk yields of full sisters and of mother and daughter. The linear regression equation for the relation of the milk yields of half sisters when these cows have common mothers is given by:

$$\text{Second half sisters' milk yield} = 12064 + 0.381 \text{ first half sister's milk yield} \quad (41)$$

From this equation the probable average milk yields of the half sisters are given in table 88. The probable milk yields of full sisters are repeated for its comparative value.

The milk yield of the first daughter is given in the first column of this table. The second column presents the average milk yield of the second daughter. These daughters are half sisters from the same dam. The milk yields of the second daughters which are full sisters to those whose milk yields are stated in the first column are given in the third column. It will be noted that the milk yields of the full sisters are much nearer each other than are the milk yields of half sisters. The reason for this is fairly obvious on a little consideration. The full sisters have not only a common dam but also a common sire. The sire has a limited inheritance to give to his daughters to determine milk yield. It is reasonable to suppose that a single sire would have less variation in his inheritance than would two different sires. The table shows that such is actually the case.

This table throws some light on the question of how long it is desirable to retain a bull in service. It will be noted that if a first daughter of a cow has a milk yield of 10,000 pounds it is probable that the second daughter of this cow by the same sire will have a milk

yield of 14,229 pounds whereas the second daughter by a different sire will have a milk yield of 15,873 pounds, or an average increase of 1644 pounds as the result of changing the sire. On the other hand if the milk yield of the daughter is exceptionally high, say 30,000 pounds, the milk yield of her full sister by the same sire and out of the same dam will average 25,179 pounds whereas

TABLE 88

Milk yields of second daughters when the milk yield of the first daughter is known. Half sisters and full sisters.

FIRST DAUGHTER'S MILK YIELD	AVERAGE MILK YIELD OF SECOND DAUGHTERS. HALF SISTERS	AVERAGE MILK YIELD OF SECOND DAUGHTERS. FULL SISTERS
10,000	15,873	14,229
11,000	16,254	14,777
12,000	16,635	15,324
13,000	17,016	15,872
14,000	17,397	16,419
15,000	17,778	16,967
16,000	18,158	17,514
17,000	18,539	18,062
18,000	18,920	18,609
19,000	19,301	19,157
20,000	19,682	19,704
21,000	20,063	20,252
22,000	20,444	20,799
23,000	20,825	21,347
24,000	21,206	21,894
25,000	21,587	22,442
26,000	21,967	22,989
27,000	22,348	23,537
28,000	22,729	24,084
29,000	23,110	24,632
30,000	23,491	25,179

the milk yield of the half sister by a different sire but from the same dam, will average 23,491 pounds or an average decrease of 1688 pounds as the result of changing the sire. The conclusion is obvious, if a sire's first daughters from a given cow are low producers his second daughters are also relatively low in production. In other words if we wish further daughters from this cow we had better get another bull. But if the first daughter from the cow was exceptionally high in milk yield, the data show that the second daughter

will probably be much higher in milk yield when this same bull is used than when a different bull is bred to this same cow. Such being the case it appears much more reasonable to keep this bull in service as long as possible than to try to better him by a change.

There is also the other side of the story. The low producing daughter is equally caused by the sire and dam. In other words the "nick" was for a low producer. The data indicated above point to the desirability of changing the bull if the dam is to be kept. The question may equally well be asked if it is desirable to keep the dam. The probability is that the dam has a poor inheritance for milk yield. Under such circumstances it would seem best to dispose of such a cow as this would be likely to improve the herd.

We may examine those cases where the first daughter is of average milk production, for example 20,000 pounds age-corrected milk yield for the 365-day period. The second daughter's milk yield, a half sister, would be 19,682 pounds of milk and the milk yield of the second daughter, a full sister, would be 19,704 pounds. That is the average milk yields are practically identical for second full or half sisters when the first sister has average production. There is, however, this difference. The milk yields of all the full sisters would be more concentrated around this average, cover a less range of milk yield, than would the milk yields of the half sisters. This point is taken up in the following section.

VARIATIONS IN THE MILK YIELDS OF HALF SISTERS. THE MILK YIELDS BETWEEN WHICH 50 PER CENT OF THE SECOND HALF SISTERS ARE FOUND WHEN THE MILK YIELD OF THE FIRST HALF SISTER IS KNOWN

The range of milk yield necessary to include 50 per cent of all additional sisters when the grade of milk yield for one half sister is known, is given in table 89.

The range of milk yield which will include 50 per cent of all second daughters whose half sisters milk 20,000 pounds is from 17,103 pounds of milk to 22,261 pounds of milk or the range is 5158 pounds of milk. This range is for half sisters which have a common dam. In Chapter IX it was shown that for full sisters the range of milk yield to include 50 per cent of all second daughters is from 17,388 to 22,020 pounds of milk or the range is 4632 pounds of milk. The

range of milk yield to include 50 per cent of the second daughters for full sisters is 526 pounds closer than is the range of milk yield which is required to include 50 per cent of the milk yields of half sisters, these half sisters having a common dam. In other words our prediction of milk yield for full sisters is much closer than is our prediction of milk yield for half sisters.

TABLE 89

Milk yields of first daughters and the limits between which would be included the milk yields of 50 per cent of their half sisters

FIRST DAUGHTER'S MILK PRODUCTION	RANGE OF MILK YIELD OF 50 PER CENT OF THE SECOND DAUGHTERS. HALF SISTERS
10, 000	13, 294–18, 452
11, 000	13, 675–18, 833
12, 000	14, 056–19, 214
13, 000	14, 437–19, 595
14, 000	14, 818–19, 976
15, 000	15, 199–20, 357
16, 000	15, 579–20, 737
17, 000	15, 960–21, 118
18, 000	16, 341–21, 499
19, 000	16, 722–21, 880
20, 000	17, 103–22, 261
21, 000	17, 484–22, 642
22, 000	17, 865–23, 023
23, 000	18, 246–23, 404
24, 000	18, 627–23, 785
25, 000	19, 008–24, 166
26, 000	19, 388–24, 546
27, 000	19, 769–24, 927
28, 000	20, 150–25, 308
29, 000	20, 531–25, 689
30, 000	20, 912–26, 070

There are many other uses to which these tables may be put. It is interesting to note that the milk yields of the second daughters are low, when the milk yield of the first daughter is low, while the milk yields of the second daughters are high, when the milk yield of the first daughter is high. These daughters have a common dam. The previous study of milk yield shows that the dam materially influences the productivity of the daughter. It is of prime importance, therefore, to know the milk yield of the dams for it is possible

to improve half sisters as much by the careful selection of the dam as by the use of a good bull.

MILK YIELDS BETWEEN WHICH 99 PER CENT OF THE SECOND HALF SISTERS ARE FOUND WHEN THE MILK YIELD OF THE FIRST HALF SISTER IS KNOWN

In the previous section we have presented the limits of the probable range of variation in milk yield. In this section the probable maximum limits for this variation are found. The determination of this range is similar to that for the 50 per cent range or that for the 99 per cent range in the other sections.

The range of milk yield necessary to include 99 per cent of the daughters from a single dam but by different sires is indeed a wide one. Thus, a cow producing 14,000 pounds of milk might have a sister at nearly the top of the breed in her production or she might have a sister of very low production. With this range of milk yield required to cover even 99 out of every 100 such pairs of half sisters, it is no wonder that sometimes at least the breeder becomes discouraged at even guessing what the offspring from any given mating will do for her own milk yield. However, it must be remembered that in general these are extreme limits and that most of the daughters are much more closely concentrated.

THE DIRECT EFFECT OF THE DAM ON THE MILK YIELD OF HER DAUGHTERS

In Chapter XIII the writer has presented the correlation coefficient for the relation of the milk yield of the dam and her daughter as determined by the ordinary product moment method. A good deal of interest is attached to the indirect method of calculation as used in determining the influence of the sire on the milk yields of his daughters since here with the dam and her daughters the product moment coefficient is available for comparison with the coefficient found by the more indirect method.

It will be remembered that any one dam can fall in but one class or array of the correlation table for the relation between her milk yield and that of her daughters. In other words the daughters will form one and only one of the arrays of such a table. Had we enough daughters it would be possible to get the standard deviation of such

an array, at least approximately, by determining the standard deviation of the daughters' milk yields. In the normal correlation surface the standard deviations of the arrays bear a relation to the correlation coefficient and standard deviation of the whole table. This relation is:

$$\text{S.D.}_{\text{array}} = \text{S.D.}_{\text{table}} \sqrt{1 - r^2}$$

From this relation it is possible to determine the direct influence of the dam on the milk yield of the daughter.

TABLE 90

Milk yields of first daughters and the limits between which would lie the milk yields of 99 per cent of their half sisters

FIRST DAUGHTER'S MILK YIELD	RANGE OF MILK YIELD OF 99 PER CENT OF THE SECOND DAUGHTERS. HALF SISTERS
10, 000	6, 008–25, 738
11, 000	6, 389–26, 119
12, 000	6, 770–26, 500
13, 000	7, 151–26, 881
14, 000	7, 532–27, 262
15, 000	7, 913–27, 643
16, 000	8, 293–28, 023
17, 000	8, 674–28, 404
18, 000	9, 055–28, 785
19, 000	9, 436–29, 166
20, 000	9, 817–29, 547
21, 000	10, 198–29, 928
22, 000	10, 579–30, 309
23, 000	10, 960–30, 690
24, 000	11, 341–31, 071
25, 000	11, 722–31, 452
26, 000	12, 102–31, 832
27, 000	12, 483–32, 213
28, 000	12, 864–32, 594
29, 000	13, 245–32, 975
30, 000	13, 626–33, 356

As indicated in an earlier section this coefficient will be larger than the product moment coefficient because of the fact that the standard deviations of the individual cows are calculated around the rough observational mean and because the number of individuals influences the size of the standard deviation. The difficulty, and it appears to be a real one, is that the correlation coefficient calculated in this

manner is too large as compared with the direct method of calculation. In table 91 are given the dams' herd book numbers and the standard deviations. Only those cows with three or more daughters were used.

TABLE 91

Standard deviations of the milk yields of the daughters for those dams which had three or more daughters

DAM'S NUMBER	STANDARD DEVIATION OF DAUGHTERS' MILK YIELD	DAM'S NUMBER	STANDARD DEVIATION OF DAUGHTERS' MILK YIELD
	pounds		*pounds*
25, 173	943	73, 286	1, 247
31, 408	1, 225	77, 032	1, 247
38, 802	2, 160	77, 491	1, 090
46, 348	4, 497	77, 777	2, 357
47, 448	5, 888	78, 331	6, 164
50, 657	1, 247	79, 290	1, 633
51, 737	1, 247	76, 684	1, 414
53, 646	2, 625	82, 444	4, 190
54, 125	433	84, 455	816
54, 170	4, 082	86, 998	1, 700
55, 150	2, 315	87, 301	471
55, 194	4, 028	90, 522	4, 497
62, 230	2, 278	90, 846	4, 989
62, 337	2, 055	93, 789	2, 357
63, 667	471	98, 734	4, 867
64, 178	2,625	99, 074	1, 700
65, 465	1	99, 086	3, 300
65, 882	3, 399	100, 999	1, 633
66, 788	2, 357	101, 313	2, 357
68, 099	1, 886	106, 688	1, 299
70, 231	2, 160	106, 988	3, 300
70, 349	2, 055	107, 842	1, 414
70, 427	1, 633	108, 681	2, 055
71, 624	4, 110	110, 877	4, 320
71, 679	2, 786	112, 246	471
71, 850	1, 090	120, 022	2, 160
71, 898	471	174, 475	3, 559

From these results the average weighted standard deviation was found to be 2733 pounds. The standard deviation of the whole population was 4139 pounds. The correction factor for the standard deviations based on the number of daughters for each dam is 0.7376.

The standard deviation after applying this correction is 2733 ÷ 0.7376 = 3705. From these results the correlation coefficient is found to be 0.446. This correlation coefficient agrees closely with that of the product moment method 0.497. The indirect method clearly lends further support to the conclusion that the milk yield of the dam controls to a certain degree the milk yield which the daughter is able to give. The correlation coefficient measuring the amount of this relationship is about 0.47.[1]

The correlation coefficient for the relation of the milk yields of daughter and sire as found in Chapter X was 0.52. The difference between 0.47 and 0.52 would not be significant. Such being the case the influence of the sire on the milk yield of his daughters is clearly equal to the influence of the dam on the milk yield of her daughters and vice versa. Unfortunately it is impossible to determine the milk yield of the sire directly. This can be done only by a study of the sire's daughters' milk records. The milk yields of the dam can be measured. Such being the case, the breeder should pay the same attention to the dam in making any mating that he pays to the sire, for the milk yield of the dam tells much about the future milk yield of the daughters.

[1] The validity of the method for determining the correlation coefficients for the standard deviations corrected for number of daughters may be experimentally checked by the following experiment. Suppose that the assumption is made that the daughters are drawn from the same population as that of table 80 where the actual relation of the milk yield of daughters and dams is known. In this case the correlation coefficient is 0.497. A random sample of three is drawn and recorded. The three samples correspond to three daughters. The average weighted standard deviation of daughters was found to be 2662 for this experiment. The standard deviation of all daughters is 4325. The correction factor for the standard deviation based on samples of three is 0.7236. Or 2662, the actual standard deviation, ÷ 0.7236 = 3679, the corrected standard deviation. From the relation of these two standard deviations the correlation coefficient is found to be 0.526. As noted above the actual correlation coefficient for the table of daughters' and dams' milk yields, from which these samples of three were drawn was 0.497. The agreement between the two correlation coefficients is remarkable. The validity of the method of calculating the correlation coefficient from the standard deviations appears to be experimentally sound.

TABLE 92

Correlation surface for milk yields of half sisters. Common dam

HALF SISTERS' MILK YIELDS	HALF SISTERS' MILK YIELDS																								
	9,000–10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000–33,000	
9, 000–10, 000											1														1
10, 000				1					1																2
11, 000			2		3		1						1												7
12, 000		1				1						1		1											4
13, 000			3			6	2	6	4	3	1	1	1		1		1			1					30
14, 000				1	6	6	3	2	2	1	4	2	1	1		2				1					32
15, 000			1		2	3	2	3	4	5	5	1	2		1										29
16, 000					6	2	3	2	4	3	3	2	3	3	1	1	2								35
17, 000		1			4	2	4	4		8	6	5	1	8	2	3	1			1					50
18, 000					3	1	5	3	8	8	5	7	2	3	5	5	1	1		1					58
19, 000	1				1	4	5	3	6	5	6	4	2	4	1	2		1		1					46
20, 000				1	1	2	1	2	5	7	4	6	2	1	2	1						1			36
21, 000			1		1	1	2	3	1	2	2	2	4	4	1	2	2	1	2	1			1		33
22, 000				1		1		3	8	3	4	1	4		3	3	2	1	1			1			36
23, 000					1		1	1	2	5	1	2	1	3		3			1						21
24, 000						2		1	3	5	2	1	2	3	3	6	2			1	1		1		33
25, 000					1			2	1	1			2	2		2		1	2			1	1		16
26, 000										1	1		1	1			1				1				6
27, 000													2	1	1		2							1	7
28, 000					1	1			1	1	1		1			1									7
29, 000																1		1							2
30, 000												1		1			1								3
31, 000													1			1	1								3
32, 000–33, 000																			1						1
	1	2	7	4	30	32	29	35	50	58	46	36	33	36	21	33	16	6	7	7	2	3	3	1	498

SUMMARY

This section presents a study of the milk records of those Holstein-Friesian cows, which are out of the same dam but by a different sire. The major conclusions are:

1. The milk yield of a cow indicates the probable milk yield of her half sister, the dam being the common parent. The correlation coefficient measuring this relationship is 0.381 ± 0.033.

2. The probable variation of the milk yield of the half sisters is presented both as a whole and for different grades of milk yield.

3. The comparison of the correlation coefficients for the milk yields of half sisters with the sire as the common parent and of half sisters with the dam as common parent shows that the correlation coefficients are practically the same, 0.362 and 0.381.

4. The correlation coefficient for the milk yields of daughter and dam is determined by the indirect method of the standard deviations of the array of daughters' milk yields. The correlation coefficient is found to be 0.446. This agrees very nicely with the correlation coefficient for the direct relation of the milk yields of daughter and dam, 0.497, as found in another chapter.

5. The correlation coefficient for the relation of the sire's and daughter's milk yield is further analyzed. The correlation coefficient 0.52, measuring this relationship, agrees closely with the correlation coefficient measuring the relationship of the milk yields of the dam and daughter. These facts make it apparent that the milk yields of the daughter are jointly dependent on the sire and dam.

CHAPTER XV

On the Relation between the Butter-fat Percentage of Mother and Daughter

In organizing a herd, the individual animals should be considered, not so much for their own production, as for the possibility of their offspring's being worthy in milk yield and butter-fat percentage. For a bull this fact is particularly evident, for aside from his part in initiating the series of events leading to milk production, a bull's worth comes largely in the merit of his daughters in milk yield. The fact that the bull is able to serve many cows while each cow has one calf leads to the saying that a bull is half the herd. And so he is, but it is well to remember that for each calf he is no more than half. The dam is the other half and deserves just as much consideration as does the bull.

In an earlier section it was shown that the average milk yields of the daughters increase as the relative level of the milk yields of their dams becomes larger. The present chapter will deal with the other item of interest in connection with milk—butter-fat percentage. It is true that the variable generally considered is butter-fat. However, a little consideration of the fact that butter-fat is merely the multiple between the pounds of milk and the analysis of the milk will make it clear that it is the concentration of the fat which is the important factor. Furthermore when it is realized that in the Holstein-Friesian breed the concentration of this fat, butter-fat percentage, bears little relation to the amount of milk a cow produces it becomes clear that this variable is nearly if not entirely independent of the yield of milk, and therefore should be treated as a separate variable. The influence of the cow on the butter-fat percentage of her offspring will be studied in this section. The data include those Holstein-Friesian cows which have Advanced Registry records and whose daughters also have Advanced Registry records.

THE RELATION BETWEEN THE BUTTER-FAT PERCENTAGE OF MOTHER AND DAUGHTER IN THE HOLSTEIN-FRIESIAN ADVANCED REGISTRY

One consequence of the inheritance of butter-fat percentage is the existence of a relation between the butter-fat percentages of the mother and her daughter. Thus the daughters coming from

TABLE 93

Correlation surface showing the relation of the butter-fat percentages of mother and daughter in the Holstein-Friesian Advanced Registry

DAM'S BUTTER-FAT PERCENTAGE	DAUGHTERS'S BUTTER-FAT PERCENTAGE																				
	2.6–2.7	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5–4.6	
2.6–2.7	1			1	2																4
2.7	1	1	2	2				2													8
2.8			1		2		1	1													5
2.9		1			4	3	1	1	2												12
3.0	1			5	8	8	7	4	5	3	3	2	1								47
3.1			2		6	9	14	5	6	2	7		1		1	1					54
3.2		1	1		5	7	6	14	9	6	7	6	2		2		1	1			68
3.3	1	1		2	6	10	15	17	16	13	8	1	4	2	1	1	1	1			100
3.4		1	1	3	2	2	8	14	18	9	7	8	2		5	1					81
3.5					4	11	9	12	5	10	7	2	1	1	2	2					66
3.6		1		1		3	4	6	7	7	2	8	2	4		1	1			1	48
3.7						1	3	3	12	7	6	4	1	1				1			39
3.8						2	2	3	5	3	3	3		1	1		1	1			25
3.9					1	1	1	3	1	2	7	3		2							21
4.0				1		1		1	1	2		2				1					9
4.1					1					2		2		4	1		1				11
4.2											1	1		1	2						5
4.3						1				1				1				1			4
4.4–4.5							1					1		1				1			4
	4	6	7	15	41	59	72	86	87	67	58	43	14	18	15	7	5	6		1	611

dams of four per cent should be higher in their butter-fat percentage than the daughters coming from dams whose butter-fat percentage is three per cent. The data showing the relation of the butter-fat percentages of the pairs of mothers and daughters is given in table 93.

It is of some importance for the future use of the constants derived from this table to note that the correlation surface has the

ordinary appearance of such a surface for a correlation coefficient of about 0.4. The assumption appears reasonable that the data are drawn from a Gaussian surface. The constants showing the variations and correlations of the butter-fat percentages of mother and daughter are given in table 94.

The butter-fat percentages of daughters and their dams are slightly, although not significantly, in excess of the rest of the breed judging from the records of the whole Advanced Registry. The butter-fat percentages of mother and daughter are practically the same. The standard deviations for butter-fat percentages of mother and daughter are the same within the limits of random sampling. The coefficient of variation for the butter-fat percentages of mother and daughter are also approximately the same. These facts indicate that a fair random sample of the Advanced Registry records is represented in these data.

TABLE 94

The variation and correlation of the butter-fat percentages of mother and daughter in the Holstein-Friesian Advanced Registry

Mean butter-fat percentage, daughter	3.440 ± 0.008
Mean butter-fat percentage, dam	3.445 ± 0.009
Standard deviation of butter-fat percentage, daughter	0.311 ± 0.006
Standard deviation of butter-fat percentage, dam	0.318 ± 0.006
Coefficient of variation of butter-fat percentage, daughter	9.04 ± 0.18
Coefficient of variation of butter-fat percentage, dam	9.24 ± 0.18
Correlation coefficient between the butter-fat percentages of mother and daughter	0.413 ± 0.023

The correlation coefficient measuring the degree of interdependence of the butter-fat percentages of mother and daughter is fairly high. The butter-fat percentage found in the milk of the daughter is partially dependent on the butter-fat percentage found in the milk of her mother. In other words butter-fat percentage is transmitted from mother to daughter. The daughters of dams whose butter-fat percentage is more than 3.6 per cent are higher in their butter-fat percentages than are the daughters of dams whose butter-fat percentage is less than 3.6 per cent.

The correlation coefficient for the relation of the mother's and daughter's butter-fat percentages is slightly less than the correlation coefficient for the mother's and daughter's milk yields, 0.413

$\pm$ 0.023 to 0.497 $\pm$ 0.021. The difference is 0.084 $\pm$ 0.031. This difference is but 2.7 times the probable error and cannot therefore be considered significant.

The correlation coefficient for the relation of the butter-fat percentages of full sisters is 0.464 $\pm$ 0.032. The correlation coefficient is 0.051 $\pm$ 0.040 more than the correlation coefficient for the relation of the butter-fat percentages of mother and daughter. The difference is clearly not significant.

The correlation coefficient for the relation of the butter-fat percentages of half sisters is 0.374 $\pm$ 0.015. This correlation coefficient is 0.039 $\pm$ 0.027 less than that for the relation of the butter-fat percentages of mother and daughter. This difference is not significant. The interdependence of the full sisters' butter-fat percentages, of the mother's and daughter's butter-fat percentages, and of the half sisters' (common sire) butter-fat percentages makes them valuable, in the order indicated, for indicating the probable production of any unknown cow.

A practical consequence of the correlation between the butter-fat percentage of mother and daughter may be illustrated thus. There are 444 daughters which come from dams whose butter-fat percentage was less than 3.6 per cent. These daughters' average butter-fat percentage was 3.38. The 167 daughters from dams whose butter-fat percentage was more than 3.6 per cent had an average butter-fat percentage of 3.59 per cent. The daughters of the high test group of dams had a butter-fat percentage of 0.21 per cent in excess of the daughters which came from the lower test group of dams. That such a difference is a real one, well worthy of consideration may be seen more clearly when it is converted into pounds of butter-fat for the year. On the average, at mature production, such a difference amounts to over 40 pounds of butter-fat for each of the 167 cows from the higher testing dams.

The small breeder has a real problem hinging on this point. It is a question whether such a breeder should spend his money on a high-priced bull, and thereby, perhaps, cut down on the cows, or whether he shall spend his money on cows known to be better producers than the animals within his herd. Admittedly this question is a complex one and the argument is not always entirely conclusive. The side of it relating to milk yield has been touched on elsewhere. It may be noted that the sire and dam appear to have

an equal influence on the butter-fat percentage of their daughter. Were it possible, then, to measure directly the productive powers of the sire in butter-fat percentage it would be reasonable to expect more progress in the herd for the expenditure of a given sum by the selection of a high butter-fat percentage sire. However, it is not possible to determine the sire's worth directly. If he is a sire under four years old the only possible criteria of his productive qualities are the milk yields of his mother or his sisters. It will be shown later that the butter-fat percentage of a sire's dam is not so reliable a measure of the sire's daughters butter-fat test as is the butter-fat test of the daughters' dam. The chance taken on the probable worth of the daughters is larger when the prediction is based on the paternal granddams' butter-fat percentage than it is when the prediction is based on the dam's butter-fat percentage. In view of this fact it would appear as if greater progress would be made by the small breeder in increasing the butter-fat percentage of his cows if he devoted more time to the character of the cows forming his herd. This is especially true if this breeder is able to associate himself with others in a purchase and use of a *tested, disease-free* bull.

THE RELATION OF THE BUTTER-FAT PERCENTAGES OF MATURE DAUGHTERS AND MATURE MOTHERS IN COMPARISON WITH THE BUTTER-FAT PERCENTAGE OF IMMATURE DAUGHTERS AND THEIR IMMATURE MOTHERS

As noted in a former section, Rietz,[1] in his important contribution to the inheritance of butter-fat arrives at the conclusion that butter-fat is inherited more strongly in "mature" animals than in "immature" cows. As the evidence for this conclusion has been cited and discussed in some detail in the section on the relation of the milk yield of mother and daughter, it seems necessary here to present only the correlation coefficients for the butter-fat percentages of the "mature" and "immature" groups. These correlation coefficients for the 365-day records for the butter-fat percentage are given in table 95.

The correlation coefficient for the mother under four years and the daughter over four years is based on the limited number of 28.

[1] Rietz, H. L. 1909. On inheritance in the production of butter-fat. Biometrika, vol. vii, pp. 106–126.

Each of the other groups are easily within the limits of random sampling of each other. These results support the conclusion that the butter-fat percentage of the "immature" cows is as strongly inherited as that for "mature" cows. Rietz further subdivides his "immature" group. If we subdivide ours we find that for those cows under three years of age the correlation between the butter-fat percentages of mother and daughter is 0.304 ± 0.101 and for those cows with ages three to four years the correlation coefficient is 0.609 ± 0.082. These correlation coefficients do not differ significantly from each other or from those of the "mature" groups. It is unfortunate that the under three-year-old group and the three-year-to-four-year-old group were not larger. However, the results all tend to confirm the conclusion that within the limits of random sampling there is no difference between the inheritance of milk yield or butter-fat percentage in the "mature" or "immature" groups.

TABLE 95

Correlation coefficients for the butter-fat percentages of mother and daughter for certain age groups

AGE OF MOTHER	AGE OF DAUGHTER	CORRELATION COEFFICIENT FOR BUTTER-FAT PERCENTAGE
Over 4 years	Over 4 years	0.427 ± 0.042
Under 4 years	Over 4 years	0.760 ± 0.054
Over 4 years	Under 4 years	0.374 ± 0.032
Under 4 years	Under 4 years	0.447 ± 0.058

THE AVERAGE BUTTER-FAT PERCENTAGE OF THE DAUGHTER WHEN THE BUTTER-FAT PERCENTAGE OF THE MOTHER IS KNOWN

From a practical standpoint, nothing could be of more value to the dairyman than to be able to determine exactly the milk yield and butter-fat percentage of a cow before the mating to produce her was made. It is to be hoped that some day this exactness of prediction will be reached. The data given in table 94 enable us to make a fairly long stride in this direction, for from the dam's record we are able to determine: (1) What will be the average butter-fat percentages of the daughters for those dams which have a given grade of butter-fat percentage; (2) within what range of butter-fat percentage to expect the daughters when these daughters come from dams whose butter-fat percentage is a given grade;

(3) what will be the probable limits of the daughters' range in butter-fat percentages when they come from dams of a given grade of butter-fat percentage. The average butter-fat percentages of the daughters from a given grade of butter-fat percentage in the dams are given in tabular form below. These are, of course, average productions of the daughters and not necessarily the butter-fat percentage of any one individual daughter.

Table 94 furnished the necessary data to determine the probable butter-fat percentage of the daughter when the butter-fat percentage of her mother is known. These determinations are, of course, subject to the condition of feeding and handling of the Advanced Registry. The equation for the relation of the butter-fat percentages of mother and daughter is:

$$\text{Daughter's butter-fat percentage} = 2.05 + 0.404 \text{ dam's butter-fat percentage} \tag{42}$$

Table 96 gives the average butter-fat percentages of the daughters for different grades of butter-fat percentage in the dam. The most noticeable feature of this table is that as the butter-fat percentage of the dam increases the butter-fat percentage of the daughter increases. If the butter-fat percentage of the daughters from very low record dams are examined it is found that the daughters' butter-fat percentages are 0.5 per cent higher than their parents'. On the other hand 3.4 per cent dams have daughters only slightly over 3.4 per cent. But the daughters from 4.6 butter-fat percentage dams are nearly 0.6 per cent less than their mothers. The daughters of very low parents or of very high parents tend to be respectively lower or higher than those of the rest of the breed, which means that they tend to revert toward the average production of the Holstein-Friesian breed. The really significant fact for the breeder, however, is that the daughters from low testing dams tend to be low and the daughters from high testing dams tend to be high.

Table 96 indicates a point worthy of consideration in the choice of a bull. If a bull is bred to a very low butter-fat percentage cow it is reasonable to expect the daughter to be higher testing than the dam. Under such conditions we should not attribute too good qualities to such a bull. On the other hand, a bull bred to a very high butter-fat testing cow would be expected to have a daughter whose butter-fat test is lower. Under these conditions the bull

should not be blamed too severely. The distance or range which the bull's daughters should exceed in their butter-fat percentage to make them worthy will be discussed in the following section.

If a comparison is made between the butter-fat percentages of full sisters and of mother and daughter it will be noted that full sisters tend to resemble each other more than do mothers and daughters. While the difference is not a large one it does show that

TABLE 96

Butter-fat percentages of dams and the average butter-fat percentages for their daughters

DAM'S BUTTER-FAT PERCENTAGE	EXPECTED AVERAGE BUTTER-FAT PERCENTAGE OF THE DAUGHTERS
2.6	3.10
2.7	3.14
2.8	3.18
2.9	3.22
3.0	3.26
3.1	3.30
3.2	3.34
3.3	3.38
3.4	3.42
3.5	3.46
3.6	3.50
3.7	3.54
3.8	3.58
3.9	3.62
4.0	3.66
4.1	3.70
4.2	3.75
4.3	3.79
4.4	3.83
4.5	3.87
4.6	3.91

the full sisters tend to be more nearly alike in their butter-fat percentages than do mothers and daughters. If the butter-fat percentages of half sisters be compared with those of mothers and daughters, the butter-fat percentages of mothers and daughters are found to be more alike than the butter-fat percentages of the half sisters (by the same sire but out of different dams).

In using the equation for the prediction of the milk yield of the daughter from that of the dam the assumption is made that the

regression lines are linear. The raw observational mean butter-fat percentages of the daughters for each grade of dam's butter-fat percentage are shown in figure 19.

The observational means at the extremes of the table depend on relatively few data. The fit of the regression curve is better in the middle than at the extremes. It might be possible to get a parabola which would fit the observations a little more closely than does the linear equation. This problem may be analyzed a little further. The correlation ratio for table 93 is equal to 0.452 ± 0.022.

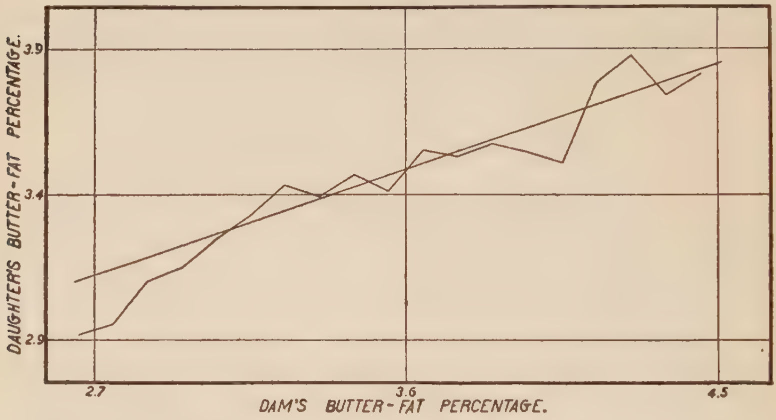

FIG. 19. BUTTER-FAT PERCENTAGES OF MOTHER AND DAUGHTER FOR THE 365-DAY RECORDS, HOLSTEIN-FRIESIAN ADVANCED REGISTRY

The correlation ratio is not significantly different from the correlation coefficient. The test for the linearity of regression is the relation between the correlation ratio squared minus the correlation coefficient squared.

$$\eta^2 - r^2 = 0.034 \pm 0.010$$

The result is 3.4 times the probable error. There is consequently some ground for the belief that a curve might fit the raw observations somewhat better than the line. However, the difference in the fit would not be very great for the test for linearity is only on the border line of significance. Therefore, in view of the advantages of a linear equation it is believed to be desirable to use it in preference to a curve.

VARIATIONS OF BUTTER-FAT PERCENTAGE. THE BUTTER-FAT PERCENTAGE BETWEEN WHICH 50 PER CENT OF THE DAUGHTERS ARE FOUND WHEN THE BUTTER-FAT PERCENTAGE OF THE DAM IS KNOWN

We should not expect a single daughter to have exactly 3.58 per cent of butter-fat in her milk when she came from a dam whose butter-fat per cent was 3.8. There is a range of butter-fat percentage within which the daughters would test for any given grade of dam. To properly select our cows and know with what we are dealing it is necessary to know something about this variation in yield. Thus if there are 100 daughters which came from dams whose butter-fat percentage was 3.8, the butter-fat percentage of about 50 of them would be between 3.39 and 3.77. The other 50 would lie outside these limits, approximately half or 25 would be above and the other half would be below. The butter-fat percentages given in the next table show the limits between which would lie the butter-fat percentage of 50 per cent of the daughters from dams of a given grade of butter-fat percentage. This range is determined from the data of table 94. The average standard deviation of any array of daughters is equal to S.D. $\sqrt{1-r^2}$ or for our data is equal to $0.311\sqrt{1-0.413^2}$ or 0.284 per cent of butter-fat. The limits of the butter-fat percentage necessary to include 25 per cent of the daughters on either side of the mean when the dam's butter-fat percentage is known are given by mean $\pm$ 0.67449×0.284. These limits are given in table 97.

From this table it will be seen that there is a fairly wide range of butter-fat percentage necessary to include even 50 per cent of the daughters of a given grade of dam. Thus for the 100 daughters of 3.2 per cent dams, a range, whose lower limit is 3.15 and whose upper limit is 3.53, would be required to include approximately 50 of them. Twenty-five of the other fifty would be expected to have a butter-fat test less than 3.15 and the other 25 would be expected to have a butter-fat test more than 3.35. It will be noted that as the grade of the butter-fat percentage of the dam increases the range likewise moves up. Thus the lower limit of the range for the cows coming from 2.6 per cent dams is 2.91 while the lower limit of the range for the cows coming from 4.6 per cent dams is 3.71.

RANGE OF BUTTER-FAT PERCENTAGE NECESSARY TO INCLUDE 99 PER CENT OF THE DAUGHTERS WHEN THE BUTTER-FAT PERCENTAGE OF THE DAM IS KNOWN

In table 97 are given the probable limits of the range in butter-fat percentage of 50 per cent of the cows coming from dams of a given grade of butter-fat percentage. Often it is important to

TABLE 97

Butter-fat percentages of dams and the limits between which would lie the butter-fat percentages of 50 per cent of their daughters

DAM'S BUTTER-FAT PERCENTAGE	RANGE OF BUTTER-FAT PERCENTAGE OF 50 PER CENT OF THE DAUGHTERS
2.6	2.91–3.29
2.7	2.95–3.33
2.8	2.99–3.37
2.9	3.03–3.41
3.0	3.07–3.45
3.1	3.11–3.49
3.2	3.15–3.53
3.3	3.19–3.57
3.4	3.23–3.61
3.5	3.27–3.65
3.6	3.31–3.69
3.7	3.35–3.73
3.8	3.39–3.77
3.9	3.43–3.81
4.0	3.47–3.85
4.1	3.51–3.90
4.2	3.55–3.94
4.3	3.59–3.98
4.4	3.63–4.02
4.5	3.67–4.06
4.6	3.71–4.10

know the probable extremes of butter-fat percentage that may be found in daughters of dams producing a given grade of butter-fat. By defining the limits as those within which 99 per cent of the daughters are found, a table may be formed to show the range for the majority of the offspring. The points limiting such a range is determined by:

$$\text{mean} \pm 2.58 \times 0.284$$

Table 98 shows these limits.

Table 98 gives the range of butter-fat percentage within which should be found 99 per cent of the daughters from any given grade of dam. This range is quite large. However, it should be remembered that even this range of butter-fat percentage is now and then exceeded by some daughter from a dam of the given grade of butter-fat percentage. It is, however, noticeable that the daughters

TABLE 98

Range of butter-fat percentage within which would be expected 99 per cent of the daughters from a given grade of dam

DAM'S BUTTER-FAT PERCENTAGE	RANGE OF BUTTER-FAT PERCENTAGE OF 99 PER CENT OF THE DAUGHTERS
2.6	2.37–3.83
2.7	2.41–3.87
2.8	2.45–3.91
2.9	2.49–3.95
3.0	2.53–3.99
3.1	2.57–4.03
3.2	2.61–4.07
3.3	2.65–4.11
3.4	2.69–4.15
3.5	2.73–4.19
3.6	2.77–4.23
3.7	2.81–4.27
3.8	2.85–4.31
3.9	2.90–4.36
4.0	2.94–4.40
4.1	2.98–4.44
4.2	3.02–4.48
4.3	3.06–4.52
4.4	3.10–4.56
4.5	3.14–4.60
4.6	3.17–4.64

from low testing dams do not have such high butter-fat percentages as those from dams of higher test. This is what inheritance means. It defines the end results of the processes of inheritance and shows us some of the steps necessary to bring about its control. The exact numerical conclusions of the data are limited by the data. However, as will be shown elsewhere, in principle the conclusions are true for another breed, the Jersey.

SUMMARY

The daughter, in the Holstein-Friesian breed and for the 365-day test, is partially dependent on her mother for the butter-fat percentage which her milk yield will test. In general the results of this study furnish another link in the chain of proof for the inheritance of butter-fat percentage. Very briefly the results are:

1. The correlation coefficient measuring the degree of dependence of the butter-fat percentage of the daughter on that of the dam is 0.413 ± 0.023.

2. The butter-fat percentages of cows from dams of a given grade of butter-fat percentage are quite variable. The ranges of butter-fat percentages to include 50 per cent of the daughters and 99 per cent of the daughters are presented.

3. The evidence indicates no difference in the inheritance of butter-fat percentage in "mature" and "immature" cows.

CHAPTER XVI

The Relation Between the Butter-fat Percentages of Cows from the Same Dam but From Different Sires. Half Sisters

Another phase of the problem of breeding for milk and butter-fat production is the question of the influence of the dam as shown by the butter-fat percentage of her daughters. Two kinds of daughters are available, full sisters and half sisters. The relation between the butter-fat percentages of full sisters has been analyzed. The relation of the butter-fat percentages of the half sisters, when the dam is the common parent, has not been examined. This relation gives another measure of the influence of the dam on the butter-fat percentage of her daughters. A comparison of the full and half sisters will also verify our previous results on the influence of the sire on the butter-fat percentage of his daughters. The whole furnishes another stone in the foundation for the proof of the inheritance of butter-fat percentage.

THE RELATION BETWEEN THE BUTTER-FAT PERCENTAGES OF A DAM'S DAUGHTERS. HALF SISTERS

The constants describing the variation and correlation of the butter-fat percentages of half sisters are given in table 99. The correlation surface from which these constants are obtained is given in table 106.

The average-butter-fat percentage for these half sisters is practically the same as that for the Advanced Registry taken as a whole. The standard deviation and the coefficient of variation are slightly, although not significantly, less than those for the whole Advanced Registry. The correlation coefficient for the relation of the butter-fat percentages of half sisters, while small, is significant. From this fact the butter-fat percentage of one half sister may be said to govern to a certain degree the butter-fat percentage of the other half sister. This resemblance may be due to the common heredity coming from

the dam or it may be due to the common environment under which the tests are made.

There is ground for the belief that the correlation coefficient for the half sisters with a common dam is too low. The records of the daughters of Artis Belle of Riverside 2D are quite exceptional. Two of these daughters, Aralia Belle Queen and Artis Belle King have butter-fat tests of 3.7 and 4.1 per cent respectively. The other daughter Lady Riverside Belle has a butter-fat test of only 2.57. In a later 7-day test this cow has a butter-fat percentage of 3 per cent. Such extreme ranges for the butter-fat tests of the daughters even though they are half sisters are not common. Figure 20 shows how materially the record of Lady Riverside Belle has influenced the regression line and consequently the correlation coefficient.

The average butter-fat percentages of the second half sisters for the given butter-fat percentages of the first half sisters are given as

TABLE 99

Variation and correlation of half sisters' butter-fat percentages in the Holstein-Friesian Advanced Registry, all data

Mean butter-fat percentage	3.44 ± 0.01
Standard deviation of butter-fat percentage	0.302 ± 0.008
Coefficient of variation of butter-fat percentage	8.78 ± 0.24
Correlation coefficient for the relation of half sisters' butter-fat percentages	0.173 ± 0.037

the irregular solid line. It is obvious from this observational line that the single record of Lady Riverside Belle materially influences all calculations. Whether the record might be an error or whether Lady Riverside Belle represents one of those extreme variants which are occasionally found, it seems unfair to use such an abnormal record in view of the general trend of the rest of the data. The data for the whole table have been calculated, however, and are given in table 99. The solid straight line represents the regression line based on these data. These constants are presented for their comparative value and will not be used further.

Dropping the record of Lady Riverside Belle, the constants shown in table 100 are derived from the data.

The correlation coefficient measuring the relation of the butter-fat percentages of half sisters with the dam as the common parent 0.221 ± 0.036, is less than the correlation coefficient measuring the

relation of the butter-fat percentages of the half sisters with the sire as the common parent, 0.374 ± 0.015. The difference, 0.374 − 0.221 = 0.153 ± 0.040, is 3.8 times the probable error and would consequently be considered as significant. However, despite the probability against random sampling as the cause of this difference in the results it is believed that random sampling is the actual cause of the difference. This conclusion is supported by a comparison of all the coefficients thus far deduced. These coefficients are found in table 101.

These studies on the correlation coefficients have favored a greater resemblance between the milk yields of the members of the different

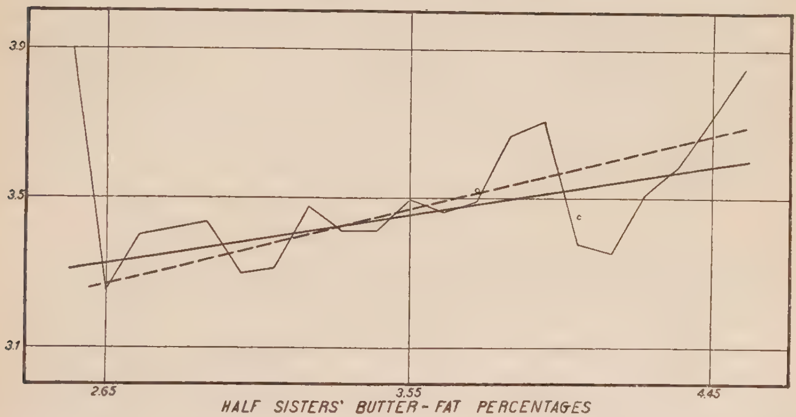

Fig. 20. Average Butter-fat Percentages of Second Half Sisters When the Butter-fat Percentage of the First Half Sister Is of a Given Grade

groups than between the butter-fat percentages of the cows of these different groups. All coefficients are strictly comparable save that for the butter-fat percentages of half sisters with common dam. Here, despite the fact that the dam appears from the other data to have equal influence with the sire, the coefficient is slightly lower than it would seem it should be. In view of this comparison, random sampling is indicated as the cause of this difference. All correlation coefficients agree in showing a fairly marked relation between the milk yields and butter-fat percentages of the different groups. It will be noted that the correlation coefficients for half sisters are

less than those for mother and daughter, and for full sisters, agreeing with the expectation on the basis of inheritance of milk yields and butter-fat percentages from both sire and dam.

Before turning to the data on the prediction of the butter-fat percentage of the second half sisters when the butter-fat percentage of the first half sister is known, it seems well to examine the regression lines. Figure 20 shows the raw and fitted regression lines. The trend of the majority of the data indicates that the regression line

TABLE 100

Variation and correlation of half sisters' butter-fat percentages omitting the record of Lady Riverside Belle

Mean butter-fat percentage	3.44±0.01
Standard deviation of butter-fat percentage	0.290±0.008
Coefficient of variation of butter-fat percentage	8.42±0.23
Correlation coefficient for the relation of half sisters' butter-fat percentages	0.221±0.036

TABLE 101

Correlation coefficients for the milk yields of the different groups in comparison with the correlation coefficients of the butter-fat percentages for these groups

GROUP	BUTTER-FAT PERCENTAGE	MILK YIELD
Half sisters, common sire	0.374±0.015	0.362±0.015
Half sisters, common dam	0.221±0.036	0.381±0.033
Full sisters	0.464±0.032	0.548±0.027
Mother and daughter	0.413±0.023	0.497±0.021

should be linear. Omitting Lady Riverside Belle the correlation ratio is found to be 0.317 ± 0.035. The correlation ratio squared minus the correlation coefficient squared is equal to 0.052 ± 0.018 or the difference is 2.9 times the probable error. Considering the irregularity of the regression line this agreement of correlation ratio and correlation coefficient is quite satisfactory. The dotted line represents the regression line found by dropping the abnormal record of Lady Riverside Belle. The observed average butter-fat percentages after this record is dropped are shown as small circles.

THE BUTTER-FAT PERCENTAGES OF THE SECOND HALF SISTERS WHEN THE BUTTER-FAT PERCENTAGE OF THE FIRST HALF SISTER IS KNOWN

The equation for the butter-fat percentages of the second half sisters when the butter-fat percentage of the first is known is equal to:

$$\text{B. F. per cent second half sister} = 2.684 + 0.221 \text{ b. f. per cent first half sister} \quad (43)$$

This equation is the one used in calculating the dotted line of figure

TABLE 102

Average butter-fat percentage of a dam's second daughters when the butter-fat percentage of the first daughter is known. Half sisters and full sisters

FIRST DAUGHTER'S BUTTER-FAT PERCENTAGE	AVERAGE BUTTER-FAT PERCENTAGE OF THE SECOND DAUGHTERS. HALF SISTERS	AVERAGE BUTTER-FAT PERCENTAGE OF THE SECOND DAUGHTERS. FULL SISTERS
2.6	3.26	3.04
2.7	3.28	3.09
2.8	3.30	3.13
2.9	3.33	3.18
3.0	3.35	3.22
3.1	3.37	3.27
3.2	3.39	3.32
3.3	3.41	3.36
3.4	3.44	3.41
3.5	3.46	3.46
3.6	3.48	3.50
3.7	3.50	3.55
3.8	3.52	3.60
3.9	3.55	3.64
4.0	3.57	3.69
4.1	3.59	3.73
4.2	3.61	3.78
4.3	3.63	3.83
4.4	3.66	3.87
4.5	3.68	3.92
4.6	3.70	3.97

20. From this equation, table 102 is obtained to show the probable butter-fat percentage of a half sister as determined from that of another. Similar information for full sisters is repeated because of its comparative value.

The average butter-fat percentage of the Holstein-Friesian Advanced Registry cattle is 3.428 per cent. Examination of the above table shows that the butter-fat percentages of the second daughters differ only slightly from the average of the breed as the first daughter's test increases. Thus when the first daughter's test is 2.6 per cent the second daughter's average test is 3.3 per cent. When the first daughter's test is 4.6 per cent the second daughter's average test is 3.7 per cent or the increase is 0.4 per cent. This increase in the test is low in amount. It seems probable from other results that it would be somewhat greater if it were possible to obtain 365-day records on the whole breed.

If the data for full sisters are contrasted with those for half sisters it is to be noted that the record of a first sister predicts the average of a second full sister much closer to its own record than it does the average of the half sister. To illustrate this point concretely, the first daughter's butter-fat percentage is assumed to be 2.6 per cent, the full sisters' average butter-fat percentage is 3 per cent whereas the half sisters' average butter-fat percentage is 3.26 or in other words the average of the full sister is 0.26 per cent of butter-fat closer to the first sister than is that of the half sister. This fact holds for the high butter-fat percentage as well as for the low butter-fat percentage. When the butter-fat percentage of the first sister is high, 4.5, the average butter-fat percentage of the full sister is 3.92 and for the half sister is 3.68 or the full sister is 0.24 per cent of butter-fat nearer the high testing daughter than is the half sister. The practical application is obvious. When the breeder finds a mating that produces one or more high butter-fat testing daughters let him hold to it and perpetuate it by repeating the mating as often as possible. On the other hand if the breeder finds his bull giving low testing daughters, the bull should be dropped unless there is some other good reason for keeping him. It is also a question if the cow producing daughters which are low testing should not also be dropped.

VARIATIONS IN THE BUTTER-FAT PERCENTAGES OF HALF SISTERS. THE BUTTER-FAT PERCENTAGES BETWEEN WHICH 50 PER CENT OF THE SECOND HALF SISTERS ARE FOUND WHEN THE BUTTER-FAT PERCENTAGE OF THE FIRST HALF SISTER IS KNOWN

The range of butter-fat percentage necessary to include 50 per cent of all second sisters when the grade of butter-fat percentage for

the first half sisters is known, is given in table 103. This range is determined from the data of table 100 by the following formula:

$$0.67449 \times 0.290\sqrt{1 - 0.221^2} = 0.191$$

From the table it will be seen that a range of about 0.4 per cent of butter-fat will include 50 per cent of the second half sisters. As noted earlier, the daughters of Artis Belle of Riverside 2D,

TABLE 103

The butter-fat percentage of the first daughter and the limits between which would be found the butter-fat percentages of 50 per cent of the half sisters

FIRST DAUGHTER'S BUTTER-FAT PERCENTAGE	RANGE OF BUTTER-FAT PERCENTAGE OF 50 PER CENT OF THE SECOND DAUGHTERS. HALF SISTERS
2.6	3.07–3.45
2.7	3.09–3.47
2.8	3.11–3.49
2.9	3.13–3.52
3.0	3.16–3.54
3.1	3.18–3.56
3.2	3.20–3.58
3.3	3.22–3.61
3.4	3.25–3.63
3.5	3.27–3.65
3.6	3.29–3.67
3.7	3.31–3.69
3.8	3.33–3.72
3.9	3.36–3.74
4.0	3.38–3.76
4.1	3.40–3.78
4.2	3.42–3.80
4.3	3.44–3.83
4.4	3.46–3.85
4.5	3.48–3.87
4.6	3.51–3.89

had butter-fat tests of 2.57, 3.7, and 4.1. Looking up a first daughter's test of 2.6 on table 103 we note that these other half sisters at 3.7 and 4.1 would be both outside and above the range which should include 50 per cent of the half sisters of a 2.6 per cent cow. The question comes up as to how much further out of the range are these tests 3.7 and 4.1. The data of the next table assist in answering this problem.

BUTTER-FAT PERCENTAGES BETWEEN WHICH 99 PER CENT OF THE SECOND HALF SISTERS ARE FOUND WHEN THE BUTTER-FAT PERCENTAGE OF THE FIRST HALF SISTER IS KNOWN

Two and fifty-eight hundredths times the standard deviation of the arrays gives the range on each side of the average test necessary to include 99 per cent of the daughters from any given dam. The data for this calculation is taken from table 100. The results are given in table 104.

TABLE 104

The butter-fat percentages of first daughters and the limits between which would be found the butter-fat percentages of 99 per cent of their half sisters

FIRST DAUGHTER'S BUTTER-FAT PERCENTAGE	RANGE OF BUTTER-FAT PERCENTAGE OF 99 PER CENT OF THE SECOND DAUGHTERS. HALF SISTERS.
2.6	2.53–3.99
2.7	2.55–4.01
2.8	2.57–4.03
2.9	2.60–4.06
3.0	2.62–4.08
3.1	2.64–4.10
3.2	2.66–4.12
3.3	2.68–4.14
3.4	2.71–4.17
3.5	2.73–4.19
3.6	2.75–4.21
3.7	2.77–4.23
3.8	2.79–4.25
3.9	2.82–4.28
4.0	2.84–4.30
4.1	2.86–4.32
4.2	2.88–4.34
4.3	2.91–4.37
4.4	2.93–4.39
4.5	2.95–4.41
4.6	2.97–4.43

Examination of table 104 shows that the range of butter-fat percentage necessary to include 99 per cent of the second half sisters is indeed large. If we examine this table for the first daughter of 2.6 per cent of butter-fat we note that a second daughter with a butter-fat test of 4.1 per cent is on the border line or the chance is 99 to 1 against such a wide deviation. The chance against getting such a

wide deviation as a 3.7 per cent half sister to another testing 2.6 per cent is about 10 to 1. Therefore for the two half sisters the chance against getting them when their other half sister tests 2.6 per cent is more than 900 to 1. Here again there is evidence leading to the belief that the case of Lady Riverside Belle is exceptional, justifying the action taken in omitting her record in the correlations.

THE DIRECT INFLUENCE OF THE DAM ON THE BUTTER-FAT PERCENTAGE OF HER DAUGHTERS

The correlation between the butter-fat percentages of daughter and dam may be determined from the standard deviations of the butter-fat percentages in the same manner as that used for the sires.

The standard deviations of the daughters' butter-fat percentages for all dams having three or more daughters are given in table 105.

From these data the average weighted squared standard deviation is found to be 0.04403. The standard deviation is 0.2098. The correction factor for this standard deviation (on account of the few individuals from which the individual standard deviations are determined) is 0.7376[1] or the corrected standard deviation is 0.2845. The standard deviation for the whole population is 0.3024. These data give the correlation coefficient as 0.34. This correlation coefficient is influenced by the extreme variation of the daughters of Artis Belle of Riverside 2D previously described. If the records for these daughters are excluded the correlation coefficient becomes 0.39. The correlation coefficient by the direct product moment method for the butter-fat percentages of mother and daughter is 0.413 ± 0.023. These results agree rather well. They show that in the Holstein-Friesian Advanced Registry cattle, the butter-fat percentages of the daughters depend on those of the dam.

The correlation coefficient for the butter-fat percentages of the daughter and sire was found to be 0.53. This correlation coefficient is probably not significantly larger than the correlation for the relation of the butter-fat percentages of mother and daughter, 0.413

[1] Editorial. 1915. On the distribution of the standard deviations of small samples. Appendix 1. To papers by "Student" and R. A. Fisher, Biometrika, vol. x, pp. 522–529.

Young, Andrew W. Note on the standard deviation of samples of two or three. Biometrika, vol. xi, pp. 277–280.

± 0.023, as calculated directly from the yields of the corrected correlation coefficient for mothers' and daughters' butter-fat percentages or as just calculated for the standard deviations of the arrays. Certain points may be noted concerning this difference. The cor-

TABLE 105

The standard deviations of the butter-fat percentages of the daughters for those dams which had 3 or more daughters

DAM'S NUMBER	STANDARD DEVIATION OF DAUGHTERS' BUTTER-FAT PERCENTAGES	DAM'S NUMBER	STANDARD DEVIATION OF DAUGHTERS' BUTTER-FAT PERCENTAGES
25,173	0.094	73,286	0.205
31,408	0.109	76,684	0.125
38,802	0.216	77,032	0.205
46,348	0.450	77,491	0.148
47,448	0.082	77,777	0.189
50,657	0.125	78,331	0.125
51,737	0.001	79,290	0.047
53,646	0.141	82,444	0.464
54,125	0.228	84,455	0.170
54,170	0.170	86,998	0.047
55,150	0.980	87,301	0.125
55,194	0.287	90,522	0.125
62,230	0.130	90,846	0.094
62,337	0.047	93,789	0.094
63,667	0.163	98,734	0.292
64,178	0.205	99,074	0.094
65,465	0.047	99,086	0.189
65,882	0.125	100,999	0.216
66,788	0.141	101,313	0.082
68,099	0.340	106,688	0.087
70,231	0.001	106,988	0.556
70,349	0.356	107,842	0.047
70,427	0.047	108,681	0.125
71,624	0.216	110,877	0.216
71,679	0.162	112,246	0.125
71,850	0.050	120,022	0.141
71,898	0.170	174,475	0.648

rected standard deviations of the arrays are practically the same 0.2845 to 0.2818. The difference lies in the standard deviations of the whole population. This standard deviation for the daughters of mothers is 0.3024 per cent of butter-fat and for the daughters of the sires is 0.3313 per cent of butter-fat. That is, the population of

daughters' butter-fat percentages was less scattered for the mothers than it was for the sires. It seems unlikely that this difference would be found to hold true for the whole population of Holstein-Friesian cows. On the basis of these facts it seems desirable to await other data, now being analyzed, before attempting to inter-

TABLE 106

Correlation surface showing the relation of the butter-fat percentages of half sisters. Common dam.

HALF SISTERS' BUTTER-FAT PERCENTAGES	HALF SISTERS' BUTTER-FAT PERCENTAGES																						
	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6–4.7	
2.5–2.6													1			1							2
2.6						1				1													2
2.7									1	1													2
2.8																							
2.9						3	2	1	2	1	1		1		2	1							14
3.0		1			3	2	6	6	4	5	1	1	2				1						32
3.1					2	6	4	5	7	5	3	3		1									36
3.2					1	6	5	6	9	14	9	6	3	1	1	3		2	1				67
3.3			1		2	4	7	9	16	14	9	8	5		1	2							78
3.4		1	1		1	5	5	14	14	12	14	10	5	1		1	1						85
3.5					1	1	3	9	9	14	8	1	1	3	4		1	1	1				57
3.6						1	3	6	8	10	1	2	6			1		1					39
3.7	1				1	2		3	5	5	1	6	2	1	2	2							31
3.8							1	1		1	3		1					1			1		9
3.9					2			1	1		4		2		8	1			1				20
4.0	1				1			3	2	1		1	2		1								12
4.1						1				1	1												3
4.2								2			1	1		1									5
4.3								1			1				1								3
4.4																							
4.5														1									1
4.6–4.7																							
	2	2	2		14	32	36	67	78	85	57	39	31	9	20	12	3	5	3		1		498

pret these possible differences between the sire's and dam's influence on the butter-fat percentage of their offspring. In any case the conclusion is justified from a comparison of these results that the sire and dam both play a large part in determining what the butter-fat percentage of their offspring will be.

SUMMARY

This paper presents a study of the butter-fat percentages of those Holstein-Friesian Advanced Registry cattle which are out of the same dam but by different sires.

These results show that the butter-fat percentages of the half sisters are dependent on each other to a certain degree. The correlation coefficient measuring the degree of this dependence is 0.221 $\pm$ 0.036. This correlation coefficient is the smallest correlation coefficient yet found for the relation of either milk yield or butter-fat percentage between sisters or between daughters and their parents.

The variation of the half sisters' milk yields is presented both as a whole and for the limits of 50 and 99 per cent of the population.

The correlation coefficient for the butter-fat percentages of mother and daughter is calculated from the daughters' butter-fat percentages. The resulting coefficient 0.39, furnishes further proof of the dependence of a cow's butter-fat percentage on that of her mother.

CHAPTER XVII

On the Relation between the Milk Yield of the Daughter and the Butter-fat Percentage of the Dam or between the Butter-fat Percentage of the Daughter and the Milk Yield of the Dam

The assertion is frequently made that a high milk yield means a low butter-fat percentage and a low milk yield a high butter-fat percentage. As shown earlier, there is little basis for this remark where the individual Holstein-Friesian cow is concerned, the correlation being so small that casual inspection would probably pass it by unnoticed. For some of the other breeds, particularly the Channel Island breeds, such a relation exists.[1] The question naturally arises as to whether there is any cross-association between daughter and dam, for these variables—milk yield of daughter and butter-fat percentage of dam, butter-fat percentage of daughter and milk yield of dam. Such an association would not be expected on inheritance grounds unless the linkage between the factors causing the increase or decrease of the two characters were very close, if not absolutely the same. If the linkage is as tight as this, the cross-correlations of daughter's and dam's records should be very large and negative in size. The succeeding tables show these correlations.

Another point arises in this connection—the influence of environment on the record of the individual. Data have been presented to show the large correlations which exist between the milk yields of daughter and dam and between the butter-fat percentage of daughter and dam. If these correlations are due to care, feeding, in other words, environment, it would be supposed that similar correlations would be found for the cross-correlations, that is, for the milk yields with the butter-fat percentages of daughter and dam. The absence of such correlations is a strong point in the argument that the environ-

[1] Gaines, W. L., and Davidson, F. A. 1923. Relation between percentage fat content and yield of milk. Illinois Agri. Exp. Station, Bul. 245, pp. 577–621.

ment has had relatively little effect on the results herein presented. Table 107 gives the correlation surface for the relation of the milk yields of the daughters with the butter-fat percentages of the dams.

The correlation surface for the relation of the butter-fat percentages of the daughters and the milk yields of their dams is given in table 108.

TABLE 107

Correlation surface showing the relation between the milk yields of the daughters and the butter-fat percentages of the dams

DAM'S BUTTER-FAT PERCENTAGE	DAUGHTER'S MILK YIELD																								
	10,000–11,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000–34,000	
2.6–2.7							1		2					1											4
2.7						1	2	1			1	2	1												8
2.8					1							1			2	1									5
2.9				1	1	1	1	2	1		3					1	1								12
3.0			1	2	1	4	2	3	7	6	7	3	5	2		1		1	1		1				47
3.1				2	3	5	4	5	6	8	4	4	3	6	1		1		1		1				54
3.2		3	2	4	4	6	6	9	3	3	7	1	3	1	7	4		1	2	1				1	68
3.3		1	4	4	6	8	14	9	10	10	9	6	5	4	2	3	1	1	2				1		100
3.4	1	1	2	6	4	10	6	10	5	9	5	3	3	2	4		3	2	4			1			81
3.5			1	1	3		4	5	9	7	7	7	5	8	3	1	2	2	1						66
3.6		1	1	2	2	3	2	6	2	3	4	7	4	4	1	3	1	2							48
3.7			1	2	3	2	2	7	1	4	4	5		4		1	3								39
3.8					2	1	3	1	2	3	1	2		3	1	1	1	2			1	1			25
3.9	2					1	1	3	2	6	1			1	3			1							21
4.0				1				1	2	1		1	1		1		1								9
4.1					1			2		2		1	2		1	1	1								11
4.2									1	1						2			1						5
4.3								1	1						1	1									4
4.4–4.5					2			1	1																4
	3	6	12	25	33	42	48	66	55	63	53	43	32	36	27	20	15	12	12	1	3	2	1	1	611

The means, standard deviations, and coefficients of variation for these cows are repeated from the previous tables on these data. The correlation coefficients are new. Table 109 presents these data.

Figure 21 shows the regression lines for table 107 and figure 22, the regression lines for table 108.

Table 109 and figures 21 and 22 make it clear that there is no relation between the milk yields of the daughters and the butter-fat percentages of their dams. The correlation coefficient is even less between the butter-fat percentages of daughters and the milk yields of the dams. Thus the popular notion that high milk yield spells low butter-fat percentage seems to have but slight foundation in so far as these relations may be transmitted from dam to daughter.

TABLE 108

Correlation surface showing the relation between the butter-fat percentages of the daughters and the milk yields of the dams

DAUGHTER'S BUTTER-FAT PERCENTAGE	DAM'S MILK YIELD																						
	10,000–11,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000–32,000	
2.6–2.7				1		1		1	1														4
2.7					1			1	1	1	1	1											6
2.8						1	1		1		1	1	1					1					7
2.9			1	1		2		2			3	1	2	2					1				15
3.0				1	3	5	1	7	5	7	3	3	5		1								41
3.1	1	1	1	2	7	6	8	3	2	7	3	1	5	5	2	1	1	1		1		1	59
3.2		1	2	6	3	7	3	8	8	7	3	4	11	4	2			2	1				72
3.3			3	10	6	13	4	4	13	9	7	5	6	5				1					86
3.4	1	2	4	5	9	12	4	4	9	4	10	4	5	3	1	3	5		2				87
3.5		1	1	4	4	3	5	7	8	6	8	6	4	1	2	5				1	1		67
3.6		1	1	4	5	6	5	4	8	3	4	3	1	5	3	1	2		2				58
3.7		1	1	1	2	2	2	9	5	3	5	4	2	2	2	1			1				43
3.8	1	1	1			3	1	1	1	2	1			1						1			14
3.9	1			1	1	1	2	2	2	2	1				1	1	2	1					18
4.0						2	1	4	1	4		1			1			1					15
4.1					1	1		1		1			1			1				1			7
4.2							2		1		1									1			5
4.3	1					2	1	1												1			6
4.4																							
4.5–4.6																1							1
	5	8	15	36	42	67	40	59	66	56	51	34	43	28	15	14	10	7	7	6	1	1	611

The lack of any correlation between the milk yields of the daughters and the butter-fat percentages of the dams and vice versa is important in interpreting any possible influence of environment on the

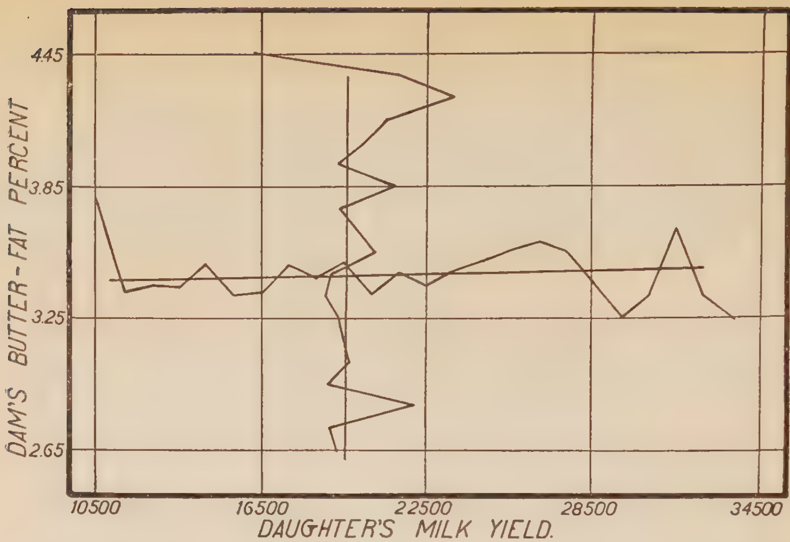

Fig. 21. Raw Regression Line (rough line) and Smoothed Regression Line (straight line) for the Relation of the Milk Yields of Daughters and the Butter-fat Percentages of Their Dams

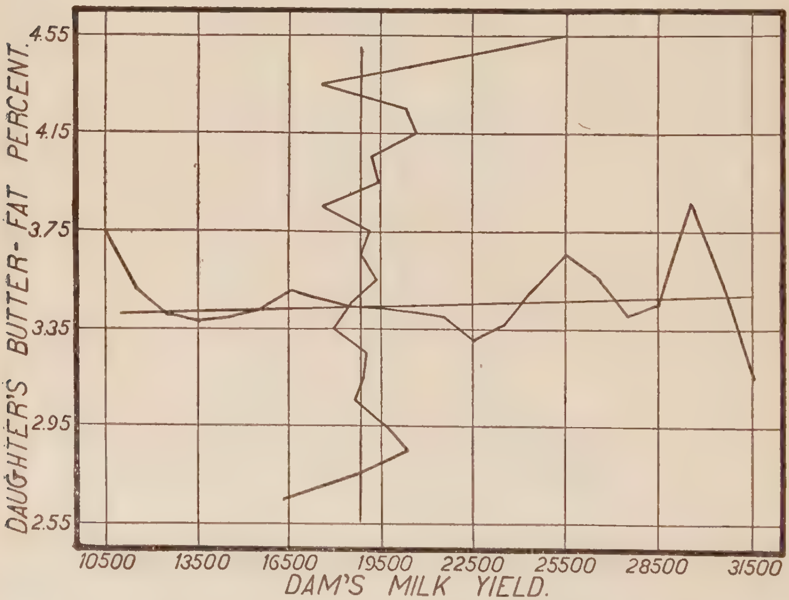

Fig. 22. Raw Regression Line and Straight Line Derived from Data of Table 109 for the Relation of the Butter-fat Percentages of the Daughters and the Milk Yields of Their Dams

relation of the milk yields of daughters and dams or the correlation of the butter-fat percentages of daughters and dams. These correlation coefficients were both rather large, 0.4 and 0.5. If this correlation was largely accounted for by heterogeneity in care and feeding, some pairs having larger milk yields because of better care than is given other pairs, then similar relations would be expected to exist for the cross-correlations. Since these cross-correlations are practically zero it appears a justifiable conclusion that the environment

TABLE 109

Constants for the variation and cross-correlation coefficients for the milk yields and butter-fat percentages of the daughters and their dams

PHYSICAL CONSTANTS	MILK YIELD	BUTTER-FAT PERCENTAGE
Daughter		
Mean	19,604±113	3.44±0.01
Standard deviation	4,132±80	0.311±0.01
Coefficient of variation	21.1±0.4	9.0±0.2
Dam		
Mean	18,803±109	3.45±0.01
Standard deviation	3,986±77	0.318±0.01
Coefficient of variation	21.2±0.4	9.2±0.2
Correlation coefficients		
Daughter's milk yield and dam's butter-fat percentage		0.070±0.027
Daughter's butter-fat percentage and dam's milk yield		0.034±0.027

has had a relatively insignificant influence on the correlation of the records of daughter and dam. In fact heredity appears to be the large element in the control of milk yield and butter-fat percentage.

SUMMARY

This chapter treats of the relations between the milk yields of the daughters and butter-fat percentages of dams or between the butter-fat percentages of the daughters and the milk yields of the dams. The correlation coefficients of these cross-relations are practically zero. In view of this fact, the popular fancy that high milk yield

means low butter-fat percentage finds no substantiation for the relations between daughters and their dams. The lack of any correlation further tends to support the view that heredity is the large element in the productivity of the cow, for if environment were a large factor we should expect at least a significant correlation for these cross-relations.

CHAPTER XVIII

Assortive Mating in the Holstein-Friesian Breed

Before the time of most of the present day breeders of dairy cattle, the principle of selection was put forth as a guide to be followed by the successful stock man. This principle applied to the milch cow is more complicated than when it is applied to beef cattle because of the fact that an actual test of the milking qualities can be made only for the female offspring. This complication greatly handicaps the dairyman. Selection of a sire before he has offspring may be based on his dam's record, on his sisters' records, or if it is so desired on the records of ancestors further back. In this section all the data presented will relate to the sire's dam's record. The chapter does not pretend to develop any new principle. It is merely taking stock of an old one, of the progress made, and of the possibilities of accomplishment. Some information on this point is desirable for at least two reasons. The first is that if assortive mating is of frequent occurrence in our breeds of dairy cattle it will seriously influence the constants presented in the preceding chapter. Again it is frequently asserted that certain of the more careful breeders have been practicing selection in their herd. To one investigating this problem the question naturally comes up as to whether or not this statement is true.

THE SELECTION PRACTICED FOR MILK YIELD

If the selection is operative it must do several things. The sire's dam's record should be higher than the rest of the breed. To be effective for the individual breeder the record of the sire's dam would be larger than the record of the cows to which the sire is bred. Study of the Holstein-Friesian 365-day records shows that the average record of the sire's dams for the 124 cases available was 20,024 pounds age-corrected milk. The average of the records for the cows to which the sire was bred was 19,395 pounds of age-corrected milk, a difference of 629 pounds of milk in favor of the sire's dams.

This difference is in the right direction, but is too small in amount to accomplish the results desired. There is practically no difference in the butter-fat percentage of the sire's dam and the cow to which he is bred. The averages are 3.48 and 3.46 respectively. These differences are not so large as they would be expected to be. The data for this comparison are taken from table 110.

The correlation surfaces from which these constants are derived are given in tables 111 and 112.

Table 110 shows that the mean milk yields of paternal granddams and dams do not differ significantly from each other. The standard deviations of the paternal granddams' and dams' milk

TABLE 110

Average milk yields and constants of variation for the paternal granddams' and the dams' milk productions and butter-fat percentages

	DAMS	PATERNAL GRANDDAMS
Mean milk yield	19,395±256	20,024±257
Standard deviation of milk yield	4,223±181	4,236±181
Coefficient of variation of milk yield	21.8±1.0	21.2±1.0
Correlation coefficient between the milk yields of dam and paternal granddam		0.142±0.059
Mean butter-fat percentage	3.48±0.02	3.46±0.02
Standard deviation of butter-fat percentage	0.350±0.015	0.320±0.014
Coefficient of variation of butter-fat percentage	10.1±0.4	9.2±0.4
Correlation coefficient between the butter-fat percentages of dam and paternal granddam		0.001±0.061

yields are practically the same. The coefficients of variation are also practically the same. The correlation coefficient between the milk yields of the paternal granddams and the dams indicates a a slight selection. Thus, while it is true that some selection is going on, this selection is not so great as first thought would suggest. It seems probable that if it were more severe it would be for the best interests of the breed. The severity of the selection may best be shown by the following example. A sire is purchased for breeding to certain cows. The sire's dam has a record. If the selection is severe it would be expected that the better the cow's records the better the sire's dam's record ought to be; for it does not seem as if any breeder would want to breed a 25,000-pound

cow to a bull whose dam had a record of but 15,000-pounds. The average production of the paternal granddam for each grade of milk production for the dam may be obtained from the regression equation as derived from table 110. This equation is:

$$\text{Sire's dam's milk yield} = 17264 + 0.142 \text{ dam's milk yield} \qquad (44)$$

Figure 23 gives the comparison of the raw means with the average milk yield as derived from this equation. The average milk records

TABLE 111

Correlation surface for the relation of the milk yields of the paternal granddam and the dam

PATERNAL GRANDDAM'S MILK YIELD (POUNDS)	DAM'S MILK YIELD (POUNDS)																				
	10,000–11,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000–30,000	
12,000–13,000		1																			1
13,000													1								1
14,000						3			3	1	2		2								11
15,000					1						1		1								3
16,000				2	1	1	3				6	1	1		3	3	1			1	23
17,000				1				2		1		1	2			1					8
18,000		4			1					2		2		1							10
19,000				1	1			1	2				3			1					9
20,000				1	2	2	1	1	3	3	2			1	1						17
21,000	1			1	1		1		3			2			1						10
22,000						2		1			1			2							6
23,000			2			2	1							1							6
24,000																					
25,000												1						1			2
26,000																1					1
27,000						1			2				1	1			1			1	7
28,000						1			1		1				2						5
29,000												1	1	1							3
30,000–31,000															1						1
	1	5	2	6	7	12	6	5	14	7	13	8	12	7	8	6	2	1		2	124

for the sire's dam and the cows to which the sire is bred are shown in table 113.

Table 113 shows a small difference to exist between the milk yield of a sire's dam when he is bred to 10,000-pound cows and the

milk yield of a sire's dam when he is bred to 30,000-pound cows. On this basis the sires bred to high milking cows are slightly better than those bred to low milking cows. The difference is not very marked. Thus the dams of sires bred to 10,000-pound cows had an averge production of 18,687 pounds of milk whereas the dams of sires bred to 30,000-pound cows had an average production of 21,533 pounds, or the difference in favor of the sire's dams bred

TABLE 112

Correlation surface for the relation of the butter-fat percentages of paternal granddam and dam

GRANDDAM'S BUTTER-FAT PERCENTAGE (PATERNAL)	DAM'S BUTTER-FAT PERCENTAGE																		
	2.7–2.8	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4–4.5	
2.7–2.8					1		2												3
2.8																			
2.9							1		1	1	2		1						6
3.0					1	3	1			1						2	1		9
3.1	2		1		1	3		1	2	1									11
3.2					1		3	2	1	1	2								10
3.3		1					2	1	1	3	3				1				12
3.4	2				1	1	1	1	1	1	1	1					1	1	12
3.5	1			6	1	1		3	3	1		1	1						18
3.6					1	2	1	5	1		2	1	1		3				17
3.7		2			1	1	1		1	1		1							8
3.8					2			1	1	5									9
3.9							2	1			2								5
4.0							1		1										2
4.1																			
4.2																			
4.3								1											1
4.4–4.5									1										1
	5	3	1	6	10	11	15	16	14	15	12	4	3		4	2	2	1	124

to the 30,000-pound cows is only 2846 pounds of milk. If there were no selection practiced within the breed the averages of the dams of the two sets of sires would be the same. Since the differences are so nearly the same the selection of sires is not severe.

This point is illustrated in figure 23. The raw mean milk yields of the paternal granddams for each grade of milk yield of the dams are irregular, covering quite a large range. There is a slight upward

trend to these average yields. This is shown by the line derived from the equation to these raw average yields of the paternal granddams.

The variation in production of the sire's dams is also large in amount. Thus the range of milk yield necessary to include 50

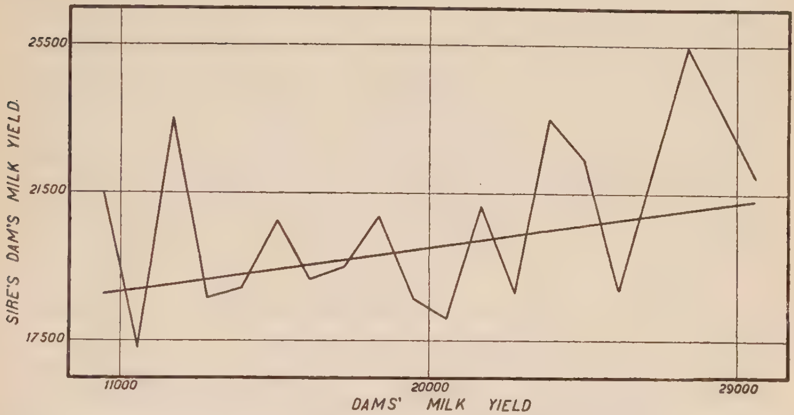

Fig. 23. Average Milk Yield of Paternal Granddam for Each Grade of Milk Yield of the Dam

TABLE 113

Milk records of the sire's dam and the milk records of the cows to which the sire is bred

DAM'S MILK YIELD	GRANDDAM'S (SIRE'S SIDE) MILK YIELD
10,000	18,687
14,000	19,256
18,000	19,825
22,000	20,395
26,000	20,964
30,000	21,533

per cent of the sires' dams for those sires bred to cows of 10,000 pounds production is from 15,859 to 21,515 pounds, or the range is over 5,000 pounds. A similar range exists for the dams of the sires bred to cows of higher milk yield. The range of production necessary to cover the majority of the sires' dams' milk yields for a given grade of dam to which the sire is bred is large indeed. In

fact, the range is large enough to cover practically the whole of the normal range of milk yield as it is found in Holstein-Friesian Advanced Registry cows. These facts substantiate the conclusion that, while some selection of the sire on the basis of his dam's record is going on in the breed, this selection is not at all severe. From the results obtained in a study of the milk records of the daughters of such sires it is clear that the sire's grandmother plays her part in causing the granddaughter's milk yield to be high or low. For the breeder who has cows milking say 14,000 pounds it is fine for him to have a sire whose dam is in the 25,000-pound class. On the other hand for the breeder who has 22,000-pound cows, it is not so desirable to breed them to a sire whose mother's production is 14,000 pounds, if one is able to judge by the milk yields of the daughters.

THE SELECTION PRACTICED FOR BUTTER-FAT PERCENTAGE

The data in table 110 show that there is little or no selection practiced for sires which have dams of high or low butter-fat percentage. That is, the average butter-fat percentage of the sire's dam is practically the same in average amount no matter what this sire's mate may have for her butter-fat percentage. Table 114 shows the averages of the butter-fat percentages for the sire's dams for each grade of butter-fat percentage for his mate. These averages are derived from a regression equation as determined from the data contained in table 110.

The differences between the sire's dam's average butter-fat percentage are so slight as to be negligible even for the extreme grades of butter-fat percentage of the cows to which the sire was bred. There has consequently been practically no selection of the sire's dams for their butter-fat percentage, the sire's dam having the same average butter-fat percentage whether the sire was bred to cows whose butter-fat percentage was 2.6, 4.6 or anywhere within this range. Of course the sires' dams vary in butter-fat percentages. This variation is about the same for all groups. To include 50 per cent of the sires' dams a range from 3.3 to 3.7 per cent of butter-fat is necessary. The range necessary to include 99 per cent of the cows covers that for practically the whole breed, that is, 2.6 to 4.5.

These results give concrete data on what is happening in the breeding operations of one breed. The results are believed to be based on all cows which had 365-day records up to volume 31 of the Advanced Register. They are essentially the same as the results from a similar study of the Jersey breed. Taking the breeders as a whole there is concrete evidence to support the contention that a selection of sires on the basis of their dams' record is actually taking place. The data further shows that a man who owns high milking cows selects his sire on the basis of a higher record for the sire's dam than does the man who has low milking cows. There is room for improvement in both of these directions. The sire's dam's production is indicative of the granddaughter's milk yield. Consequently, the higher this production, the better the chance that the granddaughter will herself produce higher than the average of the breed.

TABLE 114

Butter-fat percentage of the sire's dam and the butter-fat percentage of the cows to which the sire is bred

DAM'S BUTTER-FAT PERCENTAGE	AVERAGE GRANDDAM'S (SIRE'S SIDE) BUTTER-FAT PERCENTAGE
2.6	3.5
3.0	3.5
3.4	3.5
3.8	3.5
4.2	3.5
4.6	3.5

There is no evidence in the data for any selection of sires on the basis of their dams' butter-fat percentage. Butter-fat percentage is transmitted just as truly by the grandmother to her granddaughter as is milk yield from the grandmother to her granddaughter. There appears to be opportunity for further progress here.

THE RELATION BETWEEN THE MILK YIELDS OF THE PATERNAL AND MATERNAL GRANDDAMS

Another measure of assortive mating is the relation which exists between the milk yields of the two granddams—paternal and maternal. If assortive mating is present the sires which have parents of high milk yield should be bred to cows which have mothers of high milk yield and vice versa. Data on this point are surprisingly

scarce when it is limited to those cows which are in the Advanced Registry and which have their two granddams with Advanced Registry records. Tables 115 and 116 show the correlation surfaces for milk yields and butter-fat percentages of the paternal and maternal granddams.

Table 117 shows the constants derived from tables 115 and 116.

TABLE 115

Correlation surface for the milk yields of the paternal and maternal granddams in the Holstein-Friesian Advanced Registry

PATERNAL GRANDDAM'S MILK YIELD	MATERNAL GRANDDAM'S MILK YIELD																				
	10,000–11,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000–30,000	
12,000–13,000			1	1			1														3
13,000														1							1
14,000		1						1				1									3
15,000								1						1							2
16,000						3					1	1				2	1				8
17,000																					
18,000		1							1	1				1				1			5
19,000		1						1	1												3
20,000			1			2			2	3	1										9
21,000										2	2										4
22,000										1											1
23,000	1																				1
24,000																					
25,000																		1			1
26,000																					
27,000							1		2				1								4
28,000							1			1			1							2	5
29,000–30,000																			1		1
	1	3	2	1		5	3	3	6	8	4	2	2	3		2	1	2	1	2	51

The average milk yields, standard deviations of milk yield, and coefficients of variation of the paternal and maternal granddams do not differ significantly from each other. It will be noted that the milk yield of the paternal granddam is about 800 pounds more than that for the maternal granddam. When the number of bulls needed is very much less than the number of cows needed, it would

TABLE 116

Correlation surface for the butter-fat percentages of the paternal and maternal granddams in the Holstein-Friesian Advanced Registry

PATERNAL GRANDDAM'S BUTTER-FAT PERCENTAGE	MATERNAL GRANDDAM'S BUTTER-FAT PERCENTAGE													
	2.7–2.8	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9–4.0	
2.7–2.8							2							2
2.8														
2.9													1	1
3.0						2	1		1			1		5
3.1									1					1
3.2				1	1	3								5
3.3	1					1	1		1		1			5
3.4				1		1	3		1		1			7
3.5			1	2		1	2	1	1	1				9
3.6					1	1	1	2		1		1		7
3.7					1	1			1					3
3.8				2										2
3.9													1	1
4.0										1				1
4.1														
4.2														
4.3							1							1
4.4–4.5										1				1
	1		1	6	3	10	11	3	6	4	2	2	2	51

TABLE 117

Averages, constants of variation, and correlation coefficients for paternal and maternal granddams' milk yields and butter-fat percentages

	PATERNAL GRANDDAM	MATERNAL GRANDDAM
Mean milk yield	20,245±455	19,422±446
Standard deviation of milk yield	4,822±322	4,727±316
Coefficient of variation of milk yield	23.8±1.7	24.3±1.7
Mean butter-fat percentage	3.48±0.03	3.38±0.02
Standard deviation of butter-fat percentage	0.335±0.022	0.259±0.017
Coefficient of variation of butter-fat percentage	9.6±0.6	7.7±0.5

Correlation coefficient for the relation of the paternal and maternal granddams' milk yields .... 0.296±0.086

Correlation coefficient for the relation of the paternal and maternal granddams' butter-fat percentages .... 0.040±0.094

seem as though the bulls with much higher producing paternal granddams would be selected more frequently. As it is now, scarcely any selection in this regard is in progress for the 365-day Advanced Registry records. The correlation coefficient for the sires' dams' milk yields with the maternal dams' milk yields is significant. This indicates that there is a tendency to breed the cows with good producing mothers to the bulls with good producing mothers. The data are too few to make this more than a tentative conclusion.

The butter-fat percentages of the paternal granddams are slightly, although not significantly, more than the butter-fat percentages of the maternal granddams. The standard deviation and the coefficient of variation of the paternal granddams are more than the standard deviation and the coefficient of variation of the maternal granddams. These differences are large but because of their large probable errors are not significant. The correlation coefficient for the butter-fat percentages of the paternal and maternal granddams is small and not significant. Thus there is no tendency to mate cows with the high butter-fat testing dams to bulls with high butter-fat testing dams. In other words, there is no assortive mating for butter-fat percentage.

These results substantiate the others presented for the sire's dam and the cow to which the sire was bred. There is assortive mating for milk yield to a slight degree but there is no assortive mating for butter-fat percentage. In terms of selection there is some slight attention paid to the milk yield of a sire's dam in comparison with the milk yield of the cows to which he is going to be bred and to the milk yields of the dams of the cows to which he is going to be bred. This selection is far from stringent. There is no selection for butter-fat percentage.

SUMMARY

The evidence herein presented shows that there is a slight degree of assortive mating for milk yield and no assortive mating for butter-fat percentage in the Holstein-Friesian breed. The specific evidence is:

1. The milk yields of the paternal granddams are somewhat above those of the cows to which the sire is bred.

2. There is a small correlation coefficient between the milk yields of the sires' dams and the cows to which the sires are bred. This

correlation is plus indicating that the sires with dams of slightly better milk yield are bred to cows which have slightly better milk yields.

3. The butter-fat percentages of the sires' dams are slightly less on the average than the butter-fat percentages of the cows to which they are bred.

4. There is no correlation between the butter-fat percentages of the sires' dams and the butter-fat percentages of the cows to which these sires are bred.

5. A small correlation exists between the milk yields of the sire's dam and the milk yields of the dam of the cows to which he is bred.

6. No correlation exists between the butter-fat percentages of the sire's dam and the butter-fat percentage of the dam of the cow to which he is bred.

CHAPTER XIX

THE RELATIVE INFLUENCE OF ENVIRONMENT AND HEREDITY ON THE MILK YIELDS AND BUTTER-FAT PERCENTAGES

Milk production is a quantitative character. It varies from cow to cow and from lactation to lactation even for a single cow. But even with this variation, milk production curves are of rather uniform type, definite range, and shape. For any one individual the range of milk production is limited within fairly narrow bounds. To those who are breeding dairy cows or those who are interested in the solution of dairy breeding problems the question naturally arises, "What limits the production of any one cow?" In last analysis the position of any one cow in this variation curve depends on two variables: environment, in its effect on growth, development, and testing of dairy cows; and heredity, in that at the beginning of the cow's existence she receives from her sire and dam the elements for milk production. Heredity, then, is the basic element which can be changed at the will of the breeder only by changing the parents and producing another cow. The environment on the other hand is a thing by which the breeder can materially influence milk production.

In this paper an attempt will be made to measure the relative influence of these factors on milk yield. The environment is that of the Advanced Registry test. While such an environment is quite variable it is probably not quite so variable as would be found by taking the herds throughout the country as a whole.

The problem may be considered from this angle. The cows composing the Advanced Registry are scattered over the country. They are grouped into a herd here and a herd there. These cows differ in production. These herds may differ in production. Thus a herd in Massachusetts may differ from one in Maine in its milk yield. For the sake of comparison the Maine herd's average milk yield may be 20,000 pounds and that for Massachusetts may be 16,000 pounds. The difference of 4000 pounds may be due to better feeding, to better care, to better location, in fact to any or all

of the environmental factors influencing milk yield; or it may be due to better sires or dams which gave birth to the animals composing the herd, in short to heredity. It is possible to differentiate such herds. When they are so differentiated the relative part played by heredity and environment in the milk yields may be determined as follows: The combined influence of these two variables may be measured by correlating all the individuals within the herd for all the herds and without regard to pedigree. In this way a measure is obtained of the place-influence on milk yield. If, now, this place-influence is due to environment it ought to influence all animals alike. The average milk yield of the unrelated cows ought to be a satisfactory measure of the influence of feeding and other variable elements of the environment, for the successful feeder ought to be able to raise the milk yields of unrelated cows as readily as those which are related. If, then, the three possible combinations of correlations are obtained, for example the correlation of the milk yields of each full sister, and the correlation of the milk yields of each full sister with the average herd production of the unrelated cows, we are in a position to determine the relative influence of the three variables. For the correlation of the full sisters' milk yields measures the combined influence of environment and heredity, and the two other correlations measure the effect of the feeding and care in raising one herd's average above that of another. The influence of heredity may then be determined by the partial correlation coefficient for the milk yields of full sisters for a constant milk yield for the herds from which they came. It is obvious that in considering any particular pair of relatives, we are by no means considering the influence of the whole heredity, but only such influence as the common parents can transmit. On the other hand the whole environmental influence which tends to jointly influence the milk yields of the two relatives is examined under environment.

THE DIFFERENTIATION EXISTING FOR HERD MILK YIELD

There are 74 herds with more than ten yearly record cows. The average milk yield of these cows is 19,519 pounds. The standard deviation for milk yield is 4069 pounds. The average butter-fat percentage is 3.44 per cent and the standard deviation is 0.310 per cent. These data are closely similar to those of the Advanced Registry taken as a whole. The average milk yields of these herds range

from 15,033 to 25,800 pounds of milk. The range of butter-fat percentage is from 3.15 to 3.95 per cent. If the herds are differentiated either because of environment or heredity or both we should find a correlation between the milk yields and butter-fat percentages of the cows composing these herds. To illustrate, if a cow comes from the herd of a very careful feeder and the cow's milk yield is large because of this feeder's skill, then other cows coming from this herd should have their milk yields raised because of their feeder's skill. In other words, the milk yields of the cows composing such a herd should be correlated with the management of the herd. In the same way, if a herd's average milk yield is due to the influence of one bull, and if the bull's heredity is such that he causes his daughters to have a high milk yield, then the cows' milk yields in this herd should be correlated because of this heredity. If both of these combinations are at work on the milk yield, the final yields will be determined by this joint reaction. The correlation ratio for the milk yields and herds is 0.603 ± 0.011 and for the butter-fat percentage is 0.490 ± 0.013. The herds of the country are consequently differentiated as to yield. There is a high degree of relationship between the herds and the milk yields or butter-fat percentages of the individual cows in them. Figures 24 and 25 show the average milk yields and butter-fat percentages of the different herds arranged in ascending order. Table 118 gives the means, standard deviations, and correlations noted above.

Table 118 shows that a large correlation exists between the milk yields or butter-fat percentages of cows and the herds from which they come. As noted elsewhere this correlation may be due to environment, or to heredity, the environment within a herd tending to be more constant than between herds and the cows composing the herd frequently having much in the way of common heredity due to the use of a single bull and the same dams. The problem now becomes one of differentiating the influence of these two variables. This problem may be approached through the following channel, making the assumption that all animals without regard to heredity are equally responsive to environment. I determined the mean milk yields and butter-fat percentages of those cows within a herd which are neither full nor half sisters. These averages should measure quite accurately the influence of environment on the milk yield or butter-fat percentage, for, as has been shown earlier, the grand-

parents have only a relatively small effect on the productivity of their granddaughters, and the generations further removed have even less effect. With this information the partial correlation

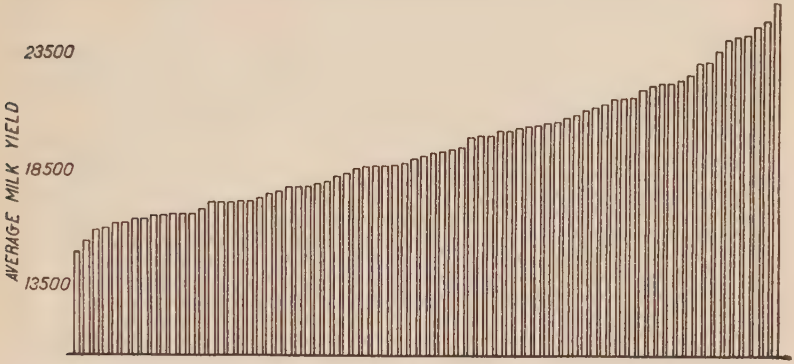

FIG. 24. AVERAGE ADVANCED REGISTRY MILK YIELDS OF THE 74 HERDS CONTAINING 10 OR MORE COWS

The records are arranged according to yield

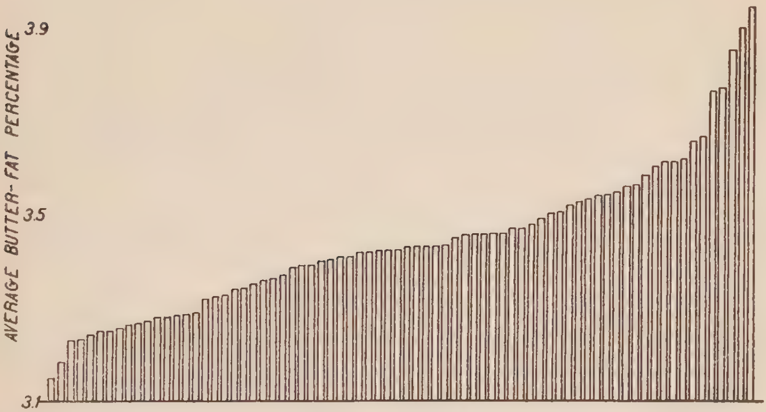

FIG. 25. AVERAGE ADVANCED REGISTRY BUTTER-FAT PERCENTAGE OF THE 74 HERDS CONTAINING 10 OR MORE COWS

The records are arranged according to the butter-fat percentage

method offers a means of solving the problem before us. If we correlate the three variables, milk yield of one sister with the milk yield of another, the milk yield of the full sister with the average

milk yield of the unrelated cows in the herd from which they came, and the milk yield of the other full sister with the milk yield of the unrelated cows[1] in the herd from which she came, we are then in a position to determine the relation of the milk yields of full sisters for a constant milk yield for the herds from which they came (constant environment as measured by its effect on milk yield). This coefficient gives a measure of the heredity, divorced from environment, in determining the milk yield of the cow. The correlation of one sister's milk yield with the average milk yield of the unrelated cows of the herd, should be the same as the correlation of the other sister's milk yield with the average milk yield of the unrelated cows of the herd. In obtaining this coefficient only the one calculation

TABLE 118

Means and standard deviations of milk yields and butter-fat percentages of 74 herds containing 10 or more Advanced Registry cows. Correlation ratios showing the relation between the herds and the milk yields and butter-fat percentages of the cows coming from them

CHARACTERS CORRELATED	MEAN	STANDARD DEVIATION	RANGE OF AVERAGES FOR THE HERDS	CORRELATION RATIO FOR HERDS AND PRODUCTIVE CAPACITY OF COWS CONTAINED THEREIN
Milk yield 74 herds..	19,519	4,069	15,033–25,800	0.603 ± 0.011
Butter-fat percentage 74 herds............	3.44	0.310	3.15–3.95	0.490 ± 0.013

is made. Table 119 gives the correlation surface for the full sisters' milk yields and the average milk yield of the unrelated cows in the herd.

The correlation surface for the unrelated cows' average butter-fat percentage and the butter-fat percentages of the full sisters is given in table 120.

Examination of table 119 shows that the herd's average for milk yield is less for the cows which are neither full nor half sisters, than for the average of all the animals in the herds. The number of full sisters which may be used to determine their relations to herd's average production are also limited because of the fact that many

[1] Unrelated cows are throughout used to designate the cows in the herd which are neither full nor half sisters.

TABLE 119

Correlation surface for the milk yields of full sisters and the herd's average milk yield for the unrelated cows (neither full nor half sisters)

HERD'S AVERAGE MILK YIELD FOR UNRELATED COWS	MILK YIELDS OF FULL SISTERS																								
	11,000–12,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000	34,000–35,000	
13,500–14,500									2	1															3
14,500					1	4		3	2		1	1													12
15,500			2		1	1						1				1									6
16,500	1	2	1	4	7	5	5	5	7	3	2	5	3	2	2	1		2			1			1	59
17,500		1	2		3	4	1	2	2	8	3	4	4		2		1	1							38
18,500	1	1	2	7	2	2	4	3	4	3	4	2	2	1	2										40
19,500						3	4	4	3	3	1	4	4	3	1										30
20,500–21,500																		1							1
	2	4	7	11	14	19	14	17	20	18	11	17	13	6	7	2	1	4			1			1	189

TABLE 120

Correlation surface for the butter-fat percentages of full sisters and the herd's average butter-fat percentage for the unrelated cows (neither full nor half sisters)

HERD'S AVERAGE BUTTER-FAT PERCENTAGE FOR UNRELATED COWS	BUTTER-FAT PERCENTAGE OF FULL SISTERS																		
	2.6–2.7	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3–4.4	
2.95–3.05	1			1			1			1									4
3.05			1				2	1	5		2	1							12
3.15					2	1	3	1	2	6									15
3.25			1		4	5	5	6	2	1	3	1		1		1			30
3.35					1	1	2	4	6	5	5	6	1	1					32
3.45		2		2	2	2	4	2	2	1				1					18
3.55		1	1	2	3	2	4	7	2	6	3	1	1		1				34
3.65			1					2	2		1			1	1				8
3.75								3	1	3									7
3.85				1	3						1	1		2	1	1			10
3.95						2			1										3
4.05								1	3	1	3	3		1				1	13
4.15								1	1										2
4.25–4.35														1					1
	1	3	4	6	15	13	21	28	27	24	18	13	2	8	3	2		1	189

of the herds have tested only cows related as either full or half sisters. The constants for the two tables are given in table 121.

The average milk yield of the herd's averages is rather low and the butter-fat percentage is slightly high. The correlation coefficient for the milk yield is small and compared with its probable error not significant. The milk yields of the full sisters are but slightly influenced by the herds from which they came. The butter-fat percentages of the full sisters are more markedly affected. The partial correlation for the milk yields of full sisters for a constant herd's average milk yield is:

$$\frac{0.548 - 0.092^2}{\sqrt{1 - 0.092^2}\sqrt{1 - 0.092^2}} \text{ or } 0.544 \pm 0.035^*$$

TABLE 121

Means and standard deviations of full sisters' milk yields and butter-fat percentages and of the herds from which they came. Correlation coefficients for full sisters and herd's average performance in milk yield and butter-fat percentage

CHARACTERS	MEAN	STANDARD DEVIATION	CORRELATION COEFFICIENT
Milk yield:			
Full sisters	19,505	3,983	
Herd's average	17,915	1,467	0.092±0.049
Butter-fat percentage:			
Full sisters	3.42	0.299	
Herd's average	3.51	0.288	0.224±0.047

The partial correlation coefficient for the butter-fat percentages of full sisters is:

$$\frac{0.464 - 0.224^2}{\sqrt{1 - 0.224^2}\sqrt{1 - 0.224^2}} \text{ or } 0.436 \pm 0.040$$

These results show that when the influence of environment on milk yield is made constant there still remains a substantial correlation between the milk yields and butter-fat percentages of full sisters. The correlation coefficient should consequently be very nearly that due to heredity. In other words heredity plays by far the larger part in determining milk yield and butter-fat percentage for these full sisters.

* The probable errors are based on the smallest number of individuals in any table for the group.

THE RELATIVE INFLUENCE OF ENVIRONMENT AND HEREDITY ON THE MILK YIELD AND BUTTER-FAT PERCENTAGE OF HALF SISTERS. THE SIRE, THE COMMON PARENT

The problem of the relative influence of feeding, care, and other environmental factors on the milk yields of half sisters may be approached in the same manner as that used for the full sisters in the preceding section. The correlation table showing the relation of the milk yields of half sisters, common sire, and the average milk yields of the unrelated cows, neither full nor half sisters, in the herds from which they came is given in table 122.

The correlation surface showing the relation of the butter-fat percentages of half sisters and the average butter-fat percentages of the unrelated cows in the herds from which they came is given in table 123.

The constants derived from tables 122 and 123 are given in table 124.

From the data of table 124 we may obtain the relation of the milk yields and butter-fat percentages of half sisters under a constant environment if we consider that the milk yields and butter-fat percentages of the unrelated cows in the herd offer a sufficient criterion of the influence of this environment on the milk yields of all the cows in the herd. Such an assumption appears to be entirely reasonable. The correlation between the milk yields of half sisters is 0.362. The correlation coefficient between the butter-fat percentages of half sisters is 0.374. The correlation coefficient for the milk yields of half sisters under a constant environment is:

$$\frac{0.362 - 0.247^2}{1 - 0.247^2} \text{ or } 0.319 \pm 0.018$$

The correlation coefficient for the butter-fat percentages of half sisters for the same constant environment in so far as it influences butter-fat percentage is:

$$\frac{0.374 - 0.138^2}{1 - 0.138^2} \text{ or } 0.362 \pm 0.017$$

These results are in accord with those for the full sisters. They show clearly that if these full or half sisters are placed in the same herd, the differentiation in their milk yields and butter-fat percentages would be marked. The variable remaining to cause this

TABLE 122

Correlation surface showing the relation of the milk yields of half sisters, common sire, and the average milk yield of the herd (unrelated cows) from which they came

HERD'S AVERAGE MILK YIELD FOR UNRELATED COWS	MILK YIELDS OF HALF SISTERS, COMMON SIRE																										
	9,000–10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000	32,000	33,000	34,000	35,000–36,000	
13,500–14,500				1					1	1	2				1													6
14,500		2	2		8	7	10	2	9	7	5	5		2	1	1												61
15,500		1				2	2		2	4	6		3	3		1												24
16,500	1	1	5	10	21	20	35	28	27	26	27	22	16	11	10	9	2	6	8	2		3	2					292
17,500			3	3	9	21	29	30	37	26	40	29	16	24	17	6	8	5	6	9	3	1						322
18,500			1	5	3	13	11	18	20	20	22	16	9	12	7	11	5	4	5	2	2	3					1	190
19,500				1	5	4	16	9	18	13	21	18	14	8	18	3	7	1	4	1	2	1	1	2	1			168
20,500					1	2	4	7	6	2	5	5	1	2	3	3		2		2	1							46
21,500									1	1		2	2	2	3	1	4	6	3		1							26
22,500									2	2	1	5	1	2			1	2			1		1					18
23,500																												
24,500–25,500								1	1																			2
	1	4	11	20	47	69	107	95	124	102	129	102	62	66	60	35	27	26	26	16	10	8	4	2	1		1	1,155

TABLE 123

Correlation surface showing the relation of the butter-fat percentages of half sisters, common sire, and the average butter-fat percentage of the unrelated cows in the herd from which they came

HERD'S AVERAGE BUTTER-FAT PERCENTAGE FOR UNRELATED COWS	BUTTER-FAT PERCENTAGE OF HALF SISTERS, SIRE'S SIDE																					
	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5–4.6	
2.85–2.95								2		2												4
2.95			1			3	6	10	5	9	4	5	1		1	1	1					47
3.05		1			3	1	3	4	7	11	5	3	2	1	1	2				1		45
3.15					1	2	5	8	8	9	6	2	3	2		1	1					48
3.25		1	1	1	12	14	15	24	28	22	23	11	9	6	2	5	3	1	1			179
3.35			1	1	3	13	19	17	22	20	19	13	10	5	10	1	2		1		1	158
3.45		2	4	8	8	14	19	20	32	28	25	12	11	6	3	3	3	1				199
3.55	1		1	2	9	18	16	28	31	24	26	21	12	11	7	2		2	4	1		216
3.65				1	2	5	5	5	13	8	13	10	6	3		2	4	4				81
3.75	1		1		1	2	1	4	6	6	3	3	1	1	1		1	1				33
3.85			1		5	2	8	7	9	10	16	8	6	7	2	4	1	2	1			89
3.95					1	2	1		3	5	1	4		1			1					19
4.05							1	4		4	2	4	2	1	2	1	2	2				25
4.15						1					2		1	1								5
4.25–4.35										1	1		1		1	2			1			7
	2	4	10	13	45	77	99	133	164	159	146	96	65	45	30	24	19	13	8	2	1	1, 155

TABLE 124

Means and standard deviations of half sisters' milk yields and butter-fat percentages; means and standard deviations of the averages of the herds from which these half sisters came; and the correlation coefficients for half sisters and the herds' average performances in milk yield and butter-fat percentage

	MEAN	STANDARD DEVIATION	CORRELATION COEFFICIENT
Milk yield:			
Half sisters..........................	19, 381	4, 160	
Herds' averages....................	18, 281	1, 660	0.247 ± 0.019
Butter-fat percentage:			
Half sisters..........................	3.44	0.315	
Herds' averages....................	3.51	0.258	0.138 ± 0.019

differentiation is the inheritance for milk yield or butter-fat percentage received from the sire and dam for the full sisters and the sire for the half sisters.

THE RELATIVE INFLUENCE OF ENVIRONMENT AND HEREDITY ON THE MILK YIELDS AND BUTTER-FAT PERCENTAGES OF HALF SISTERS. THE DAM, THE COMMON PARENT

The correlation surfaces for the relation of the milk yields and butter-fat percentages of half sisters (same dam) to the average milk yield and butter-fat percentage of the unrelated cows in the herds from which they came are given in tables 125 and 126.

TABLE 125

Correlation surface showing the relation of the milk yields of half sisters to the average milk yield of the unrelated cows in the herd from which they came

HERD'S AVERAGE MILK YIELD FOR UNRELATED COWS	MILK YIELD OF HALF SISTERS, COMMON DAM																						
	9,000–10,000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000–31,000	
13,500–14,500			1		1		2	2	2	1					1								10
14,500						1	3	2	1	1	2			1									11
15,500		1				1			1							1							4
16,500			2		3		5	2	5	6	3	1		2									29
17,500			1	2	3	2		2	7	6	3	6	3	3	2	2	2			1			45
18,500								2	3	1	2	1	2	1			1		1				14
19,500	1			1	2	4	3	3	2	2	3	2	3	5	1	4	1			1		1	39
20,500										2	3		2	1	1		1			2			12
21,500									2	1	1	1		1		1							7
22,500										1	1		3						1				6
23,500								1					1			2							4
24,500										2		1	1			1	1						6
25,500																							
26,500–27,500														1					1	1			3
	1	1	4	3	9	8	13	14	23	23	18	12	15	15	5	11	6		3	5		1	190

The average milk yields and butter-fat percentages, the standard deviations, and the coefficients of correlation derived from tables 125 and 126 are given in table 127.

The data of table 127 furnish the necessary information to derive the relation of the milk yields and butter-fat percentages of half sisters for a constant milk yield or butter-fat percentage in the herds from which they came—the effect of the environment on the herd's performance being measured by the milk yields or butter-fat percentages

of the unrelated cows in the herd. The correlation coefficients of 0.402 for the milk yield and 0.233 for the butter-fat percentage show that the environment influences the performance of these cattle. The major difference between herds in all probability is brought

TABLE 126

Correlation surface showing the relation of the butter-fat percentages of half sisters to the average butter-fat percentage of the unrelated cows in the herd from which they came

HERD'S AVERAGE BUTTER-FAT PERCENTAGE FOR UNRELATED COWS	BUTTER-FAT PERCENTAGE OF HALF SISTERS, COMMON DAM																		
	2.6–2.7	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3–4.4	
3.05–3.15	1				1	3	3	5	3	1	3								20
3.15												1							1
3.25		1		3	3	1	6	9	8	2	4	2		1	1			1	42
3.35				2	3	3	7	6	11	6	3	4	3	3	4		1		56
3.45					2	6	3	4	6	4	1							1	27
3.55					3	1	4	6	4	4	4	1	1	2	1				31
3.65							1	1		2		1		1		1	1		8
3.75–3.85										1	1			2	1				5
	1	1		5	12	14	24	31	32	20	16	9	4	9	7	1	2	2	190

TABLE 127

Means and standard deviations of half sisters' milk yields and butter-fat percentages; means and standard deviations of the averages of the herds from which these half sisters came; and the correlation coefficients for the relation of the half sisters' milk yields and butter-fat percentages with these herd-averages

	MEAN	STANDARD DEVIATION	CORRELATION COEFFICIENT
Milk yield:			
Half sisters	19,232	4,000	
Herds' averages	18,889	2,690	0.402 ± 0.035
Butter-fat percentage:			
Half sisters	3.46	0.301	
Herds' averages	3.41	0.166	0.233 ± 0.043

about by the feeding and management of the cows both before and during the test. To get the relation between the milk yields of half sisters for a constant environment it is necessary to know the raw correlations between the milk yields of these half sisters in the heterogeneous environment of the many herds in the Advanced

Registry. The correlation between the milk yields of half sisters with a common dam is 0.381. The correlation between the butter-fat percentages of half sisters in the same environment is 0.221. The partial correlation for the milk yields of half sisters for a constant herd's yield is:

$$\frac{0.381 - 0.402^2}{1 - 0.402^2} \text{ or } 0.262 \pm 0.046$$

The partial correlation coefficient for the butter-fat percentage of half sisters for a constant herd's butter-fat percentage is:

$$\frac{0.221 - 0.233^2}{1 - 0.233^2} \text{ or } 0.176 \pm 0.047$$

The correlation for the relation of these half sisters with the herd's average for the unrelated cows is quite a good deal more for both the milk yield and butter-fat percentage than the correlation coefficients for these same variables using the data for the half sisters on the sire's side or for the full sisters. Because of these large values the correlation coefficients for the milk yields or the butter-fat percentages of half sisters for a constant herd's average are reduced to values less than those for the other groups of full or half sisters (common sire). The difference between the milk yields for the half sisters on the sire's and dam's sides of the pedigree is not significant. The difference for the butter-fat percentages of the half sisters is greater, being 3.7 times the probable error and in customary parlance would be considered significant. There is a significant relation between the milk yields of half sisters with a common dam, when the milk yield of the herd is made constant. Significant relation exists for the butter-fat percentages of these half sisters when the butter-fat percentages of the herds are made constant. In fact the correlation coefficients are not significantly lowered by making the performance of the herds constant. Heredity and not the herd management would consequently be the major influence in the cows' production.

THE RELATIVE INFLUENCE OF ENVIRONMENT AND RHEEDITY ON THE MILK YIELDS AND BUTTER-FAT PERCENTAGES OF MOTHERS AND DAUGHTERS

To determine the relative influence of environment and heredity on the milk yields and butter-fat percentages of the daughters and

their dams, we obtain as before the relation of the milk yields and butter-fat percentages of these daughters and dams to the average milk yields and butter-fat percentages of the unrelated cows in the herds from which they came. This correlation measures roughly the effect of the environment, since it shows to what extent the environment raised or lowered the milk yields or butter-fat percentages of all the cows in the herd. To obtain the influence of heredity it is necessary to resort to the partial correlations of the second order. By having the necessary correlations of the milk yields and butter-fat percentages of daughters and dams, of daughters and the daughters' average herd's unrelated cows' performance, of daughters and the dams' average herd's unrelated cows' performance, of dams and the daughters' average herd's unrelated cows' performance, of dams and the dams' average herd's unrelated cows' performance, the daughters' herd's average unrelated cows' performance and the dams' herd's average unrelated cows' performance, it is possible to obtain the relation of the daughter's and dam's records for milk yield and butter-fat percentage for constant environment, at least in so far as these variables are affected by the feeding, management, etc. The ultimate results measure roughly, at least, the influence of heredity separated from the other extraneous variables. The constants of table 128 furnish the necessary information.

From these zero order correlation coefficients the correlation between the daughter's and dam's performances for a constant daughter's and dam's herd's average performance is obtained by the usual partial correlation formulae. This method of procedure is justified since all regression lines are practically linear and the correlation surfaces resemble the normal surface sufficiently closely for practical purposes. These partial correlation coefficients for the milk yields and butter-fat percentages of mother and daughter are:

Milk yield	0.394 ± 0.038
Butter-fat percentage	0.402 ± 0.036

The correlation coefficients show that in the end result, a cow's milk yield or butter-fat percentage is controlled to a very large degree by the inheritance she receives from her dam. This inheritance makes itself manifest even under the diverse environments of many different herds.

It is helpful to review briefly the data thus far presented. The influence of the herd's average performance (broadly speaking environment) should be completely accounted for by the correlation coefficient. The influence of heredity is only accounted for in so far as the particular relation is able to do it. The correlation coefficients between the cow's production and the average of the unrelated cows varies widely, 0.092 to 0.453 for milk yield and 0.066 to 0.243 for butter-fat percentage. Actually the mode of procedure to obtain some measure of the effect of the herd's average on the actual milk yield of the cow is undoubtedly subject to a large probable error. The most reasonable measure of this influence appears to be the average of the correlation coefficients, 0.3 for milk yield and 0.2 for butter-fat percentage. These correlation coefficients affect the zero order correlations of the respective relations to only a small degree, in general less than 3 times the probable error of

TABLE 128

Means and standard deviations for the milk yields and butter-fat percentages of daughters, their dams, the daughters' herd's average of unrelated cows, and the dams' herd's average of the unrelated cows. Correlation coefficients measuring the relationship for all possible combinations of these variables

	MILK YIELD		BUTTER-FAT PERCENTAGE	
	Mean	Standard deviation	Mean	Standard deviation
Daughters' performance .... (1)	19,718	4,016	3.46	0.316
Dams' performance .... (2)	18,603	3,905	3.46	0.315
Daughters' herd's average unrelated cows' performance .... (3)	18,846	2,636	3.41	0.156
Dams' herd's average unrelated cows' performance .... (4)	18,748	2,625	3.41	0.157

Correlation coefficients

	MILK YIELD	BUTTER-FAT PERCENTAGE
Daughters and dams .... (12)	0.497±0.021	0.413±0.023
Daughters and herd's average .... (13)	0.379±0.038	0.066±0.044
Daughters and dams' herd's average ... (14)	0.388±0.037	0.095±0.044
Dams and daughters' herd's average ... (23)	0.417±0.036	0.184±0.043
Dams and dams' herd's average .... (24)	0.453±0.035	0.243±0.041
Daughters' and dams' herd's average ... (34)	0.973±0.002	0.891±0.009

the difference. In view of this relatively small influence, the conclusion for all combinations appears to be that the milk yield or butter-fat percentage of the cow is largely controlled by the inheritance she receives, although at the same time the cow receiving the better care produces somewhat more milk even for the Advanced Registry cows.

PREDICTION OF MILK YIELD AND BUTTER-FAT PERCENTAGE FROM THAT OF THE RELATIVES AND HERD'S AVERAGE PERFORMANCE

From the data presented it is possible to form equations for the prediction of the milk yields and butter-fat percentages of a cow when the productivity of her relatives and the performance of the herd from which she came are known. These equations are given below. The milk yield or butter-fat percentage of one full sister may be called A, for the second full sister B and the average milk yield of the unrelated cows in the herds from whence they came C. The equations for the cow's probable milk or butter-fat percentage as determined from the performance of her full sister and the unrelated animals in the herd are:

$$\text{Milk yield } A = 0.54 \text{ milk yield } B + 0.14 \text{ milk yield } C + 6463 \tag{45}$$

$$\text{Butter-fat percentage } A = 1.42 + 0.44 \text{ butter-fat percentage } B + 0.14 \text{ butter-fat percentage } C \tag{46}$$

For half sisters with a common sire the equations are:

$$\text{Milk yield } A = 0.32 \text{ milk yield } B + 0.44 \text{ milk yield } C + 5178 \tag{47}$$

$$\text{Butter-fat percentage } A = 1.79 + 0.36 \text{ butter-fat percentage } B + 0.12 \text{ butter-fat percentage } C \tag{48}$$

Where A = milk yield or butter-fat percentage of half sister.
B = milk yield or butter-fat percentage of second half sister.
C = average milk yield or butter-fat percentage of unrelated cows in herd from which A or B came.

Similarly the equations for the half sisters with a common dam are:

$$\text{Milk yield } A = 5935 + 0.26 \text{ milk yield } B + 0.44 \text{ milk yield } C \tag{49}$$

$$\text{Butter-fat percentage } A = 1.70 + 0.18 \text{ butter-fat percentage } B + 0.33 \text{ butter-fat percentage } C \tag{50}$$

the letters having the same significance as those for half sisters with a common sire.

The equations for a daughter's probable milk yield or butter-fat percentage from that of her dam and the average of herd's unrelated cows from which each of them came are:

$$\text{Milk yield } A = 5912 + 0.42 \text{ milk yield } B + 0.32 \text{ milk yield } C - 0.01 \text{ milk yield } D \tag{51}$$

$$\text{Butter-fat percentage } A = 2.08 + 0.42 \text{ butter-fat percentage } B - 0.05 \text{ butter-fat percentage } C + 0.03 \text{ butter-fat percentage } D \tag{52}$$

The significance of the letters is:

A = the probable milk yield or butter-fat percentage of daughter.

B = milk yield or butter-fat percentage of the dam.

C = average milk yield or butter-fat percentage of the daughter's herd's unrelated cows.

D = average milk yield or butter-fat percentage of the dam's herd's unrelated cows.

By calculating these equations for various degrees of performance some idea may be gained of the amount of influence exerted by the different variables. If the lowest and the highest ranges of production are chosen the following points would be used; for the different relatives 10,000 pounds as the low point and 30,000 pounds as the high point in milk yield, 2.6 as the low point and 4.4 for the high point for the butter-fat percentage; for the herd's average production 14,000 as the low average and 23,000 as the high point in milk production, and 3.2 for the low point and 3.8 for the high point of the butter-fat percentage. When the necessary calculations are made for the equation for milk yield it is noted that the herd's average production when increased from the lowest to the highest causes an average increase in the cow's production of 3000 pounds and an average increase of the butter-fat percentage of 0.08 per cent. As would be expected the effect of the relatives varies with the kind, the influence ranging from 5200 pounds to 10,800 pounds for milk yield and from 0.32 to 0.76 of a per cent for the butter-fat percentage. The average effect of the near relative is consequently considerably more than the influence of the herd management for cows which receive the care which is normally given to Advanced Registry cows. As only the partial effect of heredity is measured by the near relatives and as the total influence of environment is taken into consideration it would appear to be a fair conclusion that while the environment has an appreciable influence on the milk

yield or butter-fat percentage of even cows as well kept as those of the Advanced Registry this influence is relatively little compared with that of heredity.

The total correlation between the milk yield or butter-fat percentage and the performance of the relatives and the performance of the herds from which they come for these two items may be derived as follows. The total correlation of a series of variables with a single variable is equal to:

$$\sqrt{1 - \frac{S.D._{1.23^2 \ldots \, x}}{S.D._1}} = R_{1 \cdot 23 \ldots \, x}$$

where $R_{1 \cdot 23 \ldots x}$ is the total correlation coefficient and the S.D. equals the respective standard deviations. The values of $R_{1 \cdot 23 \ldots x}$ for full sisters, half sisters, daughters and dams, and the herd's average performances are given in table 129.

TABLE 129

Total correlation coefficients for the milk yields and butter-fat percentages of full sisters, half sisters, and daughters for constant milk yields and butter-fat percentages of the herd's average performances for the unrelated cows and for the full sisters, half sisters, or dams

CHARACTERS CORRELATED	TOTAL CORRELATION	
	Milk	Butter-fat percentage
Full sisters	0.549	0.482
Half sisters (common sire)	0.396	0.383
Half sisters (common dam)	0.468	0.285
Daughters (dam's performance known)	0.532	0.413

The correlation coefficients of table 129 are of fair size indicating a significant influence of the two variables studied on the milk yield or butter-fat percentage of the cow. The application of these facts to practice may be made by the use of the equation given above. A little of the theoretical side may be discussed for by its discussion some light is thrown on the why of the above facts.

For the obvious reason that only cows give milk, the discussion and analysis has been limited to a comparison of the female part of the parents and the first generation offspring. The measurement of the force of heredity is prescribed by the equivalent of the chromosomes passed along by one parent or in half sisters by the equivalent

of half the chromosomes passed along by one parent. By following Wodsedalek's work the following facts may be derived. If there are 37 chromosomes in the male and 38 chromosomes in the female of cattle, then, on the average a mother and daughter would have 19 common chromosomes. In the case of two half sisters, the dam being common, there would be $9\frac{1}{2}$ common chromosomes. Half sisters, with a common sire, would have 10 common chromosomes, the sex chromosome going to every female unchanged. For full sisters, there would be 10 common chromosomes from the sire and $9\frac{1}{2}$ common chromosomes from the dam, so that there would be $19\frac{1}{2}$ common chromosomes between them. The full sisters would consequently tend to resemble each other in milk yield or butter-fat percentage more than daughters would resemble their dams, and half sisters with a common sire would tend to resemble each other more than half sisters with a common dam. These suppositions based on the chromosome behavior as already noted are borne out by the data presented above.

The next problem which may be considered is the problem of the amount of control exerted on the milk yield or butter-fat percentage of a cow by the combined action of both parents. In an earlier section it was shown that the relationship between the daughter and sire for milk yield is that measured by a correlation coefficient of 0.52 and the same relation for butter-fat percentage has a correlation coefficient of about 0.53. These coefficients are slightly higher than those for the daughter and dam. In view of the evidence just presented it seems probable that the relationship of sire and daughter due to heredity is at least that measured by a correlation coefficient of 0.45. The facts on assortive mating just presented indicate that the correlation between the milk yields of paternal granddam and dam would have relatively little influence on the combined effect of parental heredity in determining the milk yield of a cow. The correlation between the somatic performance of the paternal granddam and the dam for milk yield and butter-fat percentage was 0.142 and 0.201 respectively. Were it possible to determine the somatic performance of the sire it would follow from this low correlation and the hazards of choosing a bull on the basis of such low correlations, that the correlation between such a somatic performance of the bull and the performance of the cow to which he was mated would be much less than this figure. To be sure that we are not favoring the

hypothesis of heredity these correlations may be assumed to be those between the sire and dam. As already noted the correlations between sire and daughter are at least 0.45 for both milk yield and butter-fat percentage. The correlations between daughter and dam are 0.39 for milk and 0.40 for butter-fat percentage. With these data and the methods indicated above the combined correlation between both parents and their offspring may be determined. These total correlation coefficients are 0.59 and 0.58 respectively for correlation of the two parents' milk yields or butter-fat percentages with the milk yields or butter-fat percentages of their daughters.

In another place it was shown that the yearly milk yields and butter-fat percentages of Holstein-Friesian cows in successive lactations had the following average correlation coefficients with each other, 0.66 for milk yield and 0.72 for butter-fat percentage. The consistency of the milk yields or butter-fat percentages of a cow from one lactation to another is a measure of the influence of two variables, the heredity of the cow for milk yield and butter-fat percentage, and those elements of the environment which are constant and make for consistency in the performance from lactation to lactation. It will be noted that the correlations of the parents' with the offspring's production are only a little less than are the correlations of the cow's performances from lactation to lactation. From this, heredity seems to be the large element in governing the permanence of the performance of dairy cattle.

SUMMARY

Evidence is herein presented to show the relative influence of environment and heredity on the milk yields and butter-fat percentages of dairy cattle. The problem was approached on the basis that the milk yields and butter-fat percentages of the unrelated cows furnish a measure of the influence of the environment on the variables for all cows in the herds. Only a partial influence of heredity could be measured, depending on what combination of relatives was used. The relatives compared were full sisters, half sisters with common sire, half sisters with common dam, and daughters and dam.

The analysis of these results shows that both environment and heredity play a part in the consistency of the milk yield of near relatives. The closer the relative, genetically, the greater the inheritance transmitted and the greater the resemblance in their

performance. The proportionate effect of the measured heredity versus the environment depends on the nearness of the relatives. Where both parents were considered, the correlation of their performances with those of their daughters due to heredity was nearly 0.6 for both milk yield and butter-fat percentage. Where only the dam was considered the correlations were practically 0.4 for both milk yield and butter-fat percentage. For full sisters the correlations were 0.54 for milk yield and 0.44 for butter-fat percentage. For half sisters the correlations were 0.32, sire common parent, and 0.26 dam common parent, for milk yield and 0.36 and 0.16 for butter-fat percentage.

The environment was found to control about 4 per cent of the variation in butter-fat percentage.

These facts point to the conclusion that heredity plays the largest part in the permanence of milk yield or butter-fat percentage in any cow.

CHAPTER XX

THE INHERITANCE OF MILK YIELD AND BUTTER-FAT PERCENTAGE FROM PATERNAL GRANDSIRE TO GRANDDAUGHTER

Inheritance of milk yield or butter-fat percentage from paternal grandsire to granddaughter can be demonstrated only indirectly since it is impossible to measure the performance of the sire. The means which will be used to demonstrate the inheritance consist of: first, showing that there is a relation between the milk yields and butter-fat percentages of the granddaughters; and second, the indirect method of measuring the relation between the milk yields and butter-fat percentages of paternal grandsire and granddaughters by the comparison of the standard deviations of the total population with those for the single sires. The basis of this method is, as already noted, the fact that the grandsire must of necessity fall in only one array of the correlation table since, if we had an actual milk record on this sire, this record could fall in only one column of the table. With this fact and the known relations of the standard deviation of the arrays to the standard deviation of the whole table it is possible to calculate the correlation coefficient. In actual practice this value tends to be slightly higher than the product moment value.

THE CORRELATION OF THE MILK YIELDS AND BUTTER-FAT PERCENTAGES OF GRANDDAUGHTERS HAVING A COMMON PATERNAL GRANDSIRE

The physical constants for the granddaughters are given in table 130.

The average milk yields of these paternal granddaughters are quite a little in excess of those for the Holstein-Friesian cows taken as a whole. The standard deviation is approximately like that of the whole breed. The coefficient of variation is slightly less than that for the whole breed. The correlation between the milk yields of these granddaughters is small but distinctly significant. From a genetic standpoint the correlation is rather less than expected.

The average butter-fat percentage and the standard deviation of it, for granddaughters is rather more than might be expected on the basis of all the Holstein-Friesian records. The correlation coefficient for the relation of the granddaughters' butter-fat percentages is distinctly significant and is probably larger than that for the milk yield.

A better appreciation of what these correlation coefficients mean may be gained by forming the prediction equations for the milk yield or butter-fat percentage of one granddaughter when the milk yield of another granddaughter is known. The equation for the milk yields of the granddaughters is:

Milk yield paternal granddaughter = 18517 + 0.070 milk yield of first paternal granddaughter (53)

TABLE 130

Means, standard deviations, coefficients of variation, and correlation coefficients for the paternal granddaughters' milk yields and butter-fat percentages

PHYSICAL CONSTANTS	MILK YIELD	BUTTER-FAT PERCENTAGE
Mean	19,911±59	3.458±0.005
Standard deviation	4,133±42	0.327±0.003
Coefficient of variation	20.8±0.2	9.47±0.09
Correlation coefficient	0.070±0.014	0.176±0.014

The butter-fat percentage of a second paternal granddaughter when the butter-fat percentage of the first is known, is:

Butter-fat percentage second paternal granddaughter = 2.85 + 0.176 butter-fat percentage of first paternal granddaughter (54)

From these equations, tables may be formed to show the probable milk yields or butter-fat percentages of the second granddaughters when the milk yields or butter-fat percentages of the first granddaughters are known. This information is given in tables 131 and 132. From the data of table 130, the amount of variation in the performance of this granddaughter may be calculated. Two limiting ranges of this variation are chosen to illustrate the breadth of this variability; the range which will include 50 per cent of the granddaughters and the range that will include 99 per cent of the granddaughters when the performance of the first granddaughter is known. Table 131 gives this information for the milk yields and table 132 for the butter-fat percentages.

Table 131 shows that as the milk yield of the first granddaughter increases, the average milk yields of the second granddaughters also increase. This increase of the second granddaughter is slight even though the first may increase a great deal. Thus, when the first granddaughter has a milk yield of 10,000 pounds the second has an

TABLE 131

Average milk yields of second granddaughters (paternal grandsire) when the performance of the first granddaughter is known. Range in milk yield necessary to include 50 per cent and 99 per cent of the second granddaughters when the performance of the first granddaughter is known

MILK YIELDS OF FIRST GRANDDAUGHTER	AVERAGE MILK YIELDS OF SECOND GRANDDAUGHTERS	RANGE OF MILK YIELD NECESSARY TO INCLUDE	
		50 per cent of second granddaughters	99 per cent of second granddaughters
10, 000	19, 217	16, 436–21, 998	8, 580–29, 854
11, 000	19, 287	16, 506–22, 068	8, 650–29, 924
12, 000	19, 357	16, 576–22, 138	8, 720–29, 994
13, 000	19, 427	16, 646–22, 208	8, 790–30, 064
14, 000	19, 497	16, 716–22, 278	8, 860–30, 134
15, 000	19, 567	16, 786–22, 348	8, 930-30, 204
16, 000	19, 637	16, 856-22, 418	9, 000–30, 274
17, 000	19, 707	16, 926–22, 488	9,070–30, 344
18, 000	19, 777	16, 996–22, 558	9, 140–30, 414
19, 000	19, 847	17, 066–22, 628	9, 210–30, 484
20, 000	19, 917	17, 136–22, 698	9, 280–30, 554
21, 000	19, 987	17, 206–22, 768	9, 350–30, 624
22, 000	20, 057	17, 276–22, 838	9, 420–30, 694
23, 000	20, 127	17, 346–22, 908	9, 490–30, 764
24, 000	20, 197	17, 416–22, 978	9, 560–30, 834
25, 000	20, 267	17, 486–23, 048	9, 630–30, 904
26, 000	20, 337	17, 556–23, 118	9, 700–30, 974
27, 000	20, 407	17, 626–23, 188	9, 770–31, 044
28, 000	20, 477	17, 696–23, 258	9, 840–31, 114
29, 000	20, 547	17, 766–23, 328	9, 910–31, 184
30, 000	20, 617	17, 836–23, 398	9, 980–31, 254

average milk yield of 19,217; and when the first granddaughter has a milk yield of 30,000 pounds, the second has an average milk yield of 20,617. Or for an increase in milk yield for the first granddaughter of 20,000 pounds there was an increase in milk of only 1400 pounds for the second granddaughters. In other words, the knowledge that one granddaughter had a milk yield of 30,000 pounds would only

tell us that another granddaughter's milk yield would on the average be 1400 pounds more than that of the second granddaughter of a sire whose first granddaughter had a milk yield of 10,000 pounds. Clearly even as to average milk yields the record of a granddaughter has relatively little value in predicting the milk yield of another.

Furthermore, the variation of the milk yields of these granddaughters is very large. Column 3 gives the ranges necessary to include 50 out of every 100 granddaughters and column 4 gives the range between which should be found 99 out of every hundred granddaughters when the milk yields of one of them are those shown in column 1. These ranges are large, indicating a relatively low predictive value. From these facts it may be concluded that there is relatively little advantage in knowing the milk yields of one granddaughter if we are trying to estimate the probable production of another granddaughter. The case is even worse for cousins. The correlation coefficient for the milk yields of cousins is only 0.005 where the mean milk yield is 19,850 pounds and the standard deviation is 4210 pounds. All results are consistent in showing that the animals which are valuable for the indication of the probable milk yield of a cow must be near relatives—in fact close relatives.

The correlation for the butter-fat percentages of the granddaughters are a little higher than those for the milk yields. These higher correlation coefficients lead to a greater predictive value for the butter-fat percentages of granddaughters. The predictive value is small even for this case, however. Table 132 gives the information for the butter-fat percentages of granddaughters of the paternal sire.

As would be expected from the correlation coefficient, a record of a paternal grandsire's granddaughter for butter-fat percentage has some predictive value in indicating that of another granddaughter. For these data this value is slightly more than it is for the milk yields of these individuals. Thus the average butter-fat percentages of granddaughters when a first granddaughter has a butter-fat percentage of 2.6, is 3.31. The granddaughters with first granddaughters of 4.5 per cent have an average butter-fat percentage of 3.64 or an increase of 0.33 per cent. While this increase is small it is significant. The ranges of butter-fat percentage necessary to include 50 per cent and 99 per cent of the second granddaughters are quite large, showing that it is not possible to predict a given grand-

daughter's production with much accuracy even though the production of a fairly large group of them will have the average butter-fat percentages indicated above. Cousins are of even less value in predicting performance. The correlation coefficient for the cousin's butter-fat percentage is only 0.119, where the mean butter-fat percentage is 3.45 and the standard deviation is 0.326. This relatively

TABLE 132

Average butter-fat percentages of the second granddaughters (paternal grandsire) when the performance of the first granddaughter is known. Range in butter-fat percentage necessary to include 50 per cent and 99 per cent of the second granddaughters when the performance of the first granddaughter is known

BUTTER-FAT PERCENTAGE OF FIRST GRANDDAUGHTER	AVERAGE BUTTER-FAT PERCENTAGE OF SECOND GRANDDAUGHTERS	RANGE OF BUTTER-FAT PERCENTAGE NECESSARY TO INCLUDE	
		50 per cent of second granddaughters	99 per cent of second granddaughters
2.6	3.31	3.1–3.5	2.5–4.1
2.7	3.32	3.1–3.5	2.5–4.2
2.8	3.34	3.1–3.6	2.5–4.2
2.9	3.36	3.1–3.6	2.5–4.2
3.0	3.38	3.2–3.6	2.6–4.2
3.1	3.39	3.2–3.6	2.6–4.2
3.2	3.41	3.2–3.6	2.6–4.2
3.3	3.43	3.2–3.7	2.6–4.3
3.4	3.45	3.2–3.7	2.6–4.3
3.5	3.47	3.3–3.7	2.6–4.3
3.6	3.48	3.3–3.7	2.7–4.3
3.7	3.50	3.3–3.7	2.7–4.3
3.8	3.52	3.3–3.7	2.7–4.4
3.9	3.54	3.3–3.8	2.7–4.4
4.0	3.55	3.3–3.8	2.7–4.4
4.1	3.57	3.4–3.8	2.7–4.4
4.2	3.59	3.4–3.8	2.8–4.4
4.3	3.61	3.4–3.8	2.8–4.4
4.4	3.62	3.4–3.8	2.8–4.5
4.5	3.64	3.4–3.9	2.8–4.5

small correlation between cousins acts like a wet blanket to a popular method of pedigree writing. In this method it is customary to say, "by an *unknown bull X* brother to a *famous bull Y* whose daughters produced and tested phenomenally." These daughters would be cousins to those of the *unknown bull X*. Consequently, even though the knowledge of the phenomenal production of the *famous bull's*

daughters is to be desired, but little is gained in predicting the probable performance of the *unknown bull's* daughters since little relation exists between the accomplishments of cousins in these cattle. From all that has preceded it is consistantly true that the prediction of a cow's milk yield or butter-fat percentage must be based on very near relatives.

THE RELATION BETWEEN THE GRANDSIRE AND THE PRODUCTIVITY OF HIS GRANDDAUGHTERS

The calculation of the direct relation of the grandsire to the production of his granddaughters comes from a knowledge of the standard deviations of the production of the grandsire's granddaughters and their relation to the standard deviation of all the granddaughters. The data for the standard deviations of the grandsire's granddaughters' milk yields and butter-fat percentages are given in table 133.

From the data of table 133 the mean weighted squared standard deviation is equal to 13,636,260. This standard deviation must be corrected for the small number of individuals on which the individual standard deviations are based. This correction on the basis of the relative frequency of the individual classes is 1.1774 or the mean corrected squared standard deviation for the individual arrays of the grandsire is 16,055,333. The standard deviation for the milk yields of these granddaughters is 4133 pounds. The correlation coefficient between the grandsires and the milk yields of the granddaughters is then equal to:

$$\sqrt{1 - \frac{16{,}055{,}333}{17{,}083{,}822}} \text{ or } 0.245$$

In the same manner, the mean weighted squared standard deviation of the butter-fat percentages is equal to 0.0806124 and the corrected mean weighted squared standard deviation of the arrays of granddaughters is 0.094913. The squared standard deviation for the granddaughters is 0.107194. The correlation coefficient for the grandsire's potential butter-fat percentage and the granddaughters' butter-fat percentage is:

$$\sqrt{1 - \frac{0.094913}{0.107194}} \text{ or } 0.339$$

TABLE 133

Standard deviations of the milk yields and butter-fat percentages of grandsires having six or more daughters in the Advanced Registry

GRANDSIRE'S HERD BOOK NUMBER	STANDARD DEVIATION OF THE GRANDDAUGHTERS		GRANDSIRE'S HERD BOOK NUMBER	STANDARD DEVIATION OF THE GRANDDAUGHTERS	
	Milk yields	Butter-fat percentages		Milk yields	Butter-fat percentages
14, 634	4, 764	0.194	29, 642	4, 276	0.339
18, 726	1, 750	0.348	29, 737	2, 749	0.215
20, 735	4, 152	0.352	30, 154	3, 300	0.262
21, 033	1, 278	0.136	30, 190	4, 642	0.443
21, 226	4, 271	0.282	30, 328	1, 491	0.160
21, 318	2, 539	0.220	30. 412	4, 298	0.275
21, 366	3, 836	0.319	31, 030	3, 594	0.291
21, 724	2, 415	0.196	31, 212	4, 382	0.362
22, 128	3, 362	0.302	31, 361	1, 979	0.107
22, 235	1, 067	0.243	31, 789	3, 024	0.295
22, 241	1, 772	0.203	31, 943	4, 158	0.224
22, 372	3, 912	0.212	32, 242	2, 585	0.406
22, 394	2, 937	0.280	32, 429	2, 828	0.381
22, 779	4, 366	0.200	32, 481	2, 904	0.231
22, 894	4, 410	0.347	32, 554	4, 709	0.225
22, 909	5, 091	0.205	32, 558	3, 480	0.287
23, 102	3, 902	0.320	32, 571	2, 842	0.308
23, 109	3, 775	0.157	32, 846	2, 753	0.287
23, 154	3, 403	0.214	34, 503	3, 455	0.383
23, 224	4, 233	0.256	34, 932	3, 096	0.354
23, 260	4, 467	0.338	35, 077	4, 731	0.294
23, 300	3, 110	0.299	35, 227	3, 648	0.280
23, 446	1, 790	0.190	35, 269	4, 124	0.272
23, 971	3, 954	0.311	36, 158	2, 657	0.273
24, 762	2, 311	0.250	36, 168	4, 350	0.258
25, 368	3, 040	0.299	36, 324	3, 392	0.236
25, 467	2, 449	0.334	36, 974	3, 475	0.304
25, 700	4, 525	0.266	37, 314	3, 176	0.314
25, 755	2, 326	0.161	38, 291	3, 542	0.349
25, 982	3, 715	0.322	38, 446	3, 642	0.503
26, 025	3, 472	0.119	38, 462	3, 012	0.235
26, 239	2, 754	0.216	38, 835	500	0.291
26, 250	3, 590	0.224	38, 978	4, 155	0.192
26, 533	4, 304	0.311	39, 037	4, 571	0.293
26, 766	3, 566	0.247	39, 849	2, 763	0.236
26, 935	4, 207	0.163	40, 534	1, 700	0.189
26, 939	4, 116	0.229	40, 592	2, 217	0.208
26, 940	2, 587	0.185	41, 266	3, 933	0.247

TABLE 133—*Continued*

GRANDSIRE'S HERD BOOK NUMBER	STANDARD DEVIATION OF THE GRANDDAUGHTERS		GRANDSIRE'S HERD BOOK NUMBER	STANDARD DEVIATION OF THE GRANDDAUGHTERS	
	Milk yields	Butter-fat percentages		Milk yields	Butter-fat percentages
27,041	2,775	0.263	42,142	2,921	0.271
27,100	2,357	0.309	44,444	5,998	0.292
27,282	2,968	0.332	44,781	2,441	0.273
27,823	1,572	0.198	44,931	4,123	0.149
27,929	2,595	0.285	45,674	2,991	0.208
28,133	1,700	0.259	46,136	3,505	0.237
28,176	3,014	0.170	46,767	2,582	0.514
28,400	3,663	0.188	48,020	4,610	0.255
28,430	3,289	0.217	49,643	1,528	0.115
28,660	4,611	0.297	51,523	3,727	0.224
28,835	2,100	0.338	53,418	3,345	0.390
29,328	2,814	0.303	56,435	3,796	0.183
29,521	2,727	0.169	60,403	3,327	0.270
29,600	2,427	0.287	66,664	4,235	0.127
			72,287	2,129	0.281

These correlation coefficients are apt to be a little larger than the product moment correlation coefficients. The correlation of the granddaughters with the sire is of the order 0.25 or about half that for the relation of the sire and dam to the milk yield or butter-fat percentage of their daughters. This is in line with most other evidence on the inheritance of quantitative characters. The fact in itself is important for those who are to purchase stock where performance is the chief consideration, for under these conditions the record on a grandparent is only about half as good as the record on a parent in predicting milk yield or butter-fat percentage of a cow. In like manner the variation of the records of the granddaughters is quite a good deal larger for those where the grandparents are constant in performance than the performance of the daughters for a constant performance of the parents.

These facts shed some light on the poverty of the results of ordinary pedigree study in predicting the productivity of cattle. In pedigree study the animals controlling the milk yields of the cow pedigreed are shown to be more influential in the first and less important in the second generation. If, and the writer has some fragmentary information to substantiate this, the animals in the third generation are

correspondingly less in their influence it follows that any pedigree study based on these third generation animals or animals further removed is going to have little weight in determining the milk yield or butter-fat percentage of a cow so pedigreed. Thus it is not surprising to find the results of the pedigree study previously made, showing little of value in assisting to determine the probable production of a cow, when this study is so largely devoted to the third generation animals or those beyond in the fourth and fifth generations.

SUMMARY

This section presents the data for the inheritance of milk yield and butter-fat percentage from the paternal grandsire. Two methods are used to demonstrate this inheritance. The first consists in showing that there is an association between the milk yields and butter-fat percentages of the granddaughters and of the cousins. The correlation coefficients are: for milk yield 0.070 ± 0.014 for granddaughters and 0.005 ± 0.029 for cousins; and for butter-fat percentage 0.176 ± 0.014 for granddaughters and 0.119 ± 0.029 for cousins. The second method depends on the relation of the standard deviation of the grandsire's granddaughters' milk yields or butter-fat percentages to the standard deviation of the milk yields or butter-fat percentages of all the granddaughters. These correlation coefficients are 0.245 for milk yield and 0.339 for butter-fat percentage. From these facts it is clear that milk yield and butter-fat percentage are inherited from grandsire to granddaughter. This inheritance, when measured by the correlation coefficients, is only about one-half that found for the parents.

CHAPTER XXI

Influence of the Paternal Granddam on the Milk Yields and Butter-fat Percentages of her Granddaughters

A discussion of the inheritance received by the granddaughter from the paternal granddam follows naturally on that for the paternal grandsire. As with the grandsire two general methods are open for the analysis of this inheritance from the granddam to granddaughter; the method of measuring this inheritance by the association which may exist between the milk yields and butter-fat percentages of relatives having a common paternal granddam, and the direct correlation between the milk yields and butter-fat percentages of the paternal granddam and her granddaughters. This last is much the more satisfactory method since actual records are available for both variables. The relation of the records of granddaughters are of much importance in the selection of breeding stock since such records are frequently all that are available. Table 134 presents the data showing the association which exists between the milk yields and butter-fat percentages of granddaughters.

The average milk yield and butter-fat percentage of this group of cows is larger than those of the breed taken as a whole. The constants of variation are similar to those of the breed taken as a whole. The correlation coefficients for the milk yields and butter-fat percentages of the granddaughters show that there is a distinct relation between the performance of these relatives. These correlation coefficients are larger than are those for the granddaughters which have the grandsire in common. Part of this difference is caused by the fact that many of these granddaughters are either full or half sisters and that the effect of these cows on the correlation coefficient is more pronounced in the case of the granddaughters where the paternal granddam is common than it is in the case where the paternal grandsire is the common ancestor. The comparison on the basis of cousins is given in table 135.

The correlation coefficients for the milk yields and butter-fat percentages are larger for cousins when the granddam is the common parent than they are when the grandsire is the common parent. The differences are only on the border line of significance, however. The correlation coefficients for the cousins are only about one half as large as those for the milk yields of half sisters and about one third as large as those for full sisters or for parent and offspring. These constants plainly show the importance of records on near relatives in attempting to predict the milk yield of any cow.

TABLE 134

Constants of variation for the performance of granddaughters in milk yield and butter-fat percentage

PHYSICAL CONSTANTS	MILK YIELD	BUTTER-FAT PERCENTAGE
Mean	19,970±63	3.48±0.01
Standard deviation	4,095±45	0.325±0.004
Coefficient of variation	20.5±0.2	9.3±0.1
Correlation coefficient	0.297±0.014	0.336±0.014

TABLE 135

The correlation between the milk yields and butter-fat percentages of cousins

COUSINS PATERNAL OR MATERNAL	CORRELATION COEFFICIENTS	
	Milk yield	Butter-fat percentage
Cousins (common grandsire)	0.005±0.029	0.119±0.029
Cousins (common granddam)	0.171±0.045	0.214±0.044

Table 136 shows the prediction of the milk yields of one granddaughter when the milk yield of another granddaughter is known. This equation is based on the data of table 134.

$$\text{Milk yield second granddaughter} = 14039 + 0.297 \text{ milk yield first granddaughter} \quad (55)$$

Column 1 gives the milk yield of the first granddaughter. Column 2 gives the probable average milk yields of the second granddaughters when the first granddaughter's performance is that given in column 1. Columns 3 and 4 give the ranges of milk yield necessary to include 50 per cent and 99 per cent of the second granddaughters when the first has the milk yields shown in column 1.

These last two columns are obtained from the standard deviation of the arrays.

The milk yield of one granddaughter predicts that of another with a fair degree of accuracy. The second granddaughter of a pair where the first's milk yield is 10,000 pounds has a probable milk yield of 17,009 pounds; and the second granddaughter, where the first's milk yield is 30,000 pounds, has a probable milk yield of 22,949 pounds, or an increase of 5940 pounds of milk over the second granddaughter of the lower group. The variation of the second granddaughters' milk yields is large, the range for the 50 per cent and 99 per cent limits being nearly as extensive as those for the whole breed.

TABLE 136

Expected milk yield of second granddaughters when the milk yield of the first granddaughter is known. Range of variation necessary to include 50 per cent and 99 per cent of the second granddaughters

MILK YIELD FIRST GRANDDAUGHTER	EXPECTED MILK YIELD OF SECOND GRANDDAUGHTERS	RANGE NECESSARY TO INCLUDE	
		50 per cent of the granddaughters	99 per cent of the granddaughters
10,000	17,009	14,372–19,646	6,922–27,096
12,000	17,603	14,966–20,240	7,516–27,690
14,000	18,197	15,560–20,834	8,110–28,284
16,000	18,791	16,154–21,428	8,704–28,878
18,000	19,385	16,748–22,022	9,298–29,472
20,000	19,979	17,342–22,616	9,892–30,066
22,000	20,573	17,936–23,210	10,486–30,660
24,000	21,167	18,530–23,804	11,080–31,254
26,000	21,761	19,124–24,398	11,674–31,848
28,000	22,355	19,718–24,992	12,268–32,442
30,000	22,949	20,312–25,586	12,862–33,036

The mean butter-fat percentages and limits of the 50 and 99 per cent ranges for the second granddaughters are given in table 137. These are obtained by the same methods as those used for table 136.

Table 137 furnishes the information necessary to obtain some idea of the probable butter-fat percentage of a granddaughter from the known butter-fat percentage of another granddaughter. The data presented indicates quite clearly that the granddaughters' productivities are related even though the relation is small in amount.

THE RELATION BETWEEN THE MILK YIELDS AND BUTTER-FAT PERCENTAGES OF PATERNAL GRANDDAM AND GRANDDAUGHTER

The correlation surface for the relation of the milk yields of paternal granddams and their granddaughters is given in table 138. The correlation surface for the butter-fat percentages of granddam and granddaughter is given in table 139. The constants deduced from these tables are found in table 140.

The mean milk yields of both groups, granddaughters and granddams, are larger than those of the rest of the breed. The standard

TABLE 137

Expected mean butter-fat percentage and range necessary to include 50 and 99 per cent of the second granddaughters when the butter-fat percentage of the first granddaughter is known

BUTTER-FAT PERCENTAGE OF FIRST GRANDDAUGHTER	EXPECTED BUTTER-FAT PERCENTAGE OF SECOND GRANDDAUGHTERS	RANGE NECESSARY TO INCLUDE	
		50 per cent of the granddaughters	99 per cent of the granddaughters
2.6	3.19	3.0–3.4	2.4–4.0
2.8	3.25	3.1–3.5	2.5–4.0
3.0	3.32	3.1–3.5	2.5–4.1
3.2	3.39	3.2–3.6	2.6–4.2
3.4	3.46	3.3–3.7	2.7–4.2
3.6	3.52	3.3–3.7	2.7–4.3
3.8	3.59	3.4–3.8	2.8–4.4
4.0	3.66	3.5–3.9	2.9–4.5
4.2	3.72	3.5–3.9	2.9–4.5
4.4	3.79	3.6–4.0	3.0–4.6
4.6	3.86	3.7–4.1	3.1–4.7

deviations of these milk yields are also larger than those ordinarily found. The average butter-fat percentages and the standard deviations of them show values normal to the Holstein-Friesian breed. The correlation coefficient for the milk yields of granddaughter and granddam is about that to be expected from the known manner in which the chromosomes, carrying the factors for milk yield, would assort. From a practical standpoint, then, the milk yield of the paternal granddam indicates to some degree the probable milk yield of her granddaughter. The correlation coefficients for these milk yields are only about one half those for the parent and

offspring correlations. In other words the record of a grandparent is not of so much value as that of a parent in predicting the milk yield of the offspring.

TABLE 138

Correlation surface for the milk yields of paternal granddams and their granddaughters

MILK YIELD OF GRANDDAUGHTER	MILK YIELD OF PATERNAL GRANDDAM																						
	10,000–11,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000–32,000	
11,000–12,000				1		1	1			1		1		2									7
12,000						1			1														2
13,000				1				2	1	1	1			2									8
14,000						1	2	2	4		1	3	2										15
15,000	1						3	4	2	2	2	1	2	3		3							23
16,000			3	2	2	1	2	5	3	1	4		3	2				1		1			30
17,000			1	1	3	1	4	3	2		2	1	2							1			21
18,000					3		8			1	3	3	1			1		1		1			22
19,000					1	1	11	3	1	2	3	3	1			1		1	1		1		30
20,000				1	1	1	1	1	2	1	3	4					3	2					20
21,000			2		1	1	2	1		1	4	1	1				1	1		2	1		19
22,000					3		8				2		2	3					1	1			20
23,000					1		3			2	1		1			1	1	2				1	13
24,000					1		2			1	1		2			1		2	1	1			12
25,000							5			1						3		2	2				13
26,000							2									1							3
27,000							2					2				1		1					6
28,000							2				1	1						3				1	8
29,000															1								1
30,000																		1					1
31,000							1																1
32,000																2							2
33,000																1							1
34,000–35,000							1																1
	1		6	6	16	8	60	21	16	14	28	20	17	12	1	15	5	17	5	7	2	2	279

The correlation coefficient for the butter-fat percentages of granddam and granddaughter is not so large as it could be expected to be either from the probable distribution of the chromosomes or from the relations determined on the cousins. In any case the correlation

TABLE 139

Correlation surface for the butter-fat percentages of paternal granddams and their granddaughters

BUTTER-FAT PERCENTAGE OF GRAND-DAUGHTER	BUTTER-FAT PERCENTAGE OF PATERNAL GRANDDAM																		
	2.7–2.8	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4–4.5	
2.6–2.7	1				1					1									3
2.7	1						1												2
2.8						1	1	2			1	2							7
2.9			1	1	2	1		2	2			1							10
3.0	1				2	2	1		2	1	1	1			1			1	13
3.1				2	2	1	2	3	4	3	1	5	2	1					26
3.2	1		1	2	4	4	2	3	8	2	1	3	2	1			1		35
3.3			2	2	7	1	1	6	6	3	6	1	2	1					38
3.4				1	8	5	4		3	7	3	2	5	1				1	40
3.5			3	1	5	4	2	4	4	8	2	3		3					39
3.6			1	2	1	1		1	1	8			2						17
3.7				1	2	2	4	2		3	1		1						16
3.8		1					1		1	2			1						6
3.9						1	2	1		2			1			1			8
4.0				1		1		2	1	1	1		1						8
4.1					1			3		3								1	8
4.2										2									2
4.3–4.4										1									1
	4	1	8	13	35	24	21	29	32	47	17	18	17	7	1	1	1	3	279

TABLE 140

Constants of variation and correlation coefficients for the milk yields and butter-fat percentages of paternal granddams and granddaughters

	MILK YIELD	BUTTER-FAT PERCENTAGE
Granddaughters		
Mean	19,851±179	3.43±0.01
Standard deviation	4,429±126	0.319±0.009
Coefficient of variation	22.3±0.7	9.3±0.3
Granddams		
Mean	20,063±182	3.50±0.01
Standard deviation	4,503±129	0.319±0.009
Coefficient of variation	22.4±0.7	9.1±0.3
Correlation coefficient	0.258±0.038	0.091±0.040

coefficient shows that there is a distinct association between the milk yields of paternal granddams and granddaughters. The case for the butter-fat percentage is not so clear. The raw regression lines together with the straight line equations for the same are shown in figures 26 and 27.

From these straight line equations similar information to that previously presented for the granddaughters may be tabled for the milk yields and butter-fat percentages of granddam and grand-

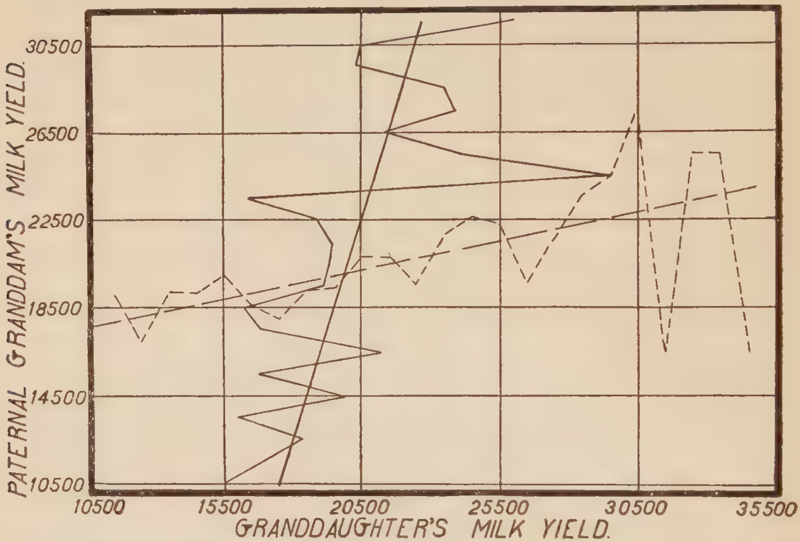

Fig. 26. Curves Showing the Relation between the Milk Yields of Paternal Granddam and Granddaughter

daughter. The equation for the probable milk yield of granddaughter from the milk yield of granddam is:

$$\text{Granddaughter's milk yield} = 14759 + 0.254 \text{ granddam's milk yield} \quad (56)$$

The equation for the butter-fat percentage of the granddaughter from that of the granddam is:

$$\text{Granddaughter's butter-fat percentage} = 3.12 + 0.090 \text{ granddam's butter-fat percentage} \quad (57)$$

Table 141 shows the average milk yields of the granddaughters for given milk yields of the granddams. The ranges necessary to include 50 per cent and 99 per cent of the granddaughters' milk yields for the different granddams' milk yields are also given.

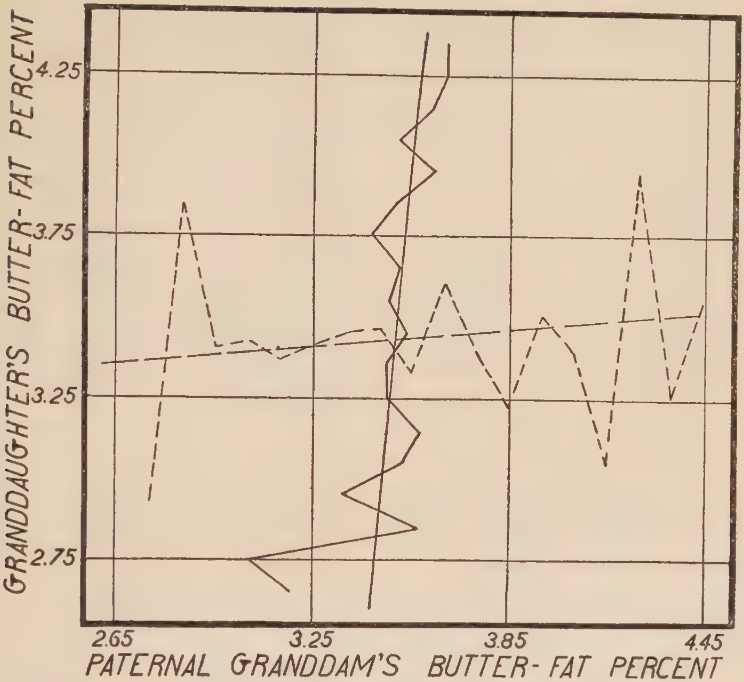

Fig. 27. Curves Showing the Relation between the Butter-fat Percentages of Paternal Granddam and Granddaughter

Table 142 gives the same information for the butter-fat percentage. As would be expected from the correlation coefficients, the milk yields of granddam and granddaughter correspond more closely to each other than do the butter-fat percentages of granddam and granddaughter.

TABLE 141

Expected milk yields of granddaughters for given milk yields of the granddams. Range of variation necessary to include 50 and 99 per cent of the granddaughters for given milk yields of the granddams

MILK YIELD OF GRANDDAMS	EXPECTED MILK YIELD OF GRANDDAUGHTERS	RANGE OF MILK YIELD NECESSARY TO INCLUDE	
		50 per cent of the granddaughters	99 per cent of the granddaughters
10,000	17,297	14,411–20,183	6,257–28,337
12,000	17,805	14,919–20,691	6,765–28,845
14,000	18,312	15,426–21,198	7,272–29,352
16,000	18,820	15,934–21,706	7,780–29,860
18,000	19,327	16,441–22,213	8,287–30,367
20,000	19,835	16,949–22,721	8,795–30,875
22,000	20,343	17,457–23,229	9,303–31,383
24,000	20,850	17,964–23,736	9,810–31,890
26,000	21,358	18,472–24,244	10,318–32,398
28,000	21,865	18,979–24,751	10,825–32,905
30,000	22,373	19,487–25,259	11,333–33,413

TABLE 142

Expected butter-fat percentage of granddaughters for given butter-fat percentages of the granddams. Range of variation necessary to include 50 and 99 per cent of the granddaughters for given butter-fat percentages of the granddams

BUTTER-FAT PERCENTAGES OF THE GRANDDAMS	EXPECTED BUTTER-FAT PERCENTAGES OF THE GRANDDAUGHTERS	RANGE OF BUTTER-FAT PERCENTAGE NECESSARY TO INCLUDE	
		50 per cent of the granddaughters	99 per cent of the granddaughters
2.6	3.35	3.1–3.6	2.5–4.2
2.8	3.37	3.2–3.6	2.6–4.2
3.0	3.39	3.2–3.6	2.6–4.2
3.2	3.40	3.2–3.6	2.6–4.2
3.4	3.42	3.2–3.6	2.6–4.2
3.6	3.44	3.2–3.7	2.6–4.3
3.8	3.46	3.2–3.7	2.6–4.3
4.0	3.48	3.3–3.7	2.7–4.3
4.2	3.49	3.3–3.7	2.7–4.3
4.4	3.51	3.3–3.7	2.7–4.3
4.6	3.53	3.3–3.7	2.7–4.4

SUMMARY

In this section a study has been made of the influence of the granddam on the milk yields and butter-fat percentages of the granddaughters. This influence is manifest in the performance of the granddaughters, the performance of the cousins, and the performance of the granddaughters in relation to the performance of the granddams. The correlation coefficients for the granddaughters are 0.297 for milk yield and 0.336 for butter-fat percentage. The correlation coefficients for the cousins are 0.171 for milk yield and 0.214 for butter-fat percentage. The correlation coefficients for the granddams' and granddaughters' productivity are 0.258 for milk yield and 0.091 for butter-fat percentage. These correlation coefficients point to the general influence of the granddam on the milk yield of her granddaughters measured by a correlation of 0.2 to 0.25 for milk yield and butter-fat percentage. In other words, the paternal grandparents are about equally potent in their influence on the milk yield and butter-fat percentages of their granddaughters and only about half as effective as the parents.

CHAPTER XXII

Influence of the Maternal Grandsire on the Milk Yields and Butter-fat Percentages of his Granddaughters

As noted elsewhere the maternal side of pedigree study is neglected in preference to the paternal side. Is this practice one based simply on custom or has it a genetic background to support it? This section and the one to follow will present the evidence for the influence of the two maternal grandparents on the milk yields and butter-fat percentages of the granddaughters.

The mode of analyzing the problem will be the same as that previously used. The relation of the milk yields and butter-fat percentages of the granddaughters, the relation of the milk yields and butter-fat percentages of the cousins, and the direct relation of the milk yields and butter-fat percentages of grandsire and granddaughter constitutes the data presented on this problem. The mean milk yields and butter-fat percentages of the granddaughters are presented in table 143. The standard deviations, coefficients of variation, and correlation coefficients are also given for these granddaughters.

The average milk yield and butter-fat percentage of this group of cows is closely similar to that of the whole Holstein-Friesian breed. The standard deviations are also similar to those of the whole breed. The groups of cows making up the granddaughters are accordingly, entirely comparable and representative of all the cows. The correlation coefficients for the milk yields and butter-fat percentages of these granddaughters are of fair size and are distinctly significant. The productivity of each granddaughter is related to that of the other in such a way that a larger milk yield or butter-fat percentage of one means a larger milk yield or butter-fat percentage of the other. How closely these records agree is shown in the next tables where the averages and variations of the second granddaughter are given for a given performance of the first granddaughter.

Table 144 is self-explanatory. Table 145 contains similar information for the butter-fat percentage of these granddaughters.

The data of tables 144 and 145 present the information necessary to determine approximately the probable performance and the possible variation of this performance for a cow when her maternal grandsire has another granddaughter with a record. The granddaughters may be full or half sisters or cousins. It is of much importance to examine the relation of the milk and butter-fat records of cousins when the maternal grandsire is the common parent. The analysis of this data is given in table 146.

TABLE 143

Constants of variation for the milk yields and butter-fat percentages of maternal granddaughters

MATERNAL GRANDDAUGHTER'S CONSTANTS OF VARIATION	MILK YIELD	BUTTER-FAT PERCENTAGE
Mean	19,278±66	3.45±0.01
Standard deviation	3,957±47	0.313±0.003
Coefficient of variation	20.5±0.3	9.1±0.1
Correlation coefficient	0.244±0.016	0.224±0.016

TABLE 144

Expected milk yields of one granddaughter when the milk yield of the first granddaughter is known. Range of variation necessary to include 50 and 99 per cent of the second granddaughters

MILK YIELD FIRST GRANDDAUGHTER	EXPECTED MILK YIELD OF SECOND GRANDDAUGHTERS	RANGE NECESSARY TO INCLUDE	
		50 per cent of the granddaughters	99 per cent of the granddaughters
10,000	17,014	14,425–19,603	7,113–26,915
12,000	17,502	14,913–20,091	7,601–27,403
14,000	17,990	15,401–20,579	8,089–27,891
16,000	18,478	15,889–21,067	8,577–28,379
18,000	18,966	16,377–21,555	9,065–28,867
20,000	19,454	16,865–22,043	9,553–29,355
22,000	19,942	17,353–22,531	10,041–29,843
24,000	20,430	17,841–23,019	10,529–30,331
26,000	20,918	18,329–23,507	11,017–30,819
28,000	21,406	18,817–23,995	11,505–31,307
30,000	21,894	19,305–24,483	11,993–31,795

The correlation coefficients for the milk yields and butter-fat percentages of cousins with common paternal dams and of cousins with common maternal grandsires are closely similar to each other.

These correlation coefficients are of fair size. The correlation coefficient for the milk yields of cousins with a common paternal grandsire is not significant, that for the butter-fat percentage is significant. No biological reason appears to exist for this difference.

TABLE 145

Expected butter-fat percentage of one granddaughter when the butter-fat percentage of the first granddaughter is known. Range of variation necessary to include 50 and 99 per cent of the second granddaughters

BUTTER-FAT PERCENTAGE OF FIRST GRANDDAUGHTER	EXPECTED BUTTER-FAT PERCENTAGE OF SECOND GRAND-DAUGHTERS	RANGE NECESSARY TO INCLUDE	
		50 per cent of the granddaughters	99 per cent of the granddaughters
2.6	3.26	3.1–3.5	2.5–4.0
2.8	3.31	3.1–3.5	2.5–4.1
3.0	3.35	3.2–3.6	2.6–4.1
3.2	3.40	3.2–3.6	2.6–4.2
3.4	3.44	3.2–3.7	2.7–4.2
3.6	3.49	3.3–3.7	2.7–4.3
3.8	3.53	3.3–3.7	2.7–4.3
4.0	3.58	3.3–3.7	2.8–4.4
4.2	3.62	3.4–3.8	2.8–4.4
4.4	3.67	3.4–3.8	2.9–4.5
4.6	3.71	3.5–3.9	2.9–4.5

TABLE 146

The correlation of the performance of cousins

KIND OF COUSIN (COMMON GRANDPARENT)	CORRELATION COEFFICIENT FOR THE PERFORMANCE	
	Milk yield	Butter-fat percentage
Paternal sire	0.005±0.029	0.119±0.029
Paternal dam	0.171±0.045	0.214±0.044
Maternal sire	0.206±0.020	0.216±0.020

In fact, it seems rather as if the productivity of the cousins should be correlated to the extent of about 0.2 or that found for the rest of the cousins. Such a supposition is possible for the butter-fat percentages of cousins with a common paternal grandsire but is statistically doubtful for the milk yields of these same cousins.

THE DIRECT RELATION OF THE MATERNAL GRANDSIRE TO THE MILK YIELDS AND BUTTER-FAT PERCENTAGES OF HIS GRANDDAUGHTERS

By using the same methods as those already described the relation between the milk yields and butter-fat percentages of maternal grandsire and granddaughters may be found. The standard deviation of the granddaughter's milk yields for each of the grandsires are given in table 147.

These standard deviations are based on grandsires having 6 to 14 granddaughters, inclusive. The average weighted squared standard deviation for the group is 10,839,820. The weighted correction factor, due to the small number of individuals on which the single standard deviations are based, is 1.2682. The corrected standard deviation is consequently 3709. With this data the correlation coefficient is found to be 0.350. This coefficient is rather large. However, when the method of derivation is considered it will be realized that the coefficient is naturally larger than the product moment coefficient. In fact, the true product moment r is likely to be somewhere near 0.2 to 0.25 corresponding to the coefficients previously found for other similar data.

In the same manner the relation of the grandsire to the butter-fat percentage of the granddaughter may be determined. Table 148 gives the standard deviations of the granddaughter's butter-fat percentages for the same sires as those of table 147.

From these data the average weighted squared standard deviation is found to be 0.06547. Applying the same correction factor due to the small number of individuals on which the single standard deviations are based, as that for milk yield, 1.2682, we have for the corrected squared standard deviation 0.08302. The correlation coefficient formed from this data is 0.390. As would be expected from the derivation of this coefficient it is rather large. The probable value of the product moment r would be more nearly 0.25 to 0.30 or a value corresponding closely with that of other data on inheritance from grandparent to granddaughter. It is of interest to gather together these coefficients for the contribution of sire and grandsires in governing the production of the granddaughters and compare them. The similarity in the method of their calculation makes it appear as though the comparison of the coefficients as they are calculated is sound. Table 149 gives this comparison.

These correlation coefficients are subject to the same influences, feeding and care of the cows and heredity. Feeding and care, environmental factors, have been shown to modify such correlation

TABLE 147

Standard deviations of milk yields of the granddaughters for each maternal grandsire

GRANDSIRE'S HERD BOOK NUMBER	STANDARD DEVIATION OF THE MILK YIELDS OF GRANDDAUGHTERS	GRANDSIRE'S HERD BOOK NUMBER	STANDARD DEVIATION OF THE MILK YIELDS OF GRANDDAUGHTERS
635*	2,514	28,565	3,055
21,724	1,286	28,982	3,590
22,187	1,826	29,027	3,510
22,233	2,010	29,303	2,718
22,235	2,478	29,500	1,700
22,699	3,399	29,588	2,840
22,881	2,967	29,600	957
22,991	3,830	30,624	1,795
23,102	3,149	30,674	639
23,153	1,543	31,072	2,625
23,260	3,417	31,212	4,357
23,301	2,267	31,387	3,543
23,366	2,561	32,110	3,464
23,450	3,726	32,322	2,673
23,538	3,562	32,408	2,867
23,971	3,758	32,492	2,805
24,954	3,606	32,554	5,249
25,166	3,727	32,655	2,807
25,467	4,740	33,437	4,460
25,865	2,749	34,114	5,735
25,982	4,832	34,467	2,055
26,025	5,047	37,852	1,247
26,646	2,267	38,446	3,201
26,936	2,268	41,206	3,414
27,041	3,659	41,266	4,610
27,868	4,992	41,751	2,537
28,176	3,625	42,147	2,560
28,400	2,749	44,034	3,416
28,430	4,561	44,444	4,346

* W. H. F. A.

coefficients to only a moderate degree. It should be kept in mind, however, that the method of calculating these coefficients is such that they are probably larger than the product moment values.

TABLE 148

Standard deviations of the butter-fat percentages of the granddaughters for each maternal grandsire

GRANDSIRE'S HERD BOOK NUMBER	STANDARD DEVIATION OF THE BUTTER-FAT PERCENTAGES OF THE GRANDDAUGHTERS	GRANDSIRE'S HERD BOOK NUMBER	STANDARD DEVIATION OF THE BUTTER-FAT PERCENTAGES OF THE GRANDDAUGHTERS
635*	0.176	28,565	0.160
21,724	0.406	28,982	0.180
22,187	0.229	29,027	0.228
22,233	0.301	29,303	0.295
22,235	0.191	29,500	0.205
22,699	0.250	29,588	0.274
22,881	0.330	29,600	0.313
22,991	0.241	30,624	0.157
23,102	0.285	30,674	0.259
23,153	0.088	31,072	0.275
23,260	0.256	31,212	0.183
23,301	0.094	31,387	0.335
23,366	0.242	32,110	0.189
23,450	0.266	32,322	0.164
23,538	0.381	32,408	0.267
23,971	0.239	32,492	0.155
24,954	0.141	32,554	0.211
25,166	0.279	32,655	0.183
25,467	0.191	33,437	0.281
25,865	0.332	34,114	0.362
25,982	0.285	34,467	0.171
26,025	0.096	37,852	0.344
26,646	0.157	38,446	0.344
26,936	0.188	41,206	0.195
27,041	0.229	41,266	0.227
27,868	0.410	41,751	0.205
28,176	0.306	42,147	0.335
28,400	0.157	44,034	0.345
28,430	0.350	44,444	0.265

* W. H. F. A.

TABLE 149

Coefficients showing the correlation between the male ancestors and the productivity of the granddaughters

MALE ANCESTOR	MILK YIELD	BUTTER-FAT PERCENTAGE
Sire	0.521	0.526
Paternal Grandsire	0.245	0.339
Maternal Grandsire	0.350	0.390

The values for the sire and daughter for both milk yield and butter-fat percentage are larger than these correlations for the relation of the grandsires and granddaughters for either of these constants. The associations between the milk yield and butter-fat percentage of the granddaughter and the maternal grandsire are larger than these same constants for the granddaughter and paternal grandsire. It should be remembered, however, that the maternal grandsires' and granddaughters' coefficients are undoubtedly subject to the larger probable error. The size of these coefficients of correlation are, on the whole, about the same as those which have been found for other quantitative data. The influence of the two grandsires appears to be approximately equal and less than that of the sire. Some appreciation of the amount of this influence may be gained by an examination of the degree of control exerted by the ancestor on the standard deviation of the progeny's milk yield or butter-fat percentage. The average coefficient of correlation for the grandsires' and granddaughters' milk yield is 0.287 and that for the butter-fat percentage is 0.363. The amount of variation eliminated by making the grandsires constant is S.D. $-$ S.D. $\sqrt{1 - r^2}$ or 4.2 per cent for the milk yields and 6.8 per cent for the butter-fat percentages. For the sire the degree of control is 14.6 per cent for the milk yield and 15 per cent for the butter-fat percentage. In other words something less than half as much control over the milk yield or butter-fat percentage is exerted by the individual grandsires as that exerted by the sire.

SUMMARY

In this section the data on the inheritance of milk yield and butter-fat percentage from the maternal grandsire to the granddaughter are presented. Three lines of evidence are presented. It is shown that a correlation exists for the milk yields and butter-fat percentage of granddaughters of 0.244 and 0.224 respectively. The correlation coefficients for the milk yields and butter-fat percentages of cousins having a common maternal grandsire, are 0.206 and 0.216. By the application of the method of average standard deviation of arrays of granddaughters for each grandsire to the standard deviation of all the granddaughters it was possible to show that a coefficient of 0.350 for milk yield and 0.390 for butter-

fat percentage exists between the maternal grandsires and granddaughters. These data point to the conclusion that the grandsire is only one-half as effective in controlling the milk yield or butterfat percentage of the granddaughter as the sire or, for that matter, the dam.

CHAPTER XXIII

The Influence of the Maternal Granddam on the Milk Yields and Butter-fat Percentages of Her Granddaughters

In the preceding section the analysis showed that the maternal grandsire had as profound an influence on the productivity of the granddaughters as either paternal grandparent. In spite of this fact the evidence shows that what selection for productivity in milk yield is accomplished, is more stringently practiced for the paternal than for the maternal side of the pedigree. No small importance can be attached to this problem. As the evidence about to be presented will show, the contribution toward the inheritance of the granddaughter by the maternal granddam is as important as the contribution of any grandparent in so far as the inheritance affects milk yield or butter-fat percentage. In view of these facts it seems reasonable to expect a revision in the methods now in vogue in the selection and breeding of dairy animals. Table 150 presents the data on the average yields, variation in yields, and correlation of the yields of granddaughters with a common maternal grandsire. The existence of a correlation between the records of granddaughters is, of course, a partial proof of the inheritance of milk yield or butter-fat percentage, the contribution of similar heredity coming from the granddam being a cause for this resemblance.

The average milk yields and butter-fat percentages of this group of cows are practically the same as those of the Holstein-Friesian Advanced Registry taken as a whole. The constants of variation are likewise normal for the breed as a whole. The correlation coefficients for the milk yields or butter-fat percentages of the granddaughters are fairly large and distinctly significant. The productivity of one of the granddaughters from a common maternal granddam controls to a fairly large degree the productivity of the other granddaughters.

The equations to predict the performance of one granddaughter from that of another are given below. From these equations and the known variations of the milk yields and butter-fat percentages tables

151 and 152 are formed. Table 151 shows the mean milk yields and the ranges of milk yield necessary to include 50 and 99 per cent of the second granddaughters when the milk yield of the first is known. Table 152 shows the same information for the butter-fat percentages.

TABLE 150

Constants of variation for the performance of the granddaughters in milk yield and butter-fat percentage

PHYSICAL CONSTANT	MILK YIELD	BUTTER-FAT PERCENTAGE
Mean	19,476±99	3.45±0.01
Standard deviation	4,168±70	0.308±0.005
Coefficient of variation	21.4 ±0.4	8.9±0.2
Correlation coefficient	0.344 ±0.021	0.258±0.022

TABLE 151

Average milk yield of second granddaughters where the milk yield of the first is known. Range of milk yield necessary to include 50 and 99 per cent of the second granddaughters

FIRST GRAND-DAUGHTER'S MILK YIELD	EXPECTED MILK YIELD OF SECOND GRANDDAUGHTERS	RANGE NECESSARY TO INCLUDE	
		50 per cent of the granddaughters	99 per cent of the granddaughters
10,000	16,216	13,576–18,856	6,117–26,315
12,000	16,904	14,264–19,544	6,805–27,003
14,000	17,592	14,952–20,232	7,493–27,691
16,000	18,280	15,640–20,920	8,181–28,379
18,000	18,968	16,328–21,608	8,869–29,067
20,000	19,656	17,016–22,296	9,557–29,755
22,000	20,344	17,704–22,984	10,245–30,443
24,000	21,032	18,392–23,672	10,933–31,131
26,000	21,720	19,080–24,360	11,621–31,819
28,000	22,408	19,786–25,048	12,309–32,507
30,000	23,096	20,456–25,736	12,997–33,195

The milk yield of the second granddaughter for a given milk yield of the first is equal to:

$$\text{Milk yield second granddaughter} = 12776 + 0.344 \text{ milk yield first granddaughter} \qquad (58)$$

Table 151 shows the information on the milk yields of the granddaughters derived from this equation.

In view of what has been previously presented the data given in table 151 should be self-explanatory. The equation for the butter-fat percentages of second granddaughters for the given butter-fat percentages of first granddaughters is given below:

$$\text{Butter-fat percentage of second granddaughter} = 2.56 + 0.258 \text{ butter-fat percentage of first granddaughter} \quad (59)$$

From this equation and the data on the standard deviation given in table 150 we obtain the information given in table 152.

TABLE 152

Average butter-fat percentage of second granddaughters when the first granddaughter has a given butter-fat percentage. Range of butter-fat percentage necessary to include 50 and 99 per cent of the second granddaughters

FIRST GRANDDAUGHTER S BUTTER-FAT PERCENTAGE	EXPECTED BUTTER-FAT PERCENTAGE OF SECOND GRANDDAUGHTERS	RANGE OF BUTTER-FAT PERCENTAGE NECESSARY TO INCLUDE	
		50 per cent of the granddaughters	99 per cent of the granddaughters
2.6	3.23	3.0-3.4	2.5–4.0
2.8	3.28	3.1–3.5	2.5–4.1
3.0	3.33	3.1–3.5	2.6–4.1
3.2	3.38	3.2–3.6	2.6–4.2
3.4	3.44	3.2–3.6	2.7-4.2
3.6	3.49	3.3–3.7	2.7–4.3
3.8	3.54	3.3–3.7	2.8–4.3
4.0	3.59	3.4–3.8	2.8–4.4
4.2	3.64	3.4–3.8	2.9–4.4
4.4	3.69	3.5–3.9	2.9–4.5
4.6	3.75	3.6–4.0	3.0–4.5

The question of the relation existing between the milk yields and butter-fat percentages of cousins follows naturally on that of the second granddaughters. There are 652 pairs of cousins with a common maternal granddam. The average milk yield for the group is 19,528 pounds of milk and the average butter-fat percentage is 3.46. The standard deviation for the milk yields is 4222 pounds and for the butter-fat percentage, 0.314. The group is consequently closely similar to that of the whole breed. The correlation coefficients for the milk yields and butter-fat percentages of these cousins are 0.234 ± 0.044 and 0.244 ± 0.044. Table 153 brings together in tabular form the results, which have been presented, on the relation of the milk yields and butter-fat percentages of cousins.

The correlation coefficients for the performance of the cousins with a common paternal grandsire are less than the rest, the milk yields significantly less, the butter-fat percentage probably not significantly less. In view of the other evidence presented there does not seem to be any biological reason for this difference, for as previously noted, the relation of the milk yields of granddaughters to their paternal grandsires is the same as the relations for the other grandparents. The six other coefficients resemble each other closely considering the data from which they are derived. In view of this fact and the other relations found for the granddaughters and grandparents it appears probable that the cousins in their productivity resemble each other to practically the same extent irrespective of what the common grandparent may be.

TABLE 153

Correlation coefficients for the milk yields and butter-fat percentages of cousins

KIND OF GRANDPARENT	MILK YIELD	BUTTER-FAT PERCENTAGE
Paternal grandsire	0.005 ± 0.029	0.119 ± 0.029
Paternal granddam	0.171 ± 0.045	0.214 ± 0.044
Maternal grandsire	0.206 ± 0.020	0.216 ± 0.020
Maternal granddam	0.234 ± 0.044	0.244 ± 0.044

THE DIRECT RELATION OF THE MATERNAL GRANDDAMS TO THE MILK YIELDS AND BUTTER-FAT PERCENTAGES OF THE GRANDDAUGHTERS

The correlation surface showing the relation of the milk yields of maternal granddam and granddaughter is given in table 154.

The correlation surface showing the relation of the butter-fat percentages of maternal granddams and granddaughters is presented in table 155.

From these tables the constants of table 156 are derived.

The mean milk yield of the granddaughters is slightly in excess of the mean milk yield of the granddams. On the other hand the mean butter-fat percentage of the granddams is larger than that of the granddaughters. Both groups are fairly representative of the Holstein-Friesian breed. The standard deviations of the milk yields are slightly high for the granddams and low for the granddaughters. The standard deviations for the butter-fat percentages are about normal.

The correlation coefficients for both the milk yields and butter-fat percentages of the granddams and granddaughters are significant and large enough to be of some importance. They show clearly that the granddam's productivity is an indication of the probable productivity of the granddaughters. In table 157 are gathered together the

TABLE 154

Correlation surface for the relation of the milk yields of maternal granddams and their granddaughters

MATERNAL GRANDDAM'S MILK YIELD	GRANDDAUGHTERS' MILK YIELD																					
	11,000–12,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000	22,000	23,000	24,000	25,000	26,000	27,000	28,000	29,000	30,000	31,000–32,000	
10,000–11,000	.				1	1	1	1														4
11,000					1	2					1											4
12,000						1			2				1				1					5
13,000		1							3	1	1	1										7
14,000					1	1		1	1	1		2	1		1							9
15,000				1			7	4	5	3	1		1									22
16,000		3				2	1							2								8
17,000			1			1	3	3	1	4	2	1	1									17
18,000			1		3	3	3	1	3	3		1		1								19
19,000			1	1			3	1	1	1	4	1	1		1			1		1	1	18
20,000						1		2	1	2	2			1		1						10
21,000						1			2		2	1										6
22,000						2		1	2	1	1	1			1	1				1		11
23,000	1		1		1						1		1									5
24,000							1						1		1			1		1	1	6
25,000						1						1	1	1								4
26,000											1			1								2
27,000				1										1			1					3
28,000											1											1
29,000					1										2					1		4
30,000–31,000									1													1
	1	4	4	3	8	16	19	14	22	16	17	9	8	7	6	2	2	2		4	2	166

coefficients showing the correlation of the grandparental performance with the performance of the granddaughters.

These correlation coefficients agree fairly well when proper weight is given to the fact that the correlations for the grandsires are derived in such a manner as to make it probable that they are slightly

TABLE 155

Correlation surface for the relation of the butter-fat percentages of maternal granddams and their granddaughters

MATERNAL GRAND-DAM'S BUTTER-FAT PERCENTAGE	GRANDDAUGHTERS' BUTTER-FAT PERCENTAGE																			
	2.5–2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3–4.4	
2.6–2.7						1														1
2.7						1														1
2.8							1													1
2.9							1													1
3.0	1				1		2	3	3	1	2	1								14
3.1					2	1	3	1	5	3	1	2								18
3.2					3		3	3	1	4	3	3	1			1		1	2	25
3.3		1	2	1	1	3	3	5	3	1	2	1				1	1			25
3.4						3	3	2	3	1	3	2	2		2	2				23
3.5					1			4	4	4	3	1	3							20
3.6				1		1	1	2	1	3				1	1			1		12
3.7								1		1	1									3
3.8							3			1	3	1	1							9
3.9								1	4	1	2	1								9
4.0										1										1
4.1											1									1
4.2																				
4.3																				
4.4											1									1
4.5–4.6													1							1
	1	1	2	2	8	10	20	22	24	21	22	12	8	1	3	4	1	2	2	166

TABLE 156

Constants of variation and correlation for the milk yields and butter-fat percentages of maternal granddams and their granddaughters

CONSTANT	MILK YIELD	BUTTER-FAT PERCENTAGE
Maternal granddams		
Mean	18, 729±227	3.42±0.02
Standard deviation	4, 331±160	0.295±0.011
Coefficient of variation	23.1 ±0.9	8.6±0.3
Granddaughters		
Mean	20, 036±209	3.39±0.02
Standard deviation	3, 986±148	0.313±0.012
Coefficient of variation	19.9±0.8	9.2±0.3
Correlation coefficient	0.307±0.047	0.192±0.050

in excess of what they would have been had they been derived by the product moment method. The evidence as a whole, points to an equal influence of the grandparents. The effect on the milk yields or butter-fat percentages as measured by the correlation coefficient appears to be equal to about 0.2 to 0.25 or a correlation coefficient equivalent to that found for most quantitative data.

Table 158 gathers together the data for a comparison of the influence of the female parents and grandparents.

TABLE 157

Correlation coefficients for the relation of the milk yields and butter-fat percentages of the grandparents and granddaughters

TYPE OF GRANDPARENT	MILK YIELD	BUTTER-FAT PERCENTAGE
Paternal grandsire	0.245	0.339
Paternal granddam	0.258	0.091
Maternal grandsire	0.350	0.390
Maternal granddam	0.307	0.192
Average	0.290	0.253

TABLE 158

Correlation coefficients for the influence of the parents and grandparents on the milk yields and butter-fat percentages of the granddaughters

TYPE OF ANCESTOR	MILK YIELD	BUTTER-FAT PERCENTAGE
Dam	0.497	0.413
Paternal grandam	0.258	0.091
Maternal granddam	0.307	0.192
Average for grandparents	0.282	0.141

Table 158 brings out the fact that the correlation coefficients of the grandparents' productivity with that of the granddaughters is half or slightly less than half that of the parents with their daughters. Returning to the formula for the reduction in the variation of performance when a given ancestor's performance is made constant, S.D. $-$ S.D. $\sqrt{1 - r^2}$, we note that for milk yield the dam reduces this variability 13.3 per cent, whereas the granddam reduces it only 4.1 per cent. For the butter-fat percentage the reduction for the dam is 9 per cent and the granddam 1 per cent on the basis

of the above figures. If we take the average of all the grandparents, table 157, the reductions for milk yields are parental 13.3 per cent to 4.3 per cent for grandparents, and for butter-fat percentage are parental 9 per cent to 3.2 per cent for grandparents. These facts bring out the point, that the grandparents control the milk yields or butter-fat percentages of their granddaughters to a much less degree than do the parents. Consequently, the record of a

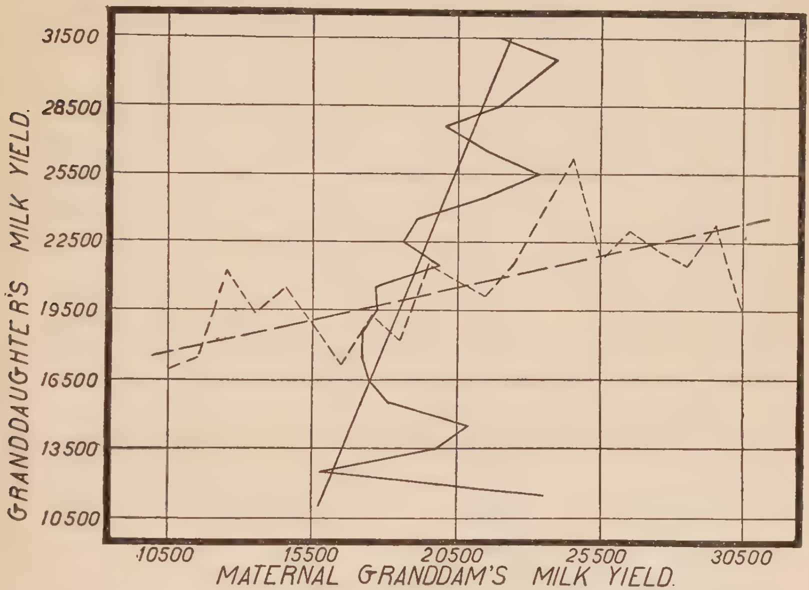

FIG. 28. THE RELATION OF THE MILK YIELDS OF MATERNAL GRANDDAM AND GRANDDAUGHTER

The raw regression lines are shown as rough lines. The straight lines are derived from the ordinary regression equations.

grandparent is of much less value in indicating the probable yield of a cow than is the record of the parents. In other words it is of fundamental importance to have records close up to the cow if we are to have progress in breeding for the supremely important economic characters, milk yield and butter-fat percentage.

From the data of table 156 we may now form the prediction equations for the milk yields and butter-fat percentages of a grand-

daughter when her maternal granddam's record is known. The equation for the milk yield is:

$$\text{Granddaughter's milk yield} = 14745 + 0.283 \text{ granddam's milk yield} \quad (60)$$

The equation for the butter-fat percentage is:

$$\text{Granddaughter's butter-fat percentage} = 2.69 + 0.204 \text{ granddam's butter fat percentage} \quad (61)$$

The regression lines for the milk yields of granddaughter and granddam as derived from table 154 are shown in figure 28. The

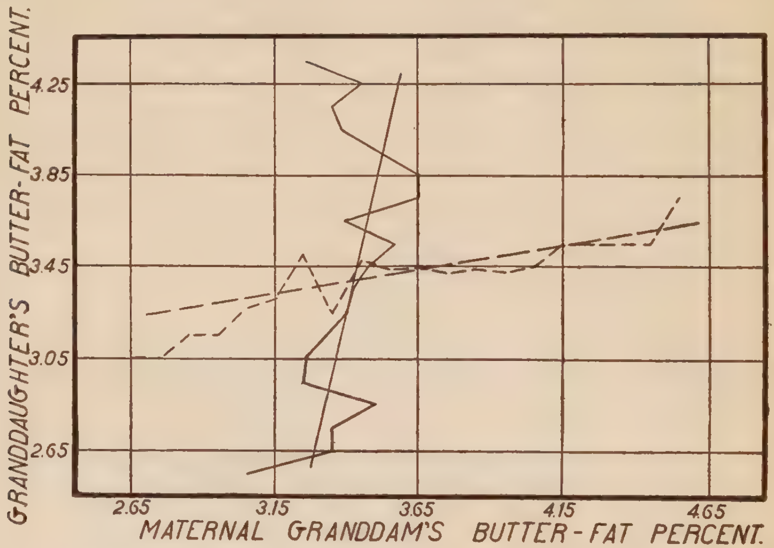

FIG. 29. THE RELATION OF THE BUTTER-FAT PERCENTAGES OF MATERNAL GRANDDAM AND GRANDDAUGHTER

The rough lines are the raw regression lines, the straight lines the lines derived from the ordinary regression equations.

regression lines for the butter-fat percentages, taken from table 155, are shown in figure 29.

Table 159 shows the average milk yields of the granddaughters for given milk yields of the granddams. The range of milk yield necessary to include 50 per cent and 99 per cent of the granddaughters is also given.

TABLE 159

Expected milk yield for a granddaughter when the milk yield of the maternal granddam is known. Range of variation necessary to include 50 and 99 per cent of the granddaughters

MILK YIELD OF MATERNAL GRANDDAM	EXPECTED MILK YIELD OF GRANDDAUGHTERS	RANGE NECESSARY TO INCLUDE	
		50 per cent of the granddaughters	99 per cent of the granddaughters
10,000	17,570	15,012–20,128	7,784–27,356
12,000	18,135	15,577–20,693	8,349–27,921
14,000	18,700	16,142–21,258	8,914–28,486
16,000	19,265	16,707–21,823	9,479–29,051
18,000	19,830	17,272–22,388	10,044–29,616
20,000	20,395	17,837–22,953	10,609–30,181
22,000	20,960	18,402–23,518	11,174–30,746
24,000	21,525	18,967–24,083	11,739–31,311
26,000	22,090	19,532–24,648	12,304–31,876
28,000	22,655	20,097–25,213	12,869–32,441
30,000	23,220	20,662–25,778	13,434–33,006

TABLE 160

Average butter-fat percentages for granddaughters when the butter-fat percentage of the maternal granddam is known. Range of variation necessary to include 50 per cent and 99 per cent of the granddaughters

BUTTER-FAT PERCENTAGE OF GRANDDAM	AVERAGE BUTTER-FAT PERCENTAGE OF GRANDDAUGHTERS	RANGE NECESSARY TO INCLUDE	
		50 per cent of the granddaughters	99 per cent of the granddaughters
2.6	3.22	3.0–3.4	2.4–4.0
2.8	3.26	3.1–3.5	2.5–4.1
3.0	3.30	3.1-3.5	2.5–4.1
3.2	3.35	3.1–3.6	2.6–4.1
3.4	3.39	3.2-3.6	2.6–4.2
3.6	3.43	3.2–3.6	2.6–4.2
3.8	3.47	3.3–3.7	2.7–4.3
4.0	3.51	3.3–3.7	2.7–4.3
4.2	3.55	3.3–3.8	2.8–4.3
4.4	3.59	3.4–3.8	2.8–4.4
4.6	3.63	3.4–3.8	2.8–4.4

Table 160 shows the same data for the butter-fat percentage.

The data contained in tables 159 and 160 appear to be self-explanatory.

SUMMARY

This chapter treats of the inheritance of milk yield and butter-fat percentage from the maternal granddam to her granddaughters. The data presented, include the relation of the milk yields and butter-fat percentages of granddaughters, cousins, and granddams with granddaughters. The correlation coefficients for the milk yields and butter-fat percentages of granddaughters are 0.344 and 0.258 respectively. These constants are 0.234 and 0.244 for cousins, and 0.307 and 0.192 for granddams and granddaughters. These data show that the granddams have a marked though small influence on the productivity of the granddaughters. The comparison of the correlation coefficients for the daughter's and dam's performance and for the granddaughter's and granddam's production shows that the parents are more than twice as effective as the grandparents in controlling the milk yield or butter-fat percentages of a cow.

CHAPTER XXIV

Résumé of the Present Data on the Inheritance of Milk Yield and Butter-fat Percentage

In the light of the best available evidence, the chromosomes and their behavior during the cell divisions just previous to the formation of the sperm or egg furnish the key to the phenomena observed in heredity. The chromosomes are considered as the bearers of the elements which determine the inheritance. In any individual there is a group of these chromosomes which reacts with the rest of the chromosomes to determine sex. These chromosomes are called the sex chromosomes. In mammals the sex chromosomes are in general one for the male and two for the female. In the female all the chromosomes including the sex chromosomes are in pairs. In the male all the chromosomes save the sex chromosomes are in pairs. The sex chromosome is single in the male for most mammals. The chromosomes are then little bundles of entities, called *factors*, each bundle going in pairs and each pair separate from the rest. These *factors* appear to be strung along the chromosomes in linear order. While the factors are distinct entities and one chromosome pair is distinct from another the reaction of the different factors on the developing body is such that a dozen or more factors may have an effect on a single minute organ. That is the body, the cow as we see it, is built by the combined action of all the factors contained in the chromosomes, while at the same time these factors are themselves distinct and remain distinct.

The factors for heredity appear to be arranged in linear order along the chromosomes. If we take any one pair of chromosomes they are known to come together in a kind of fusion sometime during the last two divisions which form the sperm or egg. It is known further that it is quite customary for a group of factors in one chromosome to cross over to the other chromosome of the pair and vice versa. In other words the factors from one chromosome may be interchanged with those of its mate in such a way that all possible recombinations of factors may ensue. The frequency

with which this recombination takes place is different for different factors and is believed to be due to the distance which may separate the factors in the chromosomes.

The evidence for the inheritance of milk yield and butter-fat percentage in common with that for many other economic characters indicates that the quantity of milk or the amount of butter-fat percentage is dependent on a number of such factors jointly. The number of chromosomes and their combination and recombination is important when viewed in the light of the above general facts of heredity. Wodsedalek[1] has presented a cytological study of the

FIG. 30. FIGURES SHOWING CERTAIN OF THE CHROMOSOME GROUPS FOUND IN CATTLE

1. Polar view of the metaphase of division in a spermatogonial cell showing thirty-six ordinary chromosomes and the single heart-shaped sex-chromosome.

2. Metaphase stages of division of the öogonia showing the sex-chromosome at the periphery of the plate. After Wodsedalek.

ovaries and testes of cattle. According to this work the male has 37 chromosomes. Of these chromosomes, one is the sex chromosome and the other 36, form 18 pairs of autosomes. The sex chromosome is received from the dam in the egg. Of the 36 chromosomes 18 are received from the sire and the 18 mates to them are received from the dam. The dam in consequence of this arrangement furnishes 19 chromosomes to the male offspring and the sire 18. For a female

[1] Wodsedalek, J. E. 1920. Studies on the cells of cattle with special reference to spermatogenesis, öogonia, and sex-determination. Biol. Bul., vol. 38, pp. 290–317.

offspring the sire contributes one sex chromosome and 18 other chromosomes and the dam one sex chromosome and 18 mates to the sire's 18 other chromosomes. The contribution of sire's chromosomes is carried by the sperm to the ripe egg. The egg carries the chromosome received from the dam. The fusion of the sperm with the egg makes the 37 chromosomes of the male and the 38 chromosomes of the female. Reproductions of certain of Wodsedalek's drawings showing the different chromosome groups are shown in figure 30.

In cattle the female sex alone gives expression to the milk producing function. We must depend on this sex to give by their milk yield a reasonable expression of their inheritance for milk yield. It will be noted from the above discussion that the sire and dam contribute an equal number of chromosomes (19) to their daughters. On a multiple factor inheritance for milk yield expectation would be an equal inheritance for milk yield and butter-fat percentage from the sire and dam. Such an equal inheritance is actually found from the records just studied. These conclusions equally apply to Masui's chromosome studies.

A daughter receives 19 chromosomes from her dam. Full sisters receive from the sire one sex chromosome and 9 other chromosomes (on the average) which are identical, a total of 10 chromosomes from the sire. From the dam these full sisters may receive either one of her two sex chromosomes or any recombination of the two. On the factor basis they would receive one-half the sex chromosome in common, and nine of the other chromosomes. Full sisters would have 19½ chromosomes in common, or we would expect them to resemble each other in their milk yields or butter-fat percentages slightly more than daughters and dams would do in theirs, since the daughter and dam have only 19 common chromosomes. Such a condition of affairs actually exists in these data. The correlation coefficients for the milk yields and butter-fat percentages of full sisters are higher than the correlation coefficients of daughter and dam.

Half sisters, with a common sire, would have a common sex chromosome and on the average 9 common chromosomes from the sire or a total of 10 common chromosomes. In view of this fact the milk yields and butter-fat percentages of these half sisters would be expected to resemble each other only about half as much as do the milk yields and butter-fat percentages of daughter and dam or full sisters. The facts show that in general such a conclusion is justified by the data.

Half sisters, where the dam is common, could receive either or any portions of their dam's two sex chromosomes, the average being half the sex chromosome in common. Of the other 36 possible chromosomes which might come from the dam, the half sisters would have 9 in common or a total of $9\frac{1}{2}$ chromosomes in common. The half sisters with a common dam would be expected to resemble each other in their milk yields and butter-fat percentages slightly less than the half sisters with a common sire. The correlation coefficients for milk yield and butter-fat percentage are slightly more for milk yield and distinctly less for butter-fat percentage. They would further be expected to resemble each other only to about half the degree that the full sisters or daughter and dam resemble each other. The correlations are less for the half sisters but not so much less as might be expected. This is partly due to the fact that environment plays a small part in raising the correlations for half sisters.

The paternal grandsire contributes no sex chromosome and 18 of the other chromosomes to his son. Two different sons would consequently have 9 chromosomes in common. Each of these sons contributes 19 chromosomes to his daughter of which no sex chromosome and 9 other chromosomes would come from the paternal grandsire. Cousins carrying only $4\frac{1}{2}$ common chromosomes each, would be expected to resemble each other in their milk yields or butter-fat percentages to only a small degree. The correlation coefficients show that cousins do resemble each other in their milk yield to only a small degree.

The data on the grandparents is not quite all that it could be for inheritance studies because of the fact that the paternal grandparents are frequently represented by one son and his daughters instead of several sons and their daughters. However, barring this fact, which perhaps should not be over emphasized, we note that the granddaughters with a common paternal grandsire have a correlation coefficient of 0.245 for milk yield and 0.339 for butter-fat percentage. The correlations for sire and daughter are 0.52 for milk and 0.53 for butter-fat percentage. There are 9 chromosomes in common for the granddaughter and her paternal grandsire whereas there are 19 between the sire and daughter. The results confirm the conclusion expected from the distribution of the chromosomes that the paternal grandsire should resemble his granddaughter only half as much as the sire should resemble his daughter.

The paternal granddam contributes the sex chromosome and 18 other chromosomes to her son. This son in turn contributes his sex chromosome and 18 others to his daughter. In other words, cousins with a common paternal granddam would have five chromosomes in common. They should consequently resemble each other slightly more than did the cousins with the common paternal grandsire. They do, the correlations being 0.171 and 0.214 for milk yield and butter-fat percentage against 0.005 and 0.119 for the same variables where the cousins have a common paternal grandsire.

The direct relation of the milk yields of the paternal granddams and granddaughters measured by the correlation coefficient is equal to 0.258 for milk yield and 0.091 for butter-fat percentage. The interrelation of the milk yields is in line with the expectation based on the number of common chromosomes between paternal granddam and granddaughter (10). The correlation coefficient for the butter-fat percentage is low when compared with that expected from the chromosome distribution or from the correlation found for the other grandparents.

The maternal grandsires contribute a sex chromosome and 18 other chromosomes to their daughters. These daughters in turn contribute one sex chromosome and 18 autosomes to their daughters. Cousins would on the average receive one half of the sex chromosome and $4\frac{1}{2}$ of the other chromosomes from the maternal grandsire. The correlation coefficient for the milk yields and butter-fat percentages of these cousins are 0.206 and 0.216 respectively, agreeing closely with those for the cousins with common paternal granddam. The direct correlations of granddaughter and maternal grandsire are equal to 0.35 and 0.39 respectively. These correlations are larger than would be expected, although it is probable that an increased size is to be expected since the grandsires transmit through relatively few dams to their daughters.

The contribution of chromosomes to cousins by the maternal granddam is $4\frac{3}{4}$ chromosomes. The correlation coefficients for these cousins, 0.234 and 0.244, is approximately the same as that for the cousins with common maternal grandsires, 0.206 and 0.216. The direct relation of maternal granddam and granddaughter is measured by correlations of 0.31 and 0.19 for milk yield and butter-fat percentage. That is, they are about what would be expected in view of the chromosome distribution.

This brief review of the mechanism of inheritance and the coordinating of it with the results obtained here for the inheritance of milk yield and butter-fat percentage shows that in general the results postulated by this mechanism and those actually obtained are in agreement.

The results may now be reviewed from the standpoint of how much of the permanence of milk yield and butter-fat percentage[2] can be accounted for by the correlations obtained for the cow and her ancestors. The average correlation between the test and retest 365-day milk yields is 0.667. The average correlation for the test and retest 365-day butter-fat percentage is 0.715. These correlations measure the degree of permanence of milk yield and butter-fat percentage in the cow. They measure the influence of those factors in environment which tend to cause permanent differences in the quantity and quality of the milk combined with the effect of heredity. The correlation for the milk yields of mother and daughter is 0.497. The correlation coefficient for the sire and daughters' milk yields (by the method of standard deviation of the daughters' milk yields) is 0.52. It is not possible to obtain the cross-correlation between sire's and dam's performance. The cross-correlation between the milk yield of paternal granddam and the dam is 0.14 and between the two granddams is 0.30. In other words, the cross-correlation decreases as the performance of the two parents is approached. It is, therefore, reasonable to suppose the correlation between sire and dam to be not more than 0.14 and probably less than this figure. The partial correlations of sire and daughter for a constant milk yield of dam is 0.50. The total correlation between the two parents and the daughters' performance in milk yield is 0.673. If now we consider the grandparents we have the following case. The average correlation between parent's and the daughter's milk yield is 0.51. The average correlation between either of the four grandparents' and their daughters' milk yields is 0.29. The correlation due to assortive mating between dam and paternal granddam is 0.14. We may also consider the assortive mating between sire and maternal grandsire and maternal granddam, between dam and paternal grandsire, between paternal grandsire and paternal granddam, and between maternal grandsire and maternal granddam to be measured by the correlation of 0.14 (it is extremely probable that it is much less

[2] See Chapter VI.

than this figure). The assortive mating of paternal granddam and maternal granddam is measured by a correlation of 0.30. It is reasonable to consider the assortive mating of paternal grandsire and maternal grandsire equal to the same correlation 0.30, although in actual fact it is quite likely to be much less. With this evidence in hand we are in a position to obtain the total effect for the combined heredity of parents and grandparents on the milk yields of the daughters. This combined influence is measured by a correlation of 0.676. The correlation coefficient between the milk yields of one lactation with those of another for the same cow is 0.667. This correlation coefficient measures those factors in the cow and her surroundings which make for permanence in performance in milk yield. These factors are heredity and constant environmental effects. The above analysis shows that the correlation for the heredity (parents and grandparents) is 0.676, in other words, practically equal to the 0.667 of test and retest records. It is, therefore, justifiable to say that the permanence in the performance of a cow is due largely to heredity.[3]

The problem may be viewed from the slightly different angle, of the relative influence of heredity and temporary environmental conditions. The correlation for test and retest records is 0.667. On the basis of 100 for the variation of milk yield, the variation remaining in milk yield after subtracting the permanent variation is $100\sqrt{1 - 0.667^2}$ or 100×0.7451 or 74.5. In other words approximately 25 per cent of the standard deviation is due to heredity. The other residual 75 per cent should not, however, be considered as due, entirely, to variable environment. It is due partially to that, but besides the changing environment, another large factor in making this variation is the random sampling of the data.

The data for the butter-fat percentage may be treated in the same way. The average parental correlation for sire or dam is 0.47. The average grandparental correlation is 0.25. The correlation for paternal granddam and dam is 0.001 and for paternal granddam and maternal granddam is 0.04, or, in other words, it is a justifiable

[3] It should be noted in all cases we are dealing with only a part of heredity, the part which would be controlled by making the milk yield of one parent, etc., constant. As we are undoubtedly dealing with a case of multiple factors it is clear that such a parent may be only partly homozygous, thus making for only a partial control of heredity.

assumption that there is no assortive mating. With these data the correlation of the heredity (parents and grandparents) with the butter-fat percentage of the daughters is found to be 0.669. For the parents alone the correlation is 0.665 with the butter-fat percentage of the daughters. The correlation between the butter-fat percentage of one lactation with that of another is 0.715. This correlation, like that for milk yield, can be interpreted as due to constant environmental conditions or to heredity. The correlation of 0.669 for heredity shows that practically all of the influence is due to heredity.

The inheritance of milk yield and butter-fat percentage is of like strength, the correlations being 0.676 and 0.669 respectively. The inheritance appears to be largely independent so far as these two variables are concerned in the Holstein-Friesian breed. Furthermore the inheritance is of practically the same strength as that found for other similar variables in other species.

The main facts may be gathered together in the following statements:

1. Milk yield and butter-fat percentage are equally inherited in Holstein-Friesian Advanced Registry cattle.

2. The total influence of the inheritance derived from the parents and grandparents on the milk yield or butter-fat percentage of the daughters is measured by a correlation of about 0.70. Inheritance consequently plays a very large part in the milk yield or butter-fat percentage of any cow.

3. Inheritance as contrasted with any knowledge of a cow's conformation is a much better indicator of the cow's milk yield. Conformation has no relation to the butter-fat percentage of the cow.

4. These studies indicate that the sire and dam are equally responsible for the milk yield and butter-fat percentage of the daughter.

5. The parent's relation to the milk yield or butter-fat percentage of the daughter is measured by a correlation coefficient of about 0.5.

6. The grandparent's relation to the milk yield or butter-fat percentage of the daughter is measured by a correlation coefficient of about 0.25.

7. From 5 and 6 we may conclude that a knowledge of the productivity of a grandparent is not more than one-half as effective in

determining the probable milk yield or butter-fat percentage of the granddaughter as knowledge of the productivity of a parent.

8. The milk yields or butter-fat percentages of full sisters are closely related to each other, the correlations being respectively 0.55 and 0.46.

9. In view of the fact that it is impossible to obtain a milk record on the sire these correlations, together with those of the daughter and dam offer perhaps, the most effective measures available to animal breeding in controlling the productivity of the resulting offspring. The combined correlation between the milk yields of a dam and a full sister with another full sister is, for milk yield 0.61, and for butter-fat percentage 0.52. The correlations differ slightly due, probably, to lack of numbers. Were an infinite population available it is quite likely the correlations would be the same and about 0.6 in value. From the data at hand the equations to handle such a problem may be determined. The correlation coefficients of daughter and dam with the performance of the full sister constant, are 0.309 for milk yield and 0.274 for butter-fat percentage.[4] The correlation coefficients for daughter and full sister, with dam's performance constant, are, 0.399 for milk yield and 0.354 for butter-fat percentage. From these data and those for the means and standard deviations of milk yields and butter-fat percentages, it is possible to make the proper equations to predict a cow's performance from the knowledge of her dam's and full sister's performance. The equation for the milk yield is:

$$\text{Daughter's milk yield} = 5827 + 0.400 \text{ full sister's milk yield} + 0.298 \text{ dam's milk yield} \tag{62}$$

The equation for the butter-fat percentage is:

$$\text{Daughter's butter-fat percentage} = 1.304 + 0.354 \text{ full sister's butter-fat percentage} + 0.266 \text{ dam's butter-fat percentage} \tag{63}$$

Table 161 presents a few of the milk yields of a full sister and her dam and the probable resulting milk yields of the daughter.

Table 162 gives similar data for the probable butter-fat percentages of a daughter when the butter-fat percentages of her dam and her full sister are known.

[4] The means and standard deviations used in calculating these coefficients are those of the whole breed, age-corrected. See Chapter IV.

Some readers will very likely think it a work of supererogation to explain the use of these tables. However, on the chance of stretching their patience, one example may not be entirely out of place. If a cow has a butter-fat percentage of 3.4 and her daughter also has her milk test 3.4, the probable butter-fat percentage of the cow's daughter, a full sister to the first, is found in the fourth column

TABLE 161

Probable milk yields of untested cows determined from the milk yields of their dams and full sisters

MILK YIELD OF FULL SISTER	MILK YIELD OF DAM					
	10,000	14,000	18,000	22,000	26,000	30,000
	PROBABLE MILK YIELD OF UNTESTED DAUGHTER					
10,000	12,807	13,999	15,191	16,383		
14,000	14,407	15,599	16,791	17,983	19,175	
18,000	16,007	17,199	18,391	19,583	20,775	21,967
22,000	17,607	18,799	19,991	21,183	22,375	23,567
26,000		20,399	21,591	22,783	23,975	25,167
30,000			23,191	24,383	25,575	26,767

TABLE 162

Probable butter-fat percentages of untested cows determined from the butter-fat percentages of their dams and full sisters

BUTTER-FAT PERCENTAGE OF FULL SISTER	BUTTER-FAT PERCENTAGE OF DAM					
	2.6	3.0	3.4	3.8	4.2	4.6
	EXPECTED BUTTER-FAT PERCENTAGE OF DAUGHTER					
2.6	2.92	3.02	3.13	3.24		
3.0	3.06	3.16	3.27	3.38	3.48	
3.4	3.20	3.30	3.41	3.52	3.62	3.73
3.8	3.34	3.44	3.55	3.66	3.76	3.87
4.2		3.59	3.70	3.81	3.91	4.01
4.6			3.84	3.94	4.05	4.16

under 3.4 of the dam and fourth row from the bottom beside 3.4 for the full sister's butter-fat percentage test, or the probable test for the untested daughter is 3.41. The values given in tables 161 and 162 have probable errors. These probable errors are equal to 2225 for milk yield and 0.181 for butter-fat percentage.

10. There is another case which is perhaps of as much practical importance as any other. A given dam has a record. It is de-

sired to purchase a bull. The bull has one recorded daughter. What is the probable production of a daughter from the record dam and by the bull in question? The second daughter of the bull would be a half sister to the sire's first daughter on the sire's side. The production of the dam and the sire's first daughter would be correlated only because of assortive mating. What the correlation would be is difficult to determine. However, if we judge by some of the other correlations determined for such assortive mating in cattle a correlation of 0.1 for either milk yield or butter-fat percentage seems reasonable. The correlation between half sisters in their milk yields and butter-fat percentages is 0.362 and 0.374 respectively. The correlation between daughter and dam in their

TABLE 163

Probable milk yields of untested cows determined from the milk yields of their dams and half sisters on the sire's side

MILK YIELD OF HALF SISTER (SIRE'S SIDE)	MILK YIELD OF DAM					
	10,000	14,000	18,000	22,000	26,000	30,000
	EXPECTED MILK YIELD OF DAUGHTER					
10,000	12,044	13,908	15,772	17,636	19,500	21,364
14,000	13,304	15,168	17,032	18,896	20,760	22,624
18,000	14,564	16,428	18,292	20,156	22,020	23,884
22,000	15,824	17,688	19,552	21,416	23,280	25,144
26,000	17,084	18,948	20,812	22,676	24,540	26,404
30,000	18,344	20,208	22,072	23,936	25,800	27,664

milk yields is 0.497 and in their butter-fat percentage is 0.413. With these data the equations for the cow's milk yield predicted from that of the dam and half sister (common sire) is found to be:

$$\text{Daughter's milk yield} = 4234 + 0.315 \text{ half sister's milk yield} + 0.466 \text{ dam's milk yield} \qquad (64)$$

The equation for the daughter's butter-fat percentage is:

$$\text{Daughter's butter-fat percentage} = 0.975 + 0.336 \text{ half sister's butter-fat percentage} + 0.380 \text{ dam's butter-fat percentage} \qquad (65)$$

Table 163 has been formed from the equation for milk yield to show some of the daughter's probable milk yields for the actual milk yields of half sister (common sire) and dam.

The uses to which this table may be put are illustrated by the following case. A cow has a dam with a milk yield of 16,000 pounds of milk and a half sister (on the sire's side) with a milk yield of 20,000 pounds, what is the probable milk yield of the cow. The expected milk yield is estimated as follows. The expected milk yields of cows with dam's of 14,000 and 18,000 pounds and for half sister's of 18,000 and 22,000 pounds are shown below.

HALF SISTER'S MILK YIELD	DAM'S MILK YIELD	
	14,000	18,000
	COW'S EXPECTED MILK YIELD	
18,000	16,428	18,292
22,000	17,688	19,552
Difference..............	1,260	1,260

As the half sister's milk yield was 20,000 pounds we want $\frac{18{,}000 - 20{,}000 = 2000}{18{,}000 - 22{,}000 = 4000}$ or 0.5 of the difference. That is 1260 × 0.5 = 630 pounds of milk is the amount to be added for 2000-pound difference between the 18,000-pound half sister and the 20,000-pound half sister to obtain the probable milk yield of the cow. Thus, for the 18,000-pound half sister the cow's probable milk would be 16,428 + 630 = 17,058 pounds and for the 22,000-pound half sister 18,292 + 630 = 18,922 pounds. The milk yield of the cow for her 16,000-pound dam would then be obtained by repeating the same procedure; 17,058 − 18,922 = 1864 × 0.5 = 932 or the expected milk yield of a cow with a dam of 16,000 pounds and half sister of 20,000 pounds is 17,058 + 932 = 17,990 pounds of milk. The probable error for the daughter's expected milk yields of table 163 is equal to 2264 pounds of milk for each cow. The record of the case illustrated is consequently equal to 17,990 ± 2264.

Table 164 presents the similar data for the probable butter-fat percentages of the daughters determined from the butter-fat percentages of their half sisters (sire's sire) and of their dams.

The probable butter-fat percentage of a cow with a dam whose butter-fat percentage is 3 and a half sister on the sire's side with a butter-fat test of 3.4 will be 3.26. The probable error of this expected butter-fat test is 0.18. In other words it is an even chance

that the cow's actual record will be between 3.26 ± 0.18 (3.08 to 3.44) or outside this range.

Practically speaking these two methods appear to offer the greatest attainable accuracy for the determination of milk yield or butter-fat percentage when the cow's dam is yet to conceive for the pregnancy which is to produce the daughter in question. The other evidence on the inheritance of milk yield or butter-fat percentage may now be reviewed in the light of these results.

TABLE 164

Probable butter-fat percentages of untested cows determined from the butter-fat percentages of their dams and half sisters on the sire's side

BUTTER-FAT PERCENTAGE OF HALF SISTERS (SIRE'S SIDE)	BUTTER-FAT PERCENTAGE OF DAM					
	2.6	3.0	3.4	3.8	4.2	4.6
	EXPECTED BUTTER-FAT PERCENTAGE OF THE UNTESTED COW					
2.6	2.84	2.99	3.14	3.29	3.44	3.60
3.0	2.97	3.12	3.27	3.42	3.57	3.73
3.4	3.11	3.26	3.41	3.56	3.71	3.87
3.8	3.23	3.39	3.54	3.69	3.84	4.00
4.2	3.37	3.53	3.68	3.83	3.98	4.14
4.6	3.51	3.66	3.81	3.96	4.12	4.27

OTHER DATA ON THE INHERITANCE OF MILK YIELD AND BUTTER-FAT PERCENTAGE

Of those studies which are available that made by Wilson[5] is one of the earliest. This paper is devoted to showing that with such a breed as the Red Danish there may be wide difference between the milk yield of the daughter and dam—that is these differences do not always blend gradually, in fact as a rule they progress by wide steps. Further an attempt is made to show that the sires in the Red Danish breed appear to be differentiated into those whose daughters are all low producers; those whose daughters may be low producers, medium producers, or high producers; and thirdly those whose daughters are all high producers. The data to support these conclusions are admittedly fragmentary and open to several criticisms. It is, however, held to show that milk yield is trans-

[5] Wilson, James. 1911. The inheritance of milk yield in cattle. Sci. Proc. Roy. Dublin Soc., vol. 13, pp. 89–112.

mitted in mendelian fashion with the heterozygote intermediate between the pure forms. The manner of grouping the data and its correction for age, etc., would seem to more or less force this conclusion.

The important results of Rietz on the inheritance of butter-fat have been frequently referred to in the progress of this paper and need not be further reviewed. They accord with the view that butter-fat is an inherited character.

Two practical experiments carried on by breeders in England are of particular interest. The object of the experiments was to cross the Jersey with the Aberdeen-Angus and to fix in the resulting offspring the hardiness of the Angus with the milk yield of the Jersey. The original crosses were made Aberdeen-Angus bull to Jersey cows. Although records were kept, no figures are cited in the paper[6] on this herd. The qualitative statement is made, however, that the F_1 cows show a high yield of milk, ranking almost as high as their Jersey dams.

In another section of England a similar cross was made by another breeder with the same objects in view. This breeder, Mr. Stevens[7] makes a similar statement in regard to the milk yield of the F_1 cows.

Kildee and McCandlish,[8] record a similar experiment which furnishes some data on the transmission of milk yield and butter-fat percentage.

They cross scrub cattle, whose milk yield averaged between 3300 and 3900 pounds, to Holstein-Friesian sires. The resulting F_1 offspring averaged 6444 pounds of milk. Crosses to Guernsey and to Jersey bulls increased the F_1 average production over that of the scrubs, by 700 and 800 pounds. The length of lactation was not strictly comparable between animals. Several bulls of each breed were used. The results point to a partial dominance for high milk yield expressed in the F_1 offspring of the Holstein-Friesian

[6] Parlour, W. 1913. Jersey-Angus cattle. Live Stock Jour. (London), vol. 77, no. 2025, p. 85.

Kuhlman, A. H. 1915. Jersey-Angus cattle. Jour. Heredity, vol. 5, no. 2, pp. 68–72.

[7] Stevens, H. D. E. 1913. Jersey-Angus cattle. Live Stock Jour. (London), vol. 77, no. 2025, p. 132.

[8] Kildee, H. H., and McCandlish, A. C. 1916. Influence of environment and breeding in increasing dairy production. Bul. 165. Iowa Agric. Exper. Sta., pp. 383–402.

sires. The data support the contention that high milk yield may be transmitted to the female offspring through the bull.

The butter-fat percentages of F_1 offspring are of equal interest. The average butter-fat percentage of the Holstein-Friesian cows is 3.43, that for the scrub dams was 4.95 and that for the F_1 cows was 4.13. Here again the sire has shown pronounced influence on the butter-fat percentage. In accordance with the other figures presented, the results show about an equal influence of sire and dam on the performance of the offspring.

The second generation cows from F_1 dams backcrossed to the pure-bred sires of the different breeds further support the view that the bull has a pronounced influence on the productivity of the offspring. The Holstein-Friesian group is more than 3000 pounds of milk in excess of the F_1 generation cows and more than 6000 pounds in excess of the original stock. In butter-fat percentage this group was further lowered over that of the scrub dams. Both parents in each generation have obviously left their mark on the offspring.

The Guernsey F_2 offspring had a milk yield much larger than the F_1; the F_1, it will be recalled was slightly more than the scrub parents in milk yield. The first generation butter-fat percentage was also still further improved. The Jersey group maintained the F_1 milk yield in the F_2 but were slightly lower in butter-fat percentage.

While the experiment was not designed primarily to obtain data on the inheritance side of milk yield and butter-fat percentage, the information which may be derived from it supports the conclusions derived from the 365-day records of the Holstein-Friesian breed.

Another experiment of the same nature as that reported by Kildee and McCandlish from which it is possible to gather some further facts on the inheritance of milk yield and butter-fat percentage is that reported by Olsen and Biggar.[9] This experiment consisted in crossing Holstein-Friesian, Jersey, and Guernsey bulls on to three grade beef cows, chiefly of Hereford and Shorthorn blood. The average production of the three parental cows was 4006 pounds of milk, testing 4.07 per cent of butter-fat. When

[9] Olsen, Thomas M., and Biggar, George C. 1922. Influence of pure-bred dairy sires. South Dakota Agric. Exper. Sta., Bul. 198, pp. 435–467.

bred to purebred Holstein-Friesian sires, these cows produced 5 F_1 cows with an average milk yield of 7182 pounds. The average butter-fat test of the F_1 cows was 3.68. These cows bred back to purebred Holstein-Friesian bulls had 4 daughters with average milk yields of 8315 pounds. The butter-test was 3.65 per cent. It is a fair assumption from our general knowledge of the Holstein-Friesian breed that the purebred Holstein-Friesian sires used in this experiment transmitted to their offspring an inheritance for milk yield far above 4000 pounds. A fair estimate of the butter-fat percentage transmitted is 3.4 per cent. In each generation, the milk yields demonstrated the influence of both parents on the performance of their daughters. The same was true of the butter-fat percentage. Each generation that the lower testing Holstein-Friesian bull was used, the result was a closer approach to the average Holstein-Friesian butter-fat test.

The results for the Jerseys show on a smaller scale (the difference between the parental milk yields being less) the same influence of both parents on the milk yields of the daughters. The butter-fat percentage was increased in each generation, thus bringing out the same facts as those shown by the Holstein-Friesian matings, that is, the influence of both parents on the butter-fat percentage of their offspring, except that the Jerseys were toward the increase of the butter-fat percentage in the offspring.

Three other lines of evidence are available on this problem. The first two are the by-products of practical breeding having as their object the combination in one animal of (1) the high milk yield of the Holstein-Friesian with the high butter-fat percentage of the Guernsey or (2) the high milk yield of the Red Danish with the high butter-fat percentage of the Jersey. The third experiment was from the first organized to furnish controlled, critical data on the inheritance of milk yield and butter-fat percentage without regard to the possible practical utility of any of the resulting animals.

The data for the first crosses, Holstein-Friesian with Guernsey reciprocally mated, as analyzed by Castle[10] show that the milk yield of 31 F_1 heifers was intermediate between that of their parents but closer to the Holstein-Friesian parent. The butter-fat percent-

[10] Castle, W. E. 1919. Inheritance of quantity and quality of milk production in dairy cattle. Proc. Nat. Acad. of Sci., vol. 5, pp. 428–434.

age of 47 of these F_1 heifers was shown by Gaines[11] to be intermediate between the two breeds. The butter-fat percentage of the straight F_2 was about that of the F_1 for 19 animals.

The second set of results comes from the analysis of the records made by the crosses of Red Danish and Jersey cattle. The crosses were made by Count F. Ahlefeldt-Laurvig and the results were analyzed by Ellinger.[12] In one case the average first 10 weeks milk yield of cows twenty-eight to thirty-five months old was 1975 pounds of milk for the Red Danish, 1568 pounds of milk for the Jersey, 1835 pounds of milk for the F_1 crossbred, 1941 pounds of milk for the second generation Red Danish backcrosses, 1635 pounds of milk for the second generation Jersey backcrosses, and 1543 pounds of milk for the $\frac{7}{8}$ Jerseys. The inheritance is clearly multiple factor inheritance. The high milk yield shows either that there is a tendency for milk yield to be increased slightly by heterosis or that the factors for high milk yield are more nearly dominant than are the factors for low milk yield.

The results for the butter-fat percentages of these cattle are as follows: Red Danish 3.56, Jersey 4.94, F_1 crossbred 4.21, $\frac{3}{4}$ Danish 4.04, $\frac{3}{4}$ Jersey 4.53, and $\frac{7}{8}$ Jersey 4.60. Clear evidence for inheritance of butter-fat percentage is found for these crosses. The inheritance is typical of multiple factor inheritance with low milk yield showing a tendency to dominance, or heterosis acting to lower the butter-fat percentage somewhat.

The third experiment, commenced by Dr. Pearl and since 1917 carried on by the writer, consists of controlled matings of three major groups of cattle. The first group, the Holstein-Friesian, represents high milk yield and low butter-fat percentage; the second group, the Jersey and Guernsey, medium milk yield and high butter-fat percentage; the third group, the Aberdeen-Angus, low milk yield and a butter-fat percentage nearly as high as that of the Jersey group. Besides these, the work has also included the Ayrshire animals with medium milk yields and a butter-fat percentage intermediate between the Holstein-Friesian and Aberdeen-Angus. A

[11] Gaines, W. L. 1923. The inheritance of fat content of milk in dairy cattle. Proceedings Amer. Soc. Animal Production, pp. 29–32.

[12] Ellinger, Tage U. 1923. The variation and inheritance of milk characters. Proceedings Nat. Acad. Sci., vol. 9, no. 4, pp. 111–116.

review of certain of these results has been published, elsewhere.[13] These results and those which have been obtained since may be summarized as follows. When the high-milking group is crossed to the medium-milking group the milk yields of the resulting crossbreds are intermediate between the two groups, approaching most closely those of the high group. When the high-milking group is crossed with the low-milking group, the resulting crossbreds have milk yields intermediate between the two groups, resembling closely that of the medium group. Two noteworthy points are made evident by the contrasting of these two groups of matings. The results indicate that the inheritance of milk yield is blending (multiple factor) inheritance and second that it is possible to make an animal, extremely heterozygous for its inheritance of milk yield, have a milk yield closely similar to that of a relatively pure strain. The exact parallel of these results is continually found in the writer's study of registered stock. The third group of mating,—the medium producers crossed with the low producers, have offspring with milk yields intermediate between the two parental groups. Here a carefully controlled experiment is giving a demonstration of the influence of both sire and dam on the milk production of the offspring.

Three groups for the butter-fat percentage are present in the experiments. The widest difference exists between the butter-fat percentages of the Holstein-Friesian and Channel Island breeds. Smaller differences are found for the Ayrshire-Jersey crosses. The results from these experiments show that the crossbreds from the high butter-fat percentage group mated with the low butter-fat percentage groups have a butter-fat percentage intermediate between the two groups, but approaching nearer the low butter-fat percentage group. The crosses of Ayrshire and Jersey give cattle whose butter-fat percentage is also intermediate between the two groups. These results point to the conclusion that butter-fat percentage is controlled in inheritance by multiple factors and that the sire and dam each play a major rôle in the butter-fat percentage of their offspring.

[13] Gowen, John W. 1920. Inheritance in crosses of dairy and beef breeds of cattle. II. On the transmission of milk yield to the first generation. Jour. Heredity, vol. xi, no. 7, pp. 300–314.

Gowen, John W. 1920. Inheritance in crosses of dairy and beef breeds of cattle. III. Transmission of butter-fat percentage to the first generation. Jour. Heredity, vol. xi, no. 8, pp. 365–376.

A good many other sources of data could be drawn upon, a bit here and a bit there, to further extend the evidence on this problem. The data are rather badly scattered, however, and much of it does not appear to have critical value. Again the conclusions to be drawn from them contain nothing essentially new or different from those already available in the data here presented. The exposition of this important problem in modern agriculture or for that matter in national health, might well rest here. However, there is one word of caution which may well be emphasized before leaving this data.

The data which are analyzed in this book to show the mode of inheritance of milk yield and butter-fat percentage are Advanced Registry data. The general conclusions derived from them are believed to have a scientific breadth sufficient to make them applicable to the breeding of dairy cattle where the aim is the maintenance or increase or the milk yield or butter-fat percentage. The same cannot be said of the equations derived from "the data for the prediction of the milk yield of the untested cow." Here the data from which the equations are derived play a very vital part. The equations are strictly applicable to only those Holstein-Friesian cows which are maintained under conditions identical with those of the Advanced Registry. They are applicable to the Holstein-Friesian Advanced Registry data themselves. The general conclusion that a knowledge of the milk yield of the dam and that of a full sister is the best practical indication of the production of an unknown cow is believed to be a fact applicable to all cattle. This knowledge is a specific fact in the Holstein-Friesian Advanced Registry data enabling us to predict the milk yield of that cow from the milk yield of her dam and full sister and to determine the limits of milk yield within which the cow's milk will undoubtedly be found to lie when the actual Advanced Registry test is made.

The writer has analyzed the milk production and butter-fat percentage of the cows in a number of herds from the standpoint of inheritance. This sort of analysis brings up a general inheritance fact which should also be taken into consideration. In an individual herd one faces a condition of breeding such that occasionally a concentration of factors for a given grade of milk yield has taken place in much the same way that such a concentration would take place by continued inbreeding. If the system of breeding is con-

tinued it results in a relatively stabilized milk yield in the offspring. In other words, were the correlation method applied to such a breed it would be found that the correlations for heredity were low while, relatively speaking, those for environment would be high. If this standardized herd's milk yield is high well and good; let the system continue until such time as it seems desirable to try to increase it. If on the other hand it is relatively low, the system of breeding has concentrated relatively low factors and a change is desirable. This change, if the production is low enough, can be made almost without fear of the results. However, as the herd's level of production is increased the difficulties increase not in direct ratio but in the writer's judgment by the square and possibly the cube. The data presented indicate that the dam's production and the production of full or half sisters are the best practical measures of the probable production of a cow. Such being the case the average production of a bull's daughter is particularly important. Unfortunately, the only generally available source of such information is the Advanced Registries where the daughters present may be selected individuals. It would certainly be better if the productivity of all a bull's daughters were known. However, until such time as such a condition may be brought about the analysis of the Advanced Registry bulls furnishes the best available data. This analysis has been presented by the writer and his associates in bulletin form and are available elsewhere.

Practical breeding then, requires a knowledge of the milk yield of every cow in the herd and of the average production of the cows in it. In the purchase of a bull to head such a herd it is equally important that the productivity of his daughters be equal to at least the average production of the herd (if the production of all the daughters is known) or better yet to be significantly greater than the average production of the herd, especially if there is any ground for the belief that the bull's daughters may have been selected.

INDEX

Sans Tache

Sans Tache

IN THE "elder days of art" each artist or craftsman enjoyed the privilege of independent creation. He carried through a process of manufacture from beginning to end. The scribe of the days before the printing press was such a craftsman. So was the printer in the days before the machine process. He stood or fell, as a craftsman, by the merit or demerit of his finished product.

Modern machine production has added much to the worker's productivity and to his material welfare; but it has deprived him of the old creative distinctiveness. His work is merged in the work of the team, and lost sight of as something representing him and his personality.

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